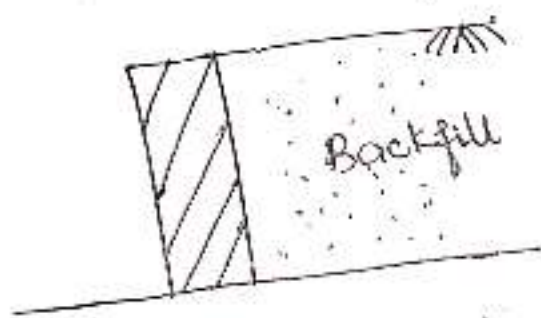


Chapter 9 EARTH PRESSURE

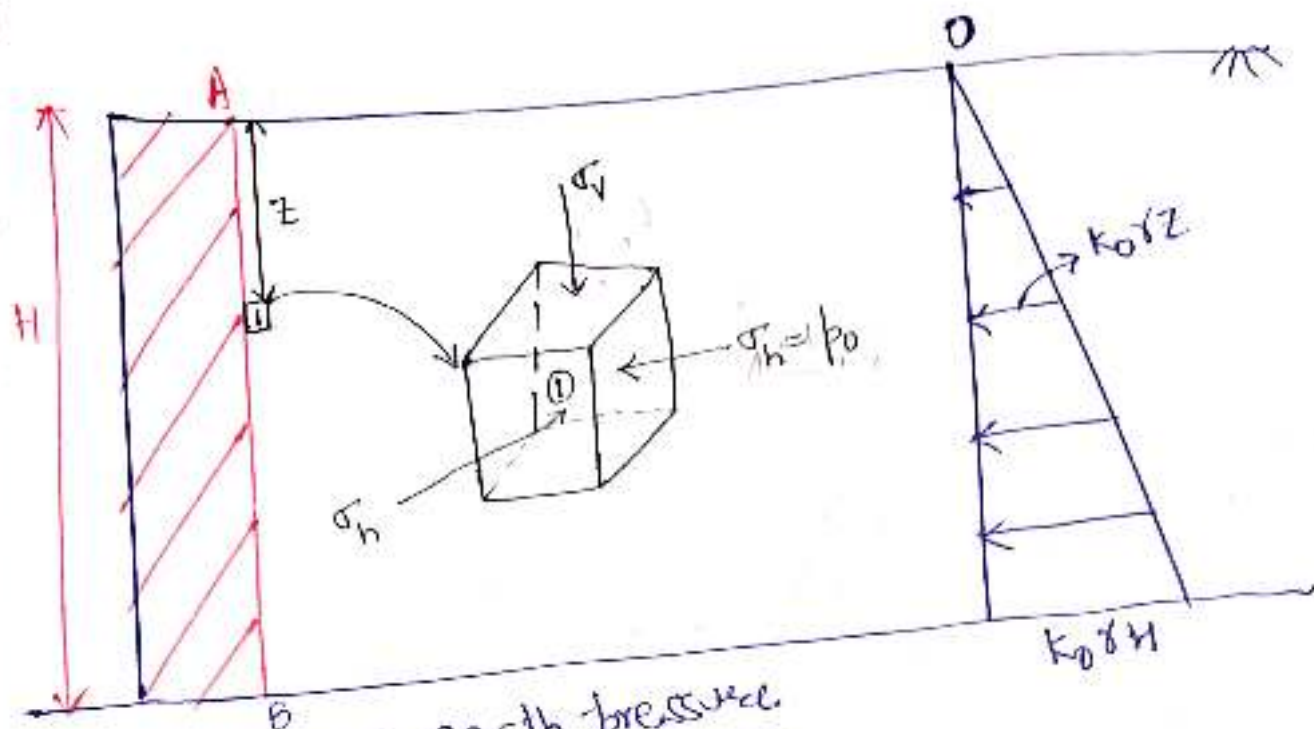


- In the designing of retaining wall, sheet pile wall and other retaining structures, pressure exerted by retained material should also be considered. This pressure exerted by retained material is termed as Earth pressure and retained material is termed as Backfill. Backfill may have its top surface horizontal or inclined.



Earth pressure at rest condition:-

- If the wall is rigid and unyielding then soil retained by it is in the state of rest and there is no displacement of rest and then pressure exerted by the soil on the wall is termed as Earth pressure at rest condition.



σ_v = vertical earth pressure.

σ_h = lateral earth pressure.

$$\frac{\sigma_h}{\sigma_v} = \text{earth pressure coefficient} = k$$

At Rest condition,

lateral strain (ϵ_h) = 0

$$\epsilon_h = \frac{\sigma_h}{E} - \mu \frac{\sigma_h}{E} - \mu \frac{\sigma_v}{E} = 0$$

$$\sigma_h (1 - \mu) = \mu \sigma_v$$

$$\frac{\sigma_h}{\sigma_v} = \frac{\mu}{1 - \mu} = k_0$$

Earth pressure coefficient at Rest condition

$$k_0 = \frac{\mu}{1 - \mu} \quad **$$

$$\therefore \frac{\sigma_h}{\sigma_v} = k_0 \Rightarrow \sigma_h = k_0 \cdot \sigma_v = p_0$$

⇒ Earth pressure at Rest condition

$$p_0 = k_0 \sigma_v^{**}$$

⇒ on the wall

- 1) At any depth z ; $\sigma_v = \gamma \cdot z$
- 2) Earth pressure $p_0 = k_0 \sigma_v = k_0 \cdot \gamma \cdot z$
- 3) At $z=0$ [Point A] $p_0 = 0$
at $z=H$ [Point B] $p_0 = k_0 \cdot \gamma \cdot H$

Note 1:-

For pure sand
For o.c soil

Jacky

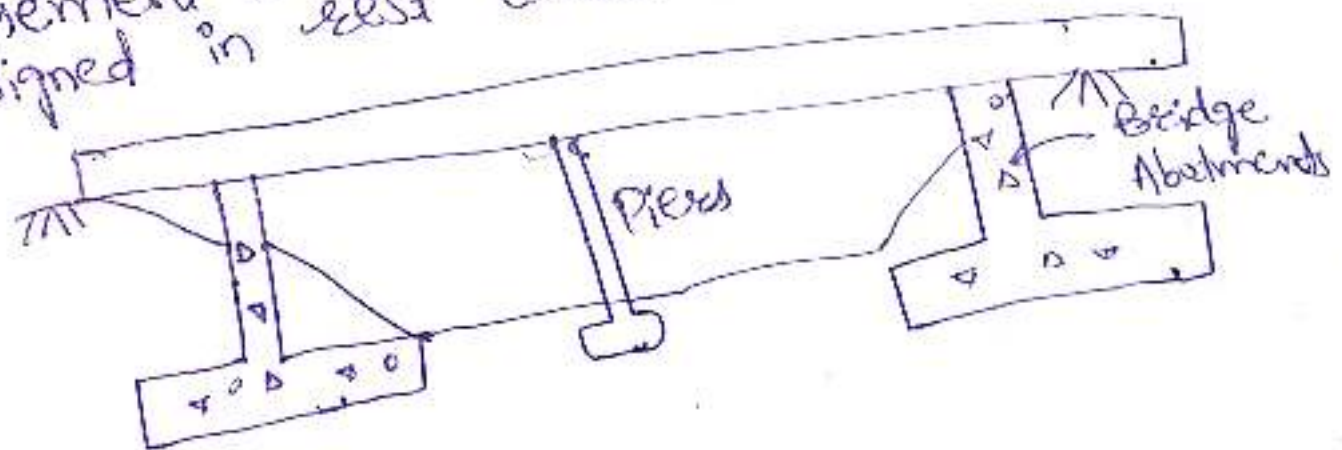
$$k_0 = 1 - \sin \phi$$

$k_0 = (k_0)_{N.C. \text{ soil}} \cdot \text{O.C.R.}$
O.C.R. = OVER consolidation ratio

Type of soil	k_0
Dense sand	0.4 - 0.45
Loose sand	0.45 - 0.5
Compacted soil	0.7 - 0.8
N.C.C	0.5 - 0.6

Note 2:-

Basement wall and Bridge Abutments are designed in rest condition.



