Stress distribution due to point load :->

Boussinesq's Theory :->

Assumptions :->

i) Soil is homogenous, isotropic, semi-infite, and elastic.

ii) Hooke's law is valid.

iii) Self wt. of the soil is neglected.

iv) Soil is unstressed initially.

v) Change in volume of soil due to application of load is neglected.

vi) The top surface of soil is free from shear stress and subjected to only point load.

vii) Distribution of the stress about the vertical axis is symmetrical.

Acc to Boussinesq's theory :-

$$\sigma_z = \underbrace{K_B}_{\text{Boussinesq's influence factor}} \cdot \frac{Q}{z^2}$$

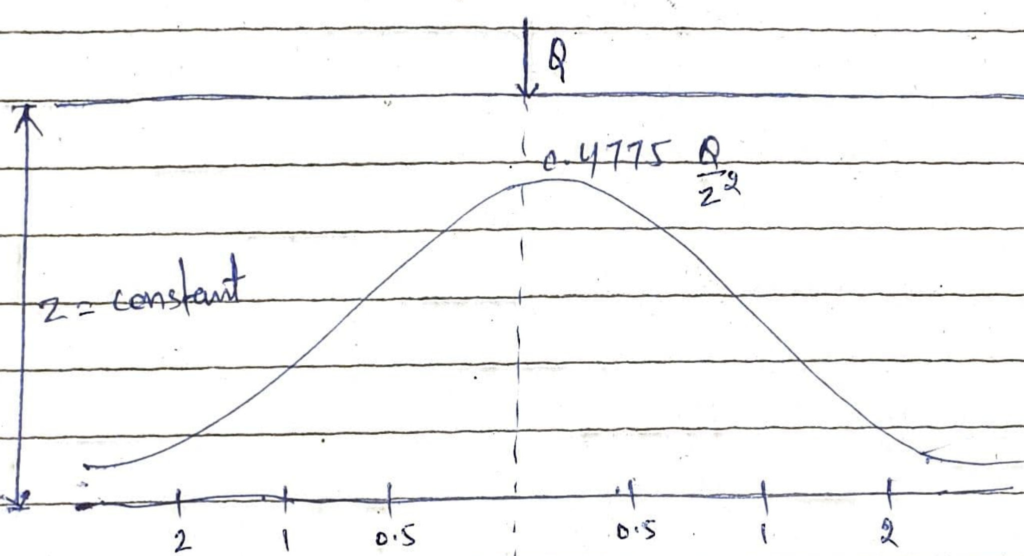
$$K_B = \frac{3}{2\pi}\left[\frac{1}{1 + \frac{r^2}{z^2}}\right]^{5/2}$$

$$\Rightarrow \boxed{\sigma_z = \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2} \cdot \frac{Q}{z^2}}$$

$$\tau_{rz} = \sigma_z \cdot \frac{r}{z}$$

⇒ Boussinesq's stress distribution theory can be analyzed for the construction of following vertical pressure distribution diagrams:→