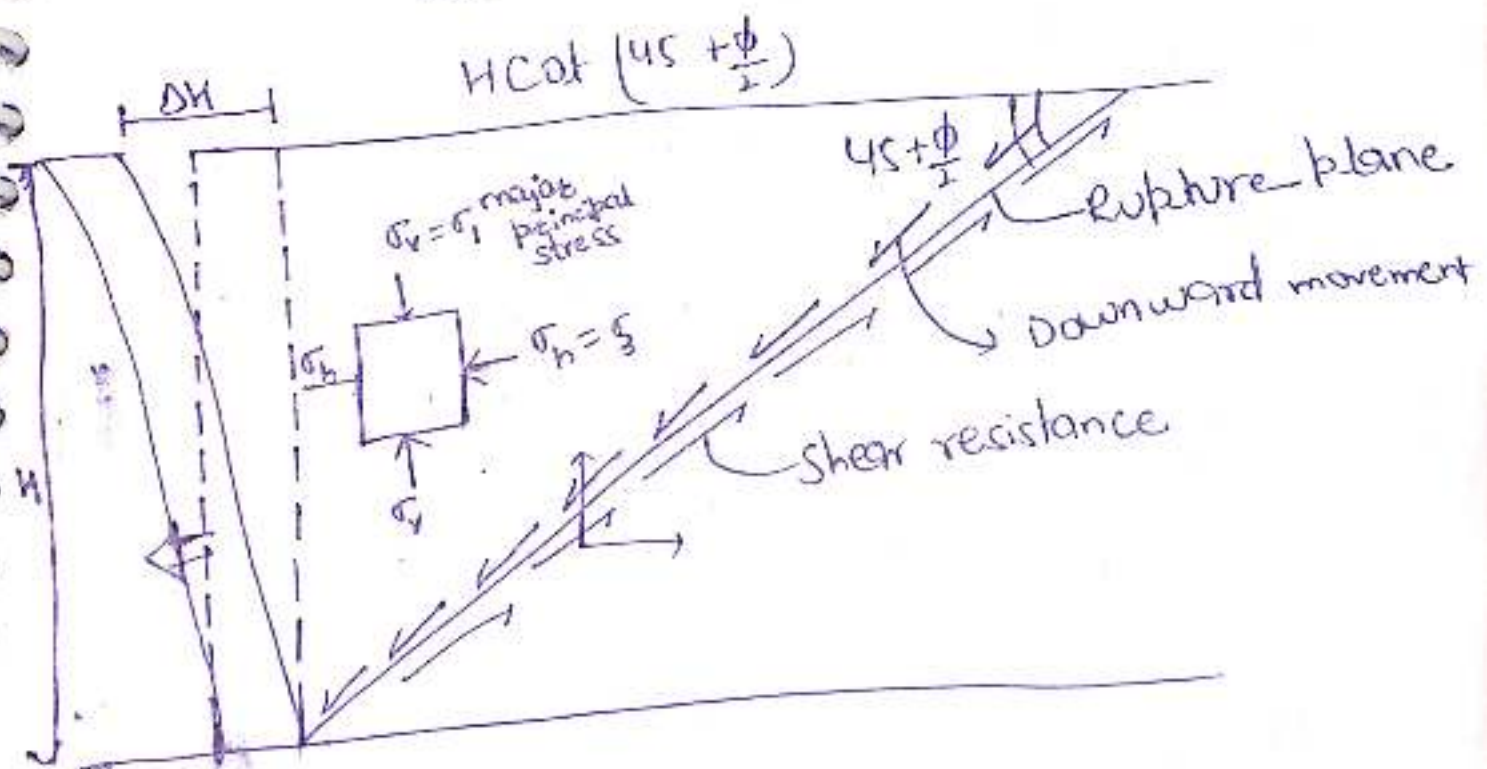
Active Earth pressure :-

- During the active stage, wall moves away from the backfill and the portion of the backfill just behind the wall leaves the rest of the soil mass and moves along with the wall. This portion of the backfill is termed as failure wedge.

- The resistance of the soil due to its shear strength is mobilized ~~at~~ upward and outward direction on the rupture surface which decreases the pressure on the wall. This decrease in pressure takes place up to an extent when entire shear resistance is mobilized. The minimum pressure acting on the wall in this stage is termed as Active Earth pressure.

- The strain required in this stage is in the order of 0.2% to 0.5%.

↓ Dense sand ↓ Loose sand



Rankine's Theory :-

For cohesionless soil
In plastic equilibrium :-

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\Rightarrow \sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\Rightarrow \sigma_v = \sigma_h \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$\Rightarrow \frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2\left(45 + \frac{\phi}{2}\right)} = k_a$$

Active earth pressure coefficient :-

$$k_a = \cot^2\left(45 + \frac{\phi}{2}\right) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\therefore \frac{\sigma_h}{\sigma_v} = k_a \Rightarrow \sigma_h = k_a \cdot \sigma_v = p_a$$

Will take forward the Rankine's Theory for cohesive soil

$$\sigma_v = \sigma_h \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

$$\Rightarrow \sigma_v \cdot \cot^2\left(45 + \frac{\phi}{2}\right) = \sigma_h + 2c \cdot \cot\left(45 + \frac{\phi}{2}\right)$$

{ $\because \cot^2\left(45 + \frac{\phi}{2}\right) = k_a$ }

$$\Rightarrow \sigma_v \cdot k_a = \sigma_h + 2c \sqrt{k_a}$$

$$\frac{\sigma_h}{\sigma_v} = \tan^2\left(45 + \frac{\phi}{2}\right) = k_p$$

Passive earth pressure coefficient

$$k_p = \tan^2\left(45 + \frac{\phi}{2}\right) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$\therefore \frac{\sigma_h}{\sigma_v} = k_p \Rightarrow \sigma_h = k_p \sigma_v = k_p$$

Being taken forward the Rankine's Theory for Cohesive Soil :-

$$\sigma_h = \sigma_v \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

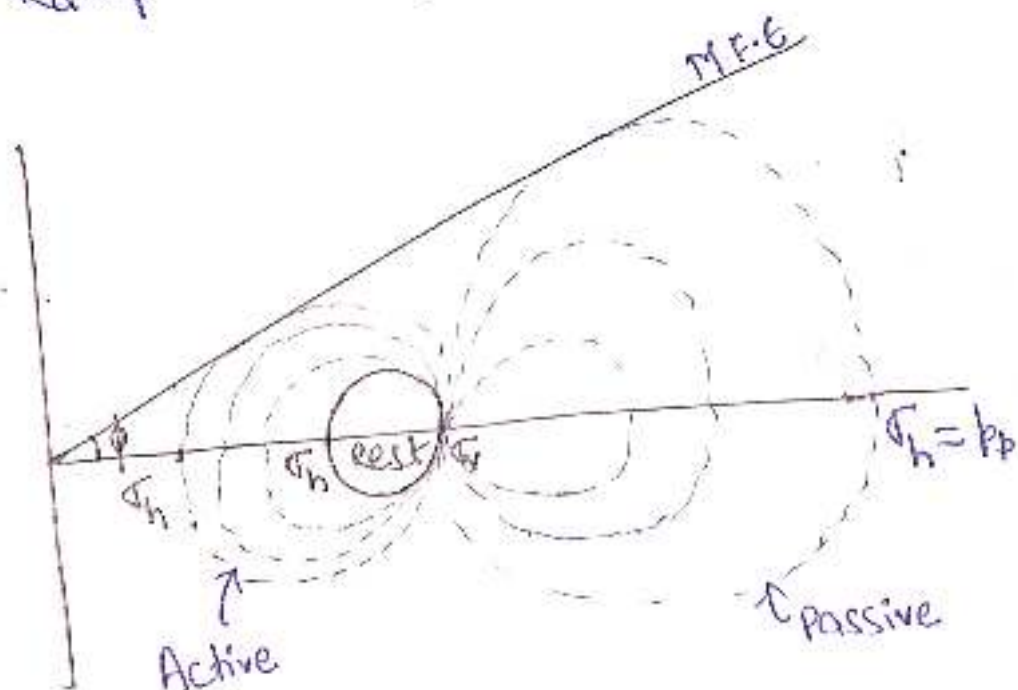
$$\sigma_h = \sigma_v k_p + 2c \sqrt{k_p} \quad \therefore \tan^2\left(45 + \frac{\phi}{2}\right) = k_p$$