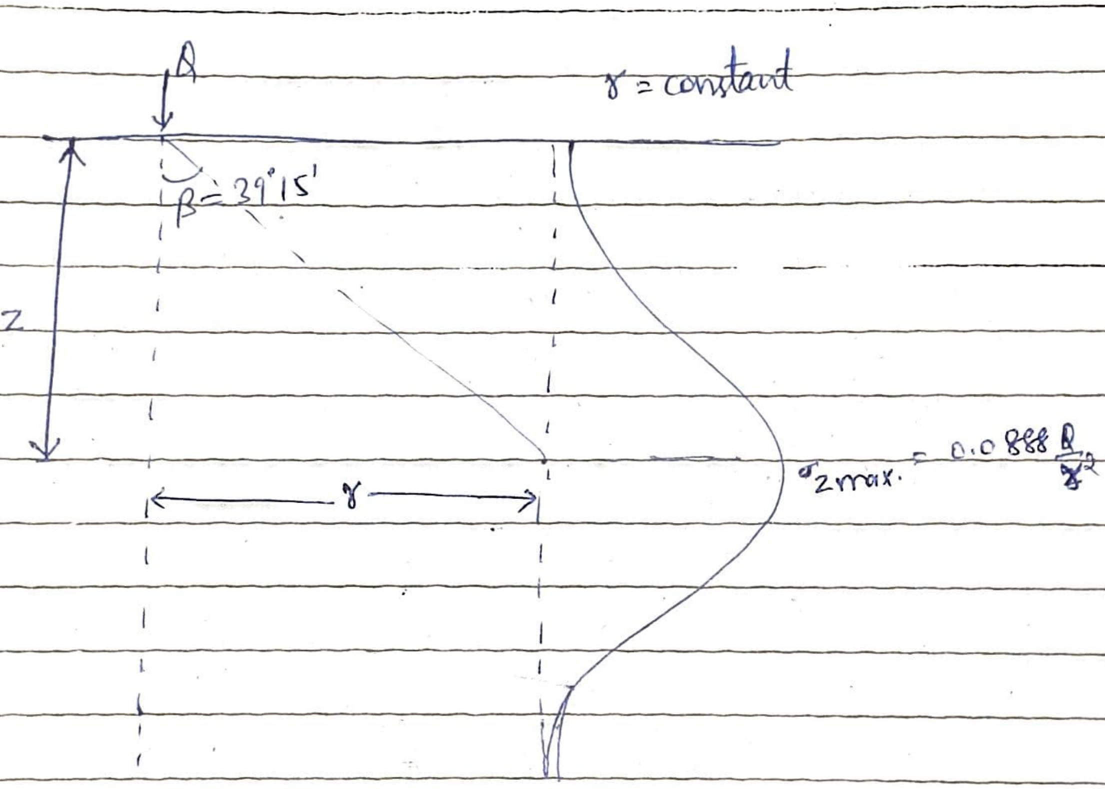
1) Vertical pressure distribution on horizontal plane at given constant depth:-



r/z	K_B	$\sigma_z = K_B \cdot \frac{Q}{z^2}$	Description
0	0.4775	$0.4775\ Q/z^2$	Max. stress
0.5	0.273	$0.273\ Q/z^2$	57% "
1	0.084	$0.084\ Q/z^2$	17.7% "
2	0.008	$0.008\ Q/z^2$	1.8% "

2) Vertical pressure distribution on vertical line at a constant radial distance :->

- Vertical pressure on vertical line at constant radial distance from point load, first increases with increase in depth, reaches its maximum value and further starts decreasing.
- The max. vertical pressure on vertical line is obtained at point of intersection of vertical plane with radial line drawn at an angle of 39°15' through the point load.



$$\sigma_z = \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2} \cdot \frac{Q}{z^2} \Rightarrow \frac{3}{2\pi}\left[\frac{z^2}{z^2+r^2}\right]^{5/2}\frac{Q}{z^2}$$

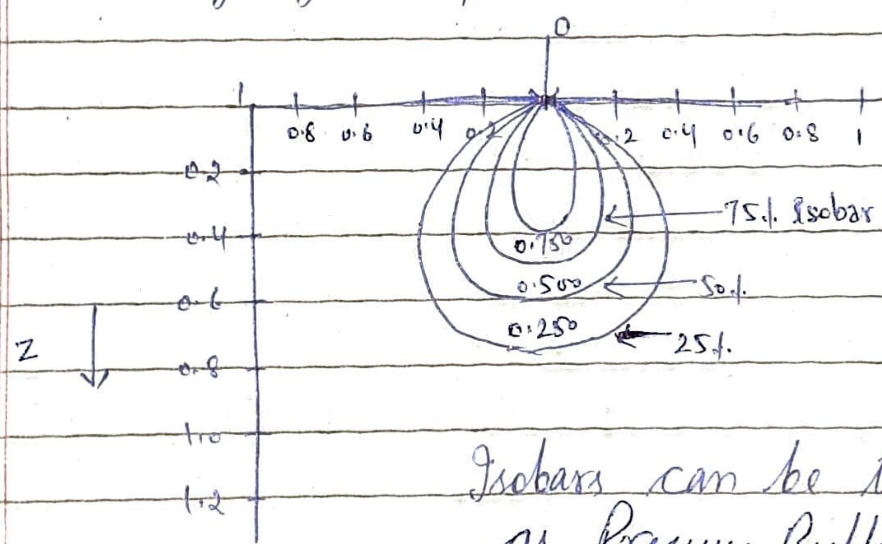
$$\boxed{\sigma_z = \frac{3Q}{2\pi}\left[\frac{z^3}{(r^2+z^2)^{5/2}}\right]}$$

For max. :-

$$\frac{\partial \sigma_z}{\partial z} = 0$$

3) Stress isobar diagram :-

- An isobar is a curve which joins points of equal vertical pressure in the soil mass.
- The stress isobar is also known as pressure bulb and is helpful in analyzing the stresses at any given point in the loaded area.



Isobars can be termed as Pressure Bulb.

$$\sigma_z = K_B \cdot \frac{Q}{z^2} = \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2} \cdot \frac{Q}{z^2}$$

for isobar, $\sigma_z = 0.75Q$

Hence,

$$0.75Q = K_B \cdot \frac{Q}{z^2}$$

$$\Rightarrow \quad \underline{K_B = 0.75z^2}$$

$$\Rightarrow \quad \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2} = 0.75z^2$$

Suppose $z = 0.4$ & solve

$$\Rightarrow \quad \boxed{r = 0.34349}$$

Westergaard's Theory :→

Assumptions :→

i) Soil is homogenous, semi-infinite & non-isotropic. It means Westergaard's theory is applicable in stratified soil.

ii) Medium is laterally reinforced with numerous closely spaced sheets of negligible thickness. It means medium is rigid horizontally and elastic vertically.

Poisson's ratio :- $\mu = \frac{\varepsilon_h}{\varepsilon_v} \to 0$

Young's modulus :-

$$\frac{E_h}{E_v} \to \infty$$

According to Westergaard :→

$$\sigma_z = K_w \frac{Q}{z^2}$$

K_w → Westergaard's Influence factor

$$K_w = \frac{1}{\pi} \left[\frac{1}{1 + \frac{2r^2}{z^2}} \right]^{3/2}$$

$$\sigma_z = \frac{1}{\pi}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}\frac{Q}{z^2}$$

For $r/z = 0$

$$\Rightarrow \sigma_z = 0.318\frac{Q}{z^2}$$

For Any $\frac{r}{z}$ (σ_z) Bousinessq $= \sigma_z$ Westergaard

$$\frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}\frac{Q}{z^2} = \frac{1}{\pi}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}\frac{Q}{z^2}$$

$$\frac{r}{z} = 1.52$$

If $\frac{r}{z} < 1.52$ $(\sigma_z)_B > (\sigma_z)_w$

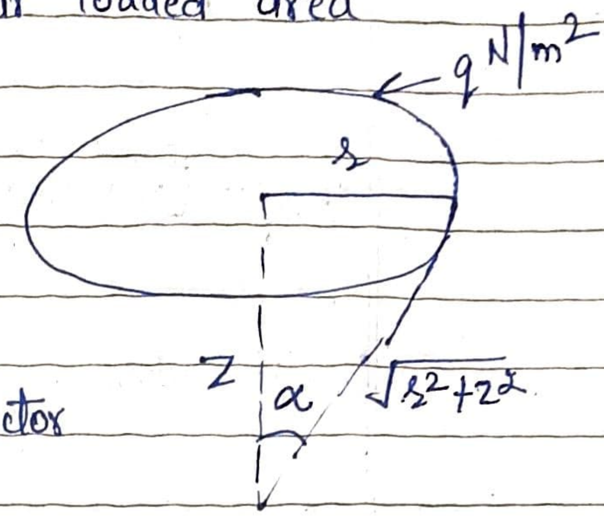
$\frac{r}{z} = 1.52$ $(\sigma_z)_B = (\sigma_z)_w$