Passive Earth pressure,

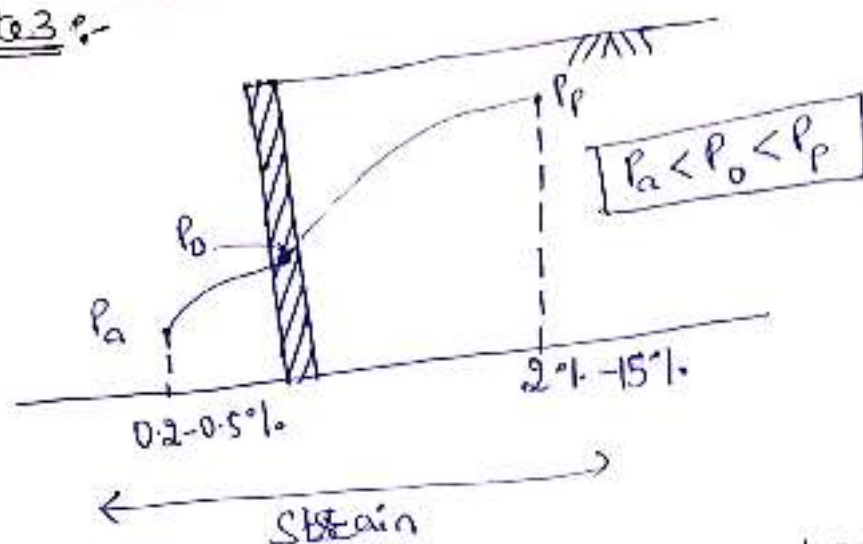
$$p_p = k_p \sigma_v + 2c \sqrt{k_p}$$

Note 1 :- $k_a \cdot k_p = 1$

Note 2 :-



Note 3:-



$$\left\{ \begin{array}{l} \text{Strain in Active} = \frac{\Delta H}{H \cot(45^\circ + \phi/2)} \approx 0.2 \text{ to } 0.5\% \\ \text{Strain in Passive} = \frac{\Delta H}{H \cot(45^\circ - \phi/2)} \approx 2 \text{ to } 15\% \end{array} \right\}$$

RANKINE'S EARTH PRESSURE THEORY:-

ASSUMPTIONS:-

- (i) Soil is homogeneous, Isotropic, semi-infinite, elastic, dry and cohesionless.
- (ii) The ground surface is plane which may be horizontal or inclined.
- (iii) The face of wall in contact with backfill is vertical and smooth.
- (iv) Soil is in the state of plastic equilibrium in passive and active earth pressure condition.
- (v) The rupture surface is plane.

Case I :- Dry / Moist cohesionless Backfill

Step 1 :- σ_v at any depth z ; $\sigma_v = \gamma z$

Step 2 :-

$$p_o = k_o \sigma_v = k_o \gamma z$$

$$p_a = k_a \sigma_v - 2c \sqrt{K_a} = k_a \gamma z$$

$$p_p = k_p \sigma_v + 2c \sqrt{K_p} = k_p \gamma z$$

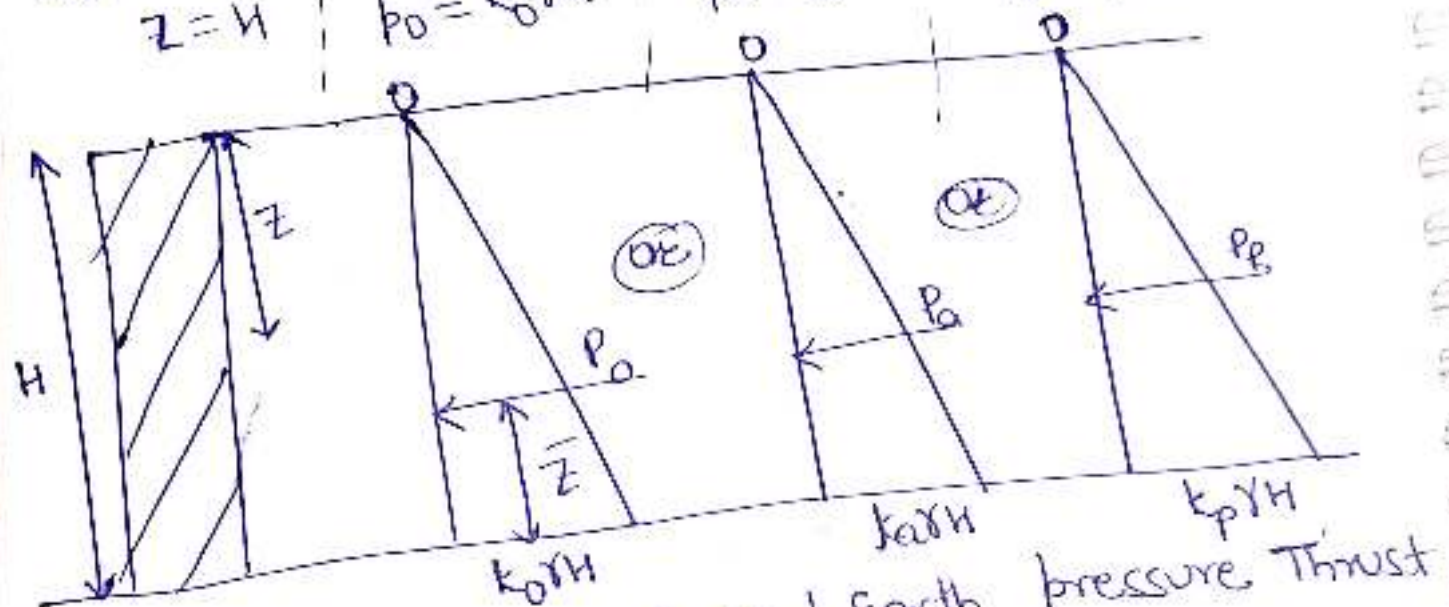
Step 3 :-

At $z=0$
 $z=H$

$p_o = 0$
 $p_o = k_o \gamma H$

$p_a = 0$
 $p_a = k_a \gamma H$

$p_p = 0$
 $p_p = k_p \gamma H$



Step 4 :-

Total earth pressure force / Earth pressure Thrust on unit length of wall :-

$$P_o = \frac{1}{2} k_o \gamma H \cdot H, \quad P_a = \frac{1}{2} k_a \gamma H^2, \quad P_p = \frac{1}{2} k_p \gamma H^2$$

Step 5 :-

Line of Action
 P will ~~not~~ act at $\bar{z} = \frac{H}{3}$ from base.