$\frac{r}{z} > 1.52$ $(\sigma_z)_B = (\sigma_z)_W$

Note:- The soil is practically non-isotropic. Hence results obtained for Westergaard theory are more realistic and closer to actual results. But practically Bossiness's is preferred over Wester. Theory. Because it gaves more conservative Results.

2. Vertical pressure Below the circular loaded area

(Z - depth below the center)



$\sigma_z = K_B q$

K_B = Bousinesque influence factor

$K_B = 1 - \cos^3\alpha$

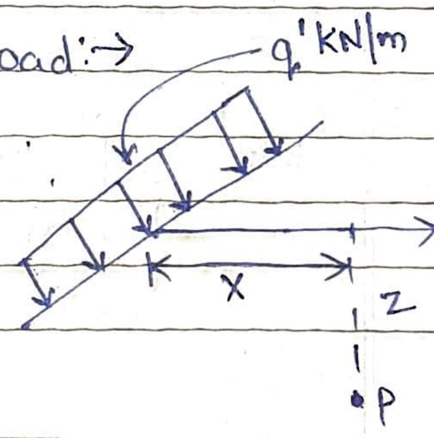
$\sigma_z = (1-\cos^3\alpha) q$, $\cos\alpha = \dfrac{z}{\sqrt{r^2+z^2}}$

$$\sigma_z = \left\{1 - \left(\frac{z}{\sqrt{r^2+z^2}}\right)^3\right\} q = \left\{1 - \frac{z^3}{(r^2+z^2)^{3/2}}\right\} q$$

$$\boxed{\sigma_z = \left\{1 - \frac{1}{\left(1+\frac{r^2}{z^2}\right)^{3/2}}\right\} q}$$

3. Vertical Pressure due to line load:→

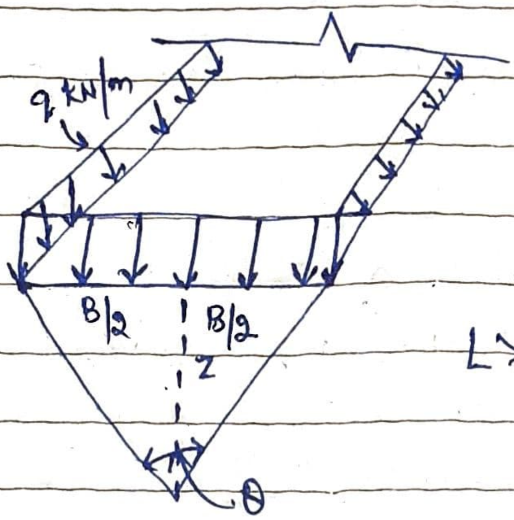
$$\sigma_z = \frac{2q'}{\pi z}\left(\frac{1}{1+\frac{x^2}{z^2}}\right)^2$$