Case II :- cohesionless backfill with uniform

Surcharge :-

Step 1 :- σ_v at any depth z ; $\sigma_v = q + \gamma z$

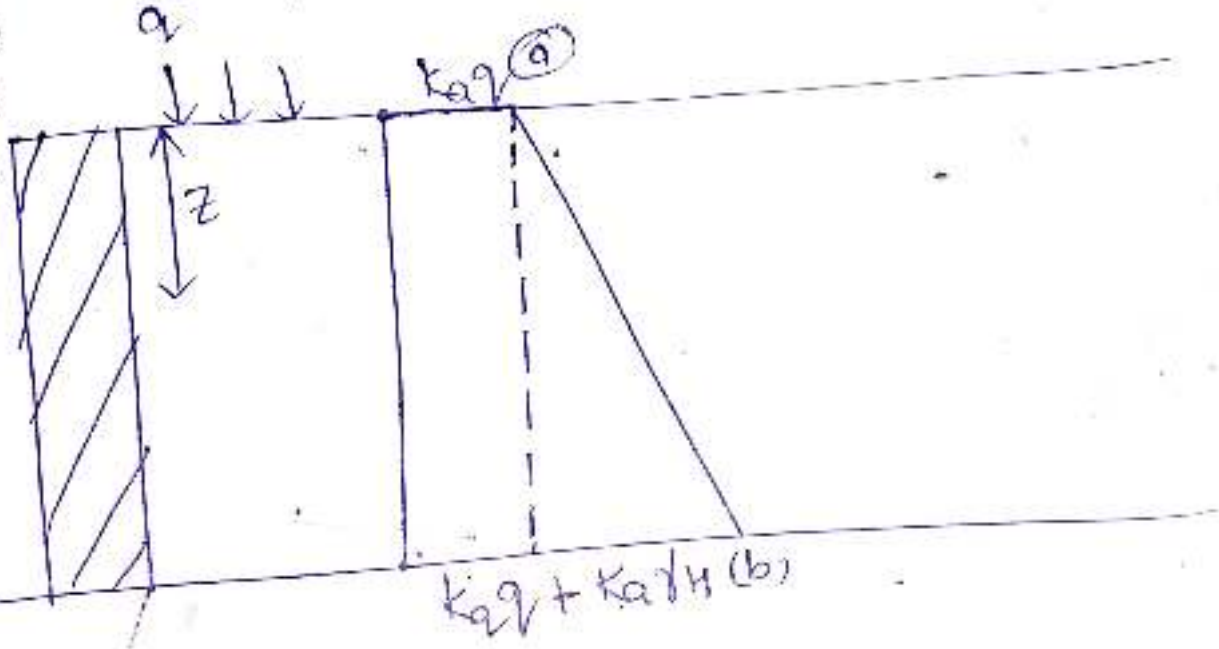
Step 2 :-

$p_o = k_o \sigma_v$	$= k_o q + k_o \gamma z$
$p_a = k_a \sigma_v - 2c \sqrt{k_a}$	$= k_a q + k_a \gamma z$
$p_p = k_p \sigma_v + 2c \sqrt{k_p}$	$= k_p q + k_p \gamma z$

Step 3 :-

At $z=0$, $p_a = k_a q$

$z=H$, $p_a = k_a q + k_a \gamma H$



Step 4 :-

$$P_a = \frac{1}{2} (a+b) H \quad \text{KN/m}$$

Steps :-

$$\bar{z} = \left(\frac{2a+b}{a+b} \right) \frac{H}{3} \quad \text{from base.}$$

Case III :- Submerged Backfill :-

Step 1 :- $\sigma_v = \bar{\sigma}_v + u$ will not be multiplied by k_a or k_p .

$$\sigma_v = \gamma' z \quad u = \gamma_w z$$

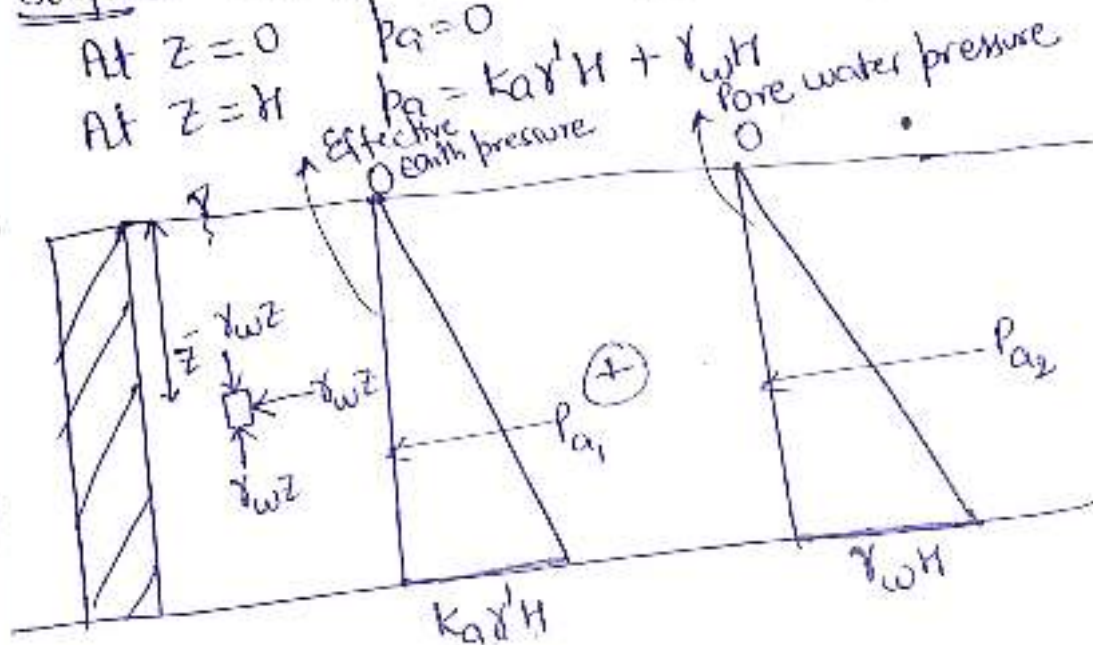
Step 2 :-

$$\begin{aligned} p_o &= k_o \bar{\sigma}_v + u &= k_o \gamma' z + \gamma_w z \\ p_a &= k_a \bar{\sigma}_v - 2z \int_0^z k_a &= k_a \gamma' z + \gamma_w z \\ p_p &= k_p \bar{\sigma}_v + 2z \int_0^z k_p &= k_p \gamma' z + \gamma_w z \end{aligned}$$

Step 3 :- effective stress before & after take γ
total stress below $\gamma_w H$ Take γ_{sat}
For pore pressure Take γ_w

At $z=0$ $p_a = 0$

At $z=H$ $p_a = k_a \gamma' H + \gamma_w H$



Step 4 :-

$$P_a = P_{a1} + P_{a2} = \frac{1}{2} k_a \gamma' H^2 + \frac{1}{2} \gamma_w H \cdot H$$

Step 5 :- $\bar{z} = \frac{H}{3}$ from base

Similarly can be calculated for Rest & Passive condition.