4. Vertical pressure Due to strip loading (Strip footing)

$$(\sigma_z)_p = [\theta + \sin\theta]\frac{q}{\pi}$$

θ = in radins

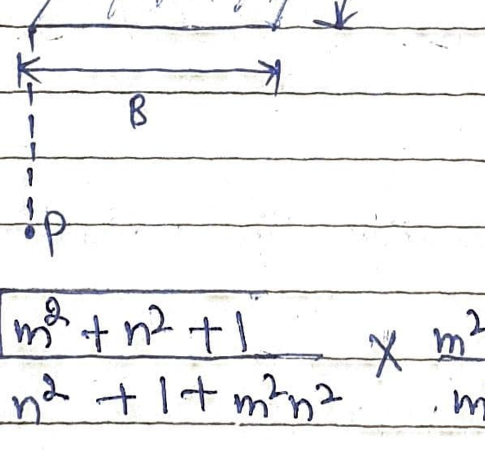


L > B

5. Vertical pressure Below the corner of Uniformly loaded Rectangular Area :→

$$(\sigma_{z=at\,p}) = K \cdot q$$

K = Influence factor



$$K = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2} \times \frac{m^2+n^2+2}{m^2+n^2+1}\right]$$

Where

$$m = \frac{B}{Z} \quad \text{and} \quad n = \frac{L}{Z}$$

m and n are Interchangeable.

$$k = \frac{1}{4\pi}\left[\frac{2mn\sqrt{A}}{A+m^2n^2} \times \frac{A+1}{A} + \tan^{-1}\left[\frac{2mn\sqrt{A}}{A-m^2n^2}\right]\right]$$

$$A = m^2 + n^2 + 1$$

Case I Point P lies inside the rectangular load area

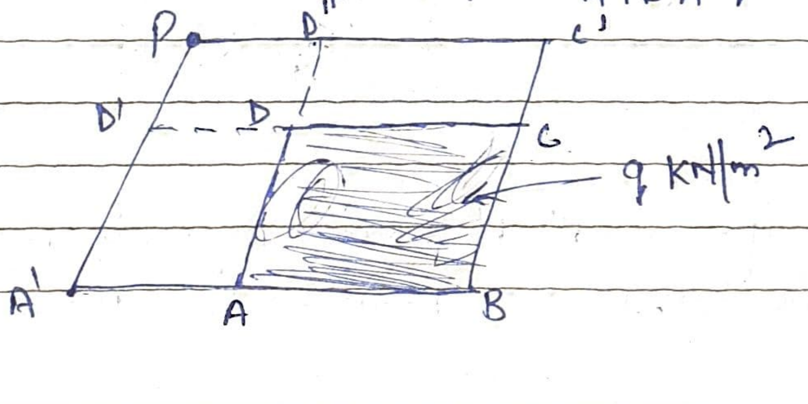
$$\sigma_z = K_1 q + K_2 q + K_3 q + K_4 q$$

$$m_1 = \frac{B_1}{Z} \quad n_1 = \frac{L_1}{Z} \quad m_2 = \frac{B_1}{Z} \quad n_2 = \frac{L_2}{Z}$$

$$m_3 = \frac{B_2}{Z} \quad n_3 = \frac{L_2}{Z} \quad m_4 = \frac{B_2}{Z} \quad n_4 = \frac{L_1}{Z}$$

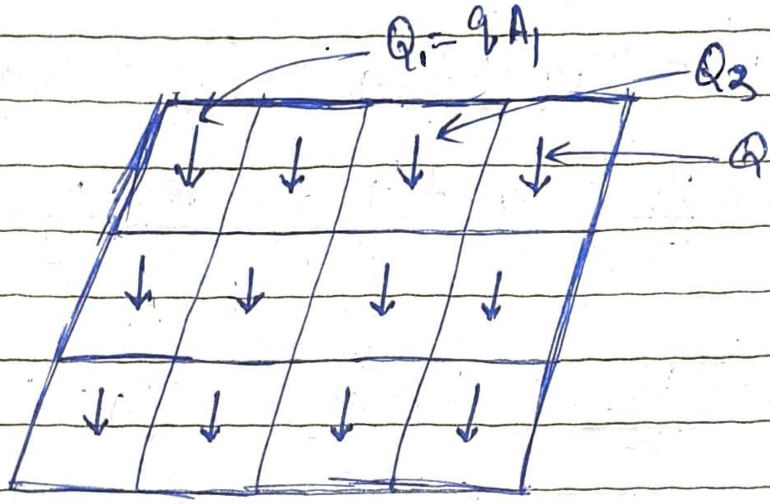
Case II Point P lies outside the rectangular loaded area:→

$$\sigma_z = K_{APC'B} \cdot q - K_{DPC'C} \cdot q - K_{A'PD''A} \cdot q + K_{D'PD''D} \cdot q$$