Note :-

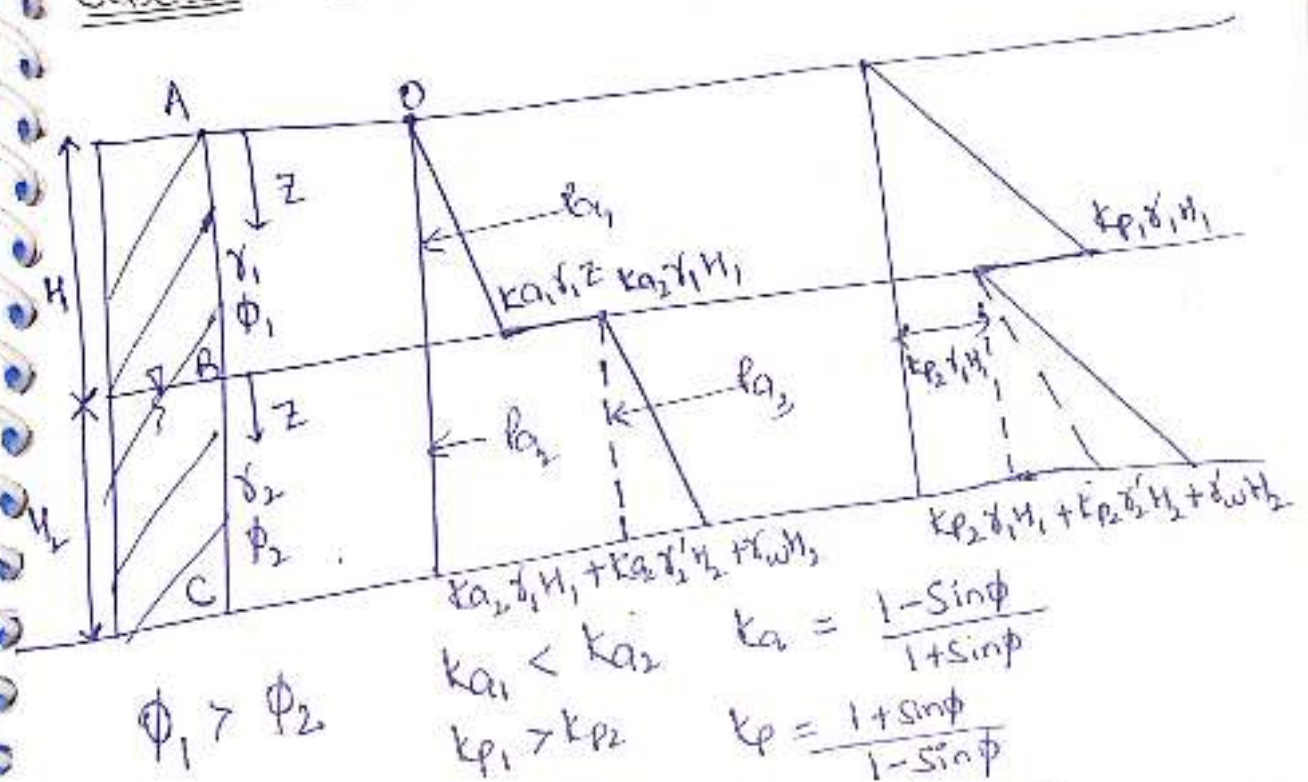
Water will exert equal pressure in all directions at particular depth ($\gamma_w z$) in

Submerged condition, which will not be multiplied by k_a or k_p (Pascal's law)

- water pressure is not considered for design of retaining wall because it may cause uneconomy in construction thus weep holes are provided in the retaining wall

- If water is present in both side of retaining wall then effect of water pressure will be cancelled out.

Case IV :- Stratified Backfill :-



gn soil ① At depth z below point A

1) $\sigma_v = \gamma_1 z$

2) $p_a = k_{a1} \sigma_v - 2c_1 \sqrt{k_{a1}} = k_{a1} \gamma_1 z$

$p_p = k_{p1} \sigma_v + 2c_1 \sqrt{k_{p1}} = k_{p1} \gamma_1 z$

$$\textcircled{3} \text{ At } z=0 \quad \left| \begin{array}{l} p_a = 0 \\ p_a = k_a \gamma_1 H_1 \end{array} \right| \quad \left| \begin{array}{l} p_p = 0 \\ p_p = k_p \gamma_1 H_1 \end{array} \right|$$

$$z = H_1 [B]$$

In soil ② At depth z below point B

$$(1) \bar{\sigma}_v = \gamma_1 H_1 + \gamma'_2 z \quad u = \gamma_w z$$

$$(2) p_a = k_{a2} \bar{\sigma}_v - 2 \int_0^z \sqrt{k_{a2}} + u = k_{a2} (\gamma_1 H_1 + \gamma'_2 z) + \gamma_w z$$

$$p_p = k_{p2} \bar{\sigma}_v + 2 \int_0^z \sqrt{k_{p2}} + u = k_{p2} (\gamma_1 H_1 + \gamma'_2 z) + \gamma_w z$$

$$\textcircled{3} \text{ At } z=0 \quad \left| \begin{array}{l} p_a = k_{a2} \gamma_1 H_1 \\ p_a = k_{p2} \gamma_1 H_1 + k_{p2} \gamma'_2 H_2 + \gamma_w H_2 \end{array} \right| \quad \left| \begin{array}{l} p_p = k_{a2} \gamma_1 H_1 \\ p_p = k_{p2} (\gamma_1 H_1 + \gamma'_2 H_2) + \gamma_w H_2 \end{array} \right|$$

$$z = H_2 [C]$$

④ Total earth pressure in $\frac{kN}{m}$

$$P_a = P_{a1} + P_{a2} + P_{a3}$$

$$P_{a1} = \frac{1}{2} \cdot k_{a1} \gamma_1 \cdot H_1 \cdot H_1 \quad \text{will act at } z_1 = H_2 + \frac{H_1}{3}$$

$$P_{a2} = k_{a2} \gamma_1 H_1 \cdot H_2$$

$$z_2 = \frac{H_2}{2}$$

$$P_{a3} = \frac{1}{2} (k_{a2} \gamma'_2 H_2 + \gamma_w H_2) \cdot H_2$$

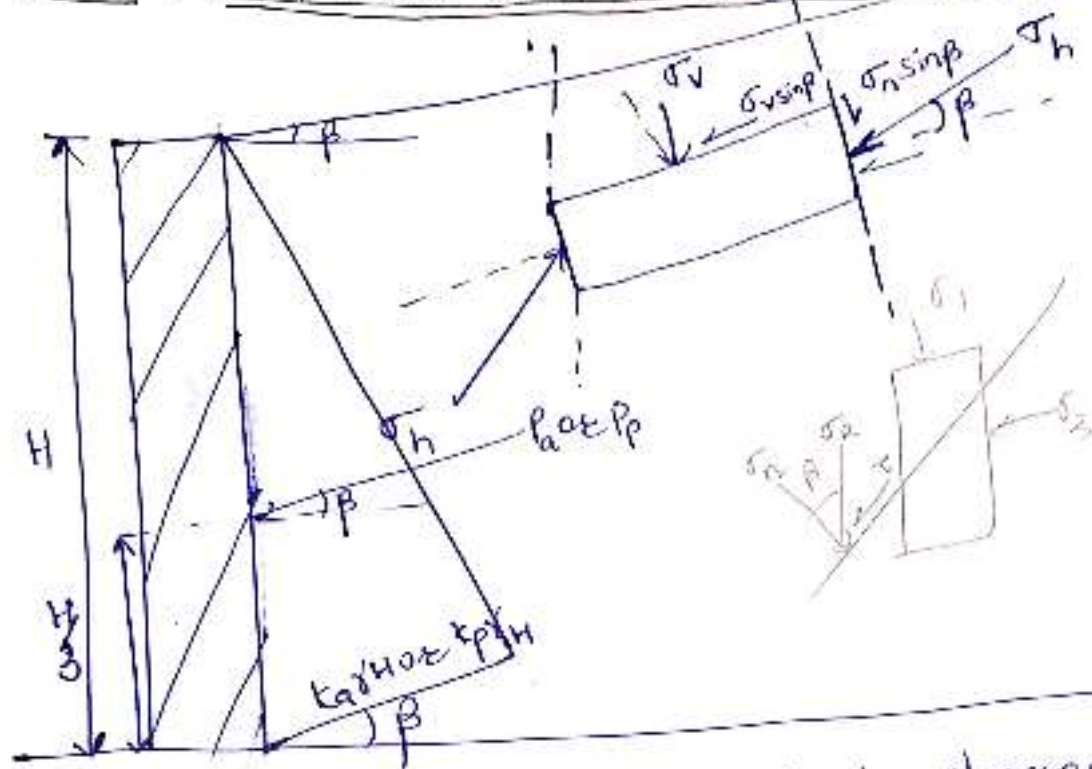
$$z_3 = \frac{H_2}{3}$$

⑤ Line of Action

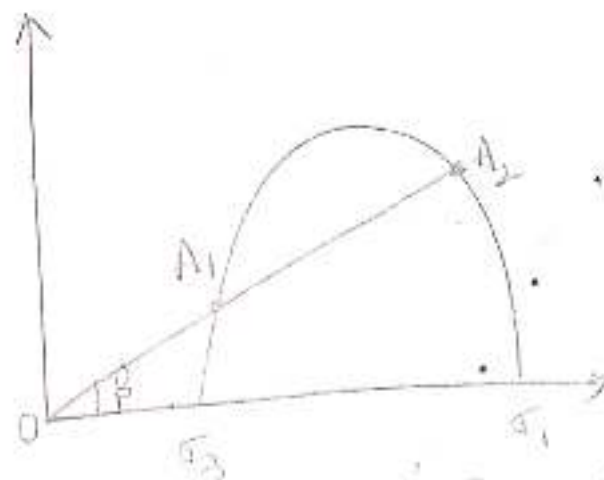
$$\bar{z} = \frac{P_{a1} z_1 + P_{a2} z_2 + P_{a3} z_3}{P_{a1} + P_{a2} + P_{a3}}$$

if $\phi_2 > \phi_1$, then diagrams will interchange.

Case I :- Soil mass with Inclined Backfill at Surcharge:-



Assumption:- σ_h and σ_v are conjugate stresses.



$$OA_1 = \sigma_h$$

$$OA_1 = \sigma_v$$

$$OA_2 = \sigma_v$$

$$OA_2 = \sigma_h$$

Active
Passive

$$\rightarrow P_a = \frac{1}{2} k_a \gamma H^2 \quad \text{or} \quad P_p = \frac{1}{2} k_p \gamma H^2$$

$\rightarrow P_a$ or P_p will act at $\frac{H}{3}$ from base at an angle of ' β ' with horizontal.

Value of k_a and k_p

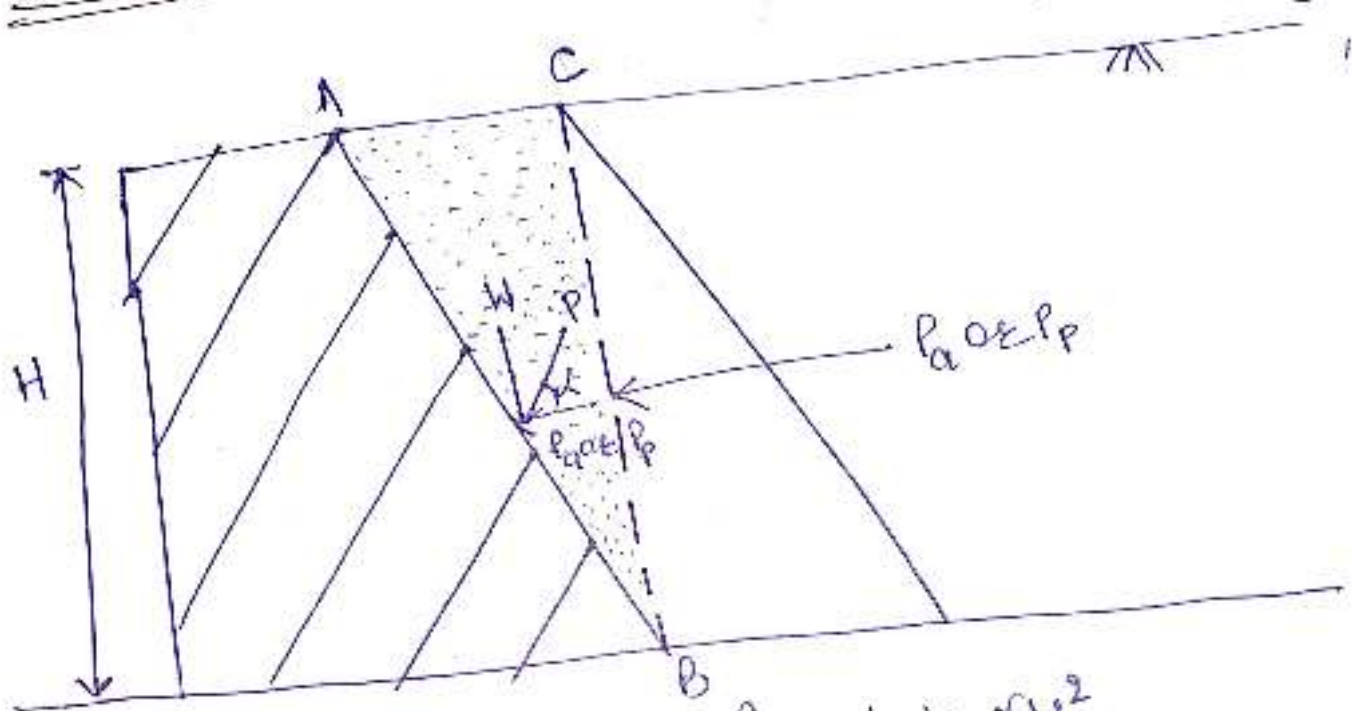
$$k_a = \cos \beta \left[\frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \right]$$

if this is formula of k_a above then here it is solved

$$k_p = \cos \beta \left[\frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}} \right]$$

Note :- $k_a \cdot k_p = \cos^2 \phi$

Cases :- Inclined wall with horizontal backfill



$$\rightarrow P_a = \frac{1}{2} k_a \gamma H^2 \quad \text{or} \quad P_p = \frac{1}{2} k_p \gamma H^2$$

$\rightarrow P_a$ or P_p will act at $\frac{H}{3}$

Resultant Earth Pressure

$$P = \sqrt{W^2 + P_a^2} \quad \text{or} \quad P = \sqrt{W^2 + P_p^2}$$

P will act at $\frac{H}{3}$ from base at an angle of α with horizontal.

$$\alpha = \tan^{-1}\left(\frac{W}{P_a}\right) \text{ or } \alpha = \tan^{-1}\left(\frac{W}{P_p}\right)$$

W $\rightarrow$ weight of wedge ABC.

Active and passive Earth Pressure for Cohesive Soil :-

- Rankine Theory was taken forward by Bell in cohesive soil. As cohesive soils are self supporting soils, active pressure exerted by them on the wall is comparatively less than cohesionless soil but passive pressure exerted by them is comparatively more than cohesionless soil.

Case I Dry / Moist Cohesive Soil (Active stage)

(1) σ_v at any depth z ; $\sigma_v = \gamma z$

(2) $p_a = K_a \sigma_v - 2c\sqrt{K_a} = K_a \gamma z - 2c\sqrt{K_a}$

(3) at $z=0$
 $z=H$

$$p_a = -2c\sqrt{K_a}$$

$$p_a = K_a \gamma H - 2c\sqrt{K_a}$$

$$p_a = K_a \gamma z_c - 2c\sqrt{K_a} = 0$$

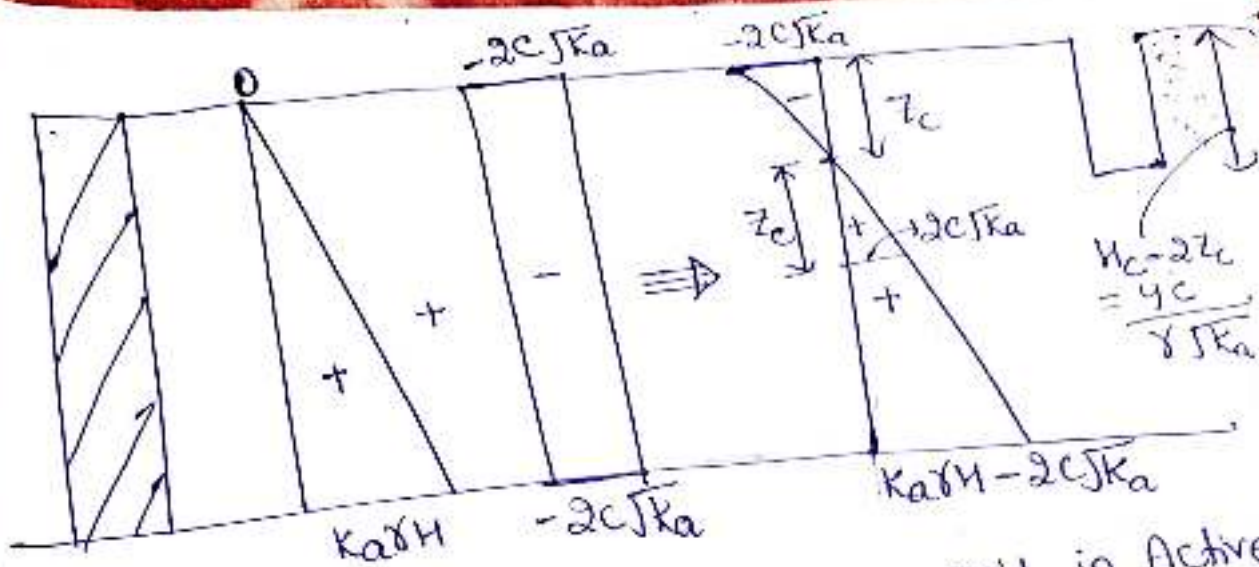
Let at z_c

$$z_c = \frac{2c\sqrt{K_a}}{K_a \gamma}$$

$$z_c = \frac{2c}{\gamma \sqrt{K_a}}$$

question

Total earth pressure at base = 0
Total earth pressure is zero at z_c
Total pressure at base = $-2c\sqrt{K_a}$



Note:-

1. Total Net pressure in cohesive soil in Active stage upto the depth q ($2z_c = H_c$) is zero hence these cohesive soils can stand with their vertical face up, upto this depth without any lateral support thereby this depth of $2z_c$ is termed as Critical depth of unsupported cut.

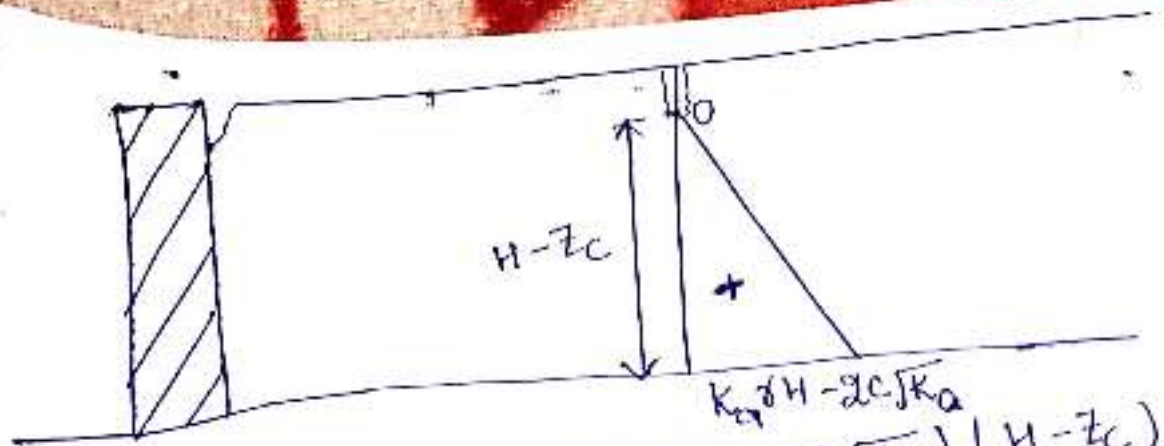
$$H_c = 2z_c = \frac{4c}{\gamma \sqrt{K_a}}$$

Note:-

2. Because of presence of tension, tension cracks may develop in cohesive soil as a result of which soil does not remain bonded to the face of the wall hence position of the soil is considered to be exert no pressure on the wall.

Step 4:- Total active earth pressure on unit length of wall

Case:- when tension cracks are developed.

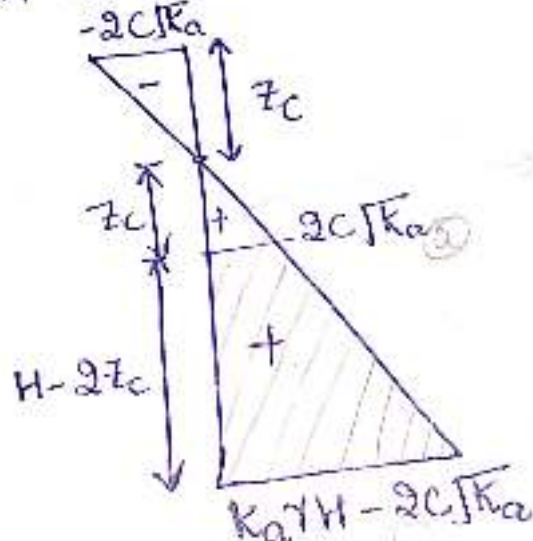


$$P_a = \frac{1}{2} (K_a \gamma H - 2C \sqrt{K_a}) (H - z_c)$$

$$\bar{z} = \frac{H - z_c}{3}$$

Case 1:-
when

tension cracks are not developed



Area
diff b/w two tension cracks

$$P_a = \left(\frac{a+b}{2} \right) (H - 2z_c)$$

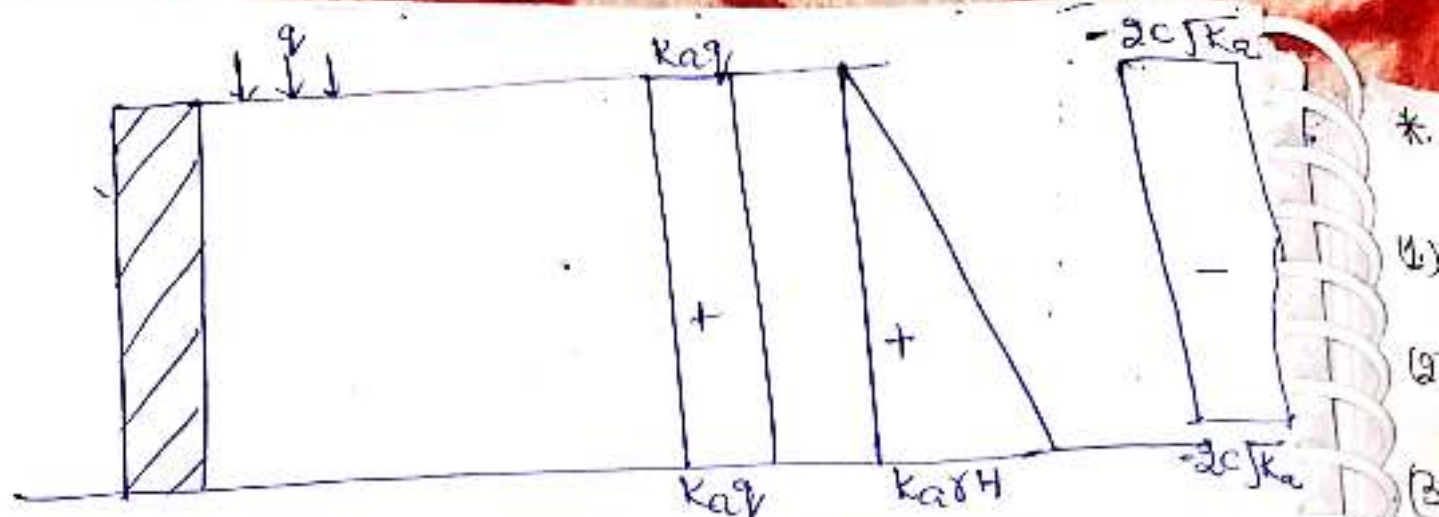
$$\bar{z} = \left(\frac{2a+b}{a+b} \right) \cdot \left(\frac{H - 2z_c}{3} \right)$$

Case 2:- Backfill with surcharge :-

$$(1.) \sigma_v = q + \gamma z$$

$$(2.) p_a = K_a \sigma_v - 2C \sqrt{K_a}$$

$$p_a = K_a q + K_a \gamma z - 2C \sqrt{K_a}$$



Depth of tension crack = $\frac{2c\sqrt{K_a} - K_a \gamma}{K_a \gamma}$

Surcharge required for no tension crack

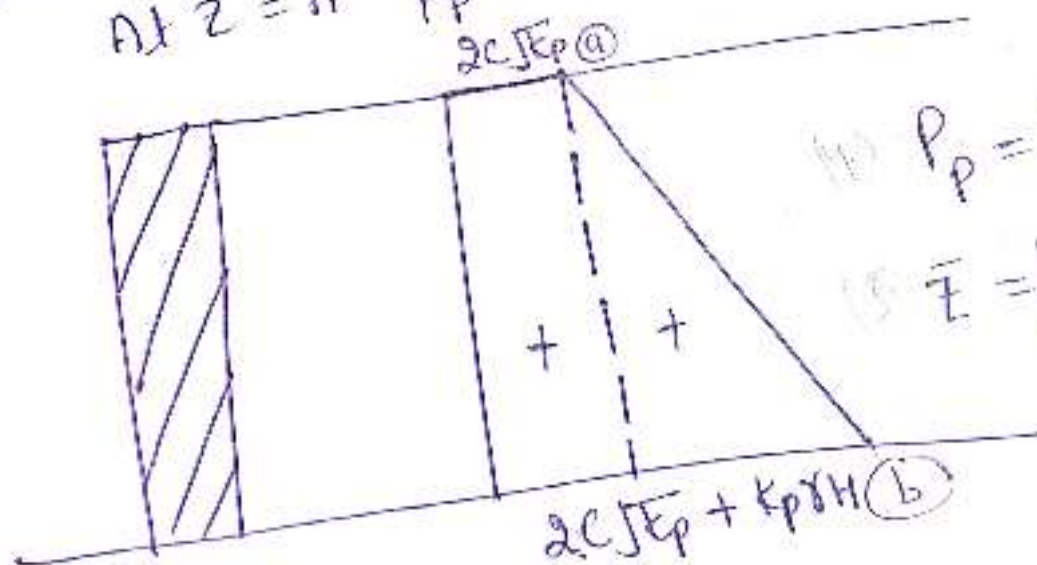
$$q = \frac{2c\sqrt{K_a}}{K_a} = \frac{2c}{\sqrt{K_a}}$$

Passive Earth Pressure in Cohesive Soil :-

(1.) $\sigma_v = \gamma z$

(2.) $p_p = K_p \sigma_v + 2c\sqrt{K_p} = K_p \gamma z + 2c\sqrt{K_p}$

(3.) At $z=0$ $p_p = 2c\sqrt{K_p}$
 At $z=H$ $p_p = K_p \gamma H + 2c\sqrt{K_p}$



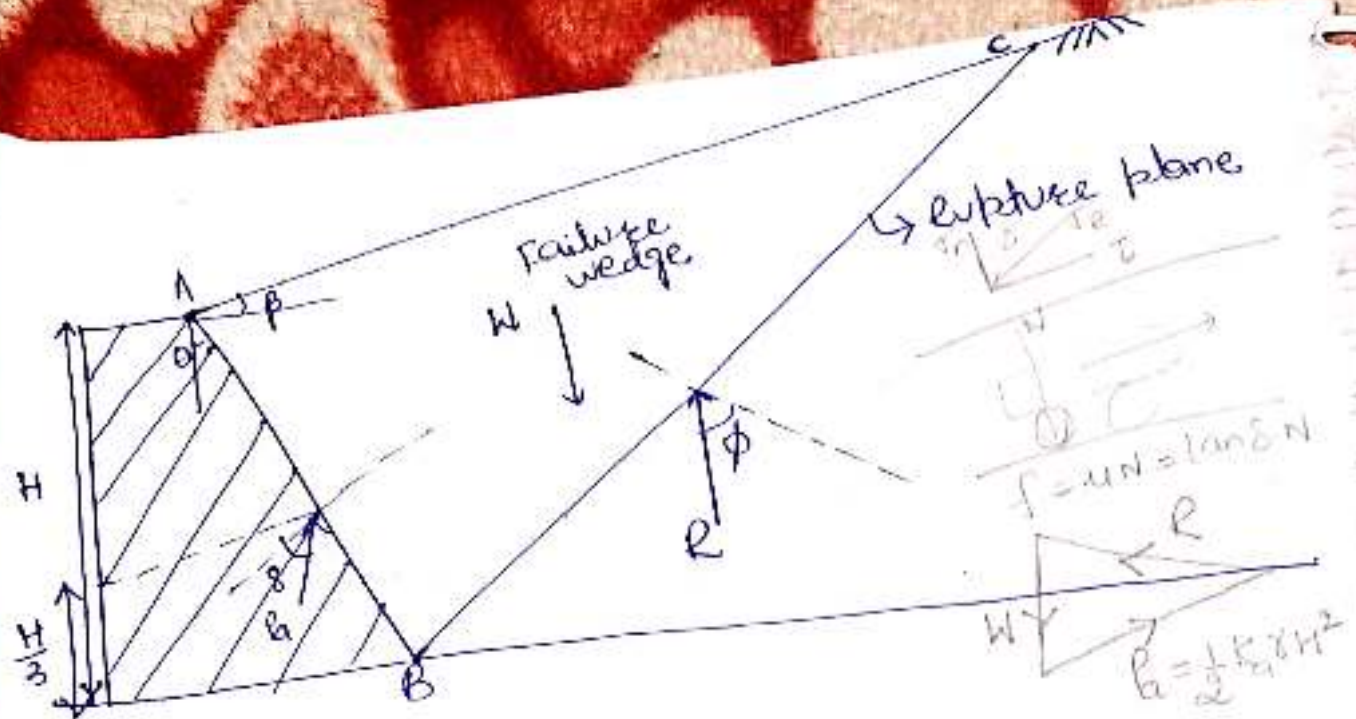
(11) $P_p = \left(\frac{a+b}{2} \right) H$

(12) $\bar{z} = \left(\frac{2a+b}{a+b} \right) \frac{H}{3}$

* COULOMB'S WEDGE THEORY:-

ASSUMPTIONS:-

- (1) Soil is homogenous, isotropic, semi-infinite, elastic, dry and cohesionless.
- (2) The face of wall in contact with backfill is vertical or inclined and is rough.
- (3) The failure wedge acts as a rigid body and stresses over it are uniformly distributed.
- (4) The failure is essentially 2-D and rupture surface is planar and passes through the heel of the wall.
- (5) The location and direction of resultant thrust b/w wall and soil is known. The point of application is taken at the lower third point of the wall by assuming triangular distribution of earth pressure.
- (6) Wedge failure is considered which is under equilibrium of 3 forces :-
 - i) Self weight of the wedge ABC.
 - ii) Resultant reaction (R) acts at downward angle of ϕ with normal to the rupture plane.
 - iii) Resultant Thrust (P_0), acts at an angle of δ with the normal to the face of the wall.



$$\rightarrow P_a = \frac{1}{2} K_a \cdot \gamma \cdot H^2$$

$\rightarrow P_a$ will act at $\frac{H}{3}$ from base at an angle of δ with normal to the face of wall.

$$K_a = \left[\frac{\sec \theta \cdot \cos(\phi - \theta)}{\sqrt{\cos(\theta + \delta