6. Approx. Method to Find Vertical Pressure

a) Equivalent Point Load Method :→ In this method the entire loaded area is divided into number of small area units and load these area Units is replaced by equivalent load acting at the Centroid of concerned area unit. Hence the entire loaded area is replaced by equivalent point loads, influence of which ~~univers~~ is considered separately to find the stress of concerned area.



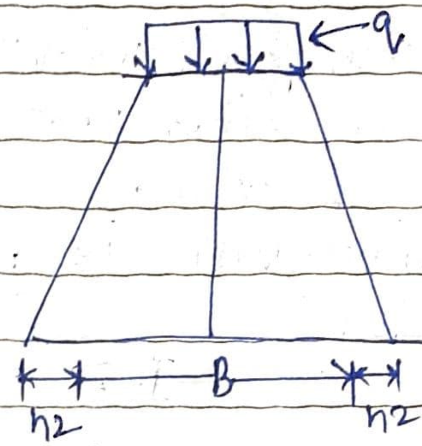
$$(\sigma_z)_p = K_{B_1}\frac{Q_1}{z^2} + K_{B_2}\frac{Q_2}{z^2} + \ldots\ldots$$

$$= \frac{1}{z^2}\sum K_{Bi}\, Q_i$$

b) Trapezoidal Method :→ In this method, load over the surface is assumed to be distributed

Over an area, shape of which is same that of loaded area but size of which increase in proportion to depth.
In this method generally 2V : 1H (1:n) (n = 0.5) distribution is considered below the loaded area.

$\sigma_z = \frac{\text{Force}}{\text{Area}}$.



Loadings		σ_z
Rectangular loading	⇒	$\frac{q(B \times L)}{(B+2nz)(L+2nz)}$
Square loading	⇒	$\frac{qB^2}{(B+2nz)^2}$
Circular loading.	⇒	$\frac{q\left(\frac{\pi}{4}D^2\right)}{\frac{\pi}{4}(D+2nz)^2} = \frac{qD^2}{(D+2nz)^2}$.
Strip loading	=	$\frac{qB}{(B+2nz)}$

$(L \simeq L+2nz)$

7. Newmark's Influence chart Method :->

In this method, soil is assumed to be homogenous semi-infinite, elastic and isotropic. It means that method not applicable to stratified soil. But greatest advantage of this method is that it can be used for any shape of loaded area.

Influence chart consist of number of concentric circles and radial lines that divides the influence chart into number of areas.

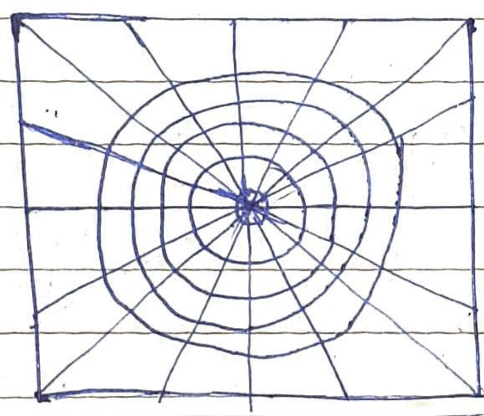
Influence value is given as

$$I.F = \frac{1}{mn}$$

Where m = number of concentric circles.
n = number of radial lines.

Genrally Newmark's chart consist of 10 circles and 20 Radial lines.

$$I.F = \frac{1}{200} = 0.005$$



$$\boxed{\sigma_z = I_f \times N_A \times q}$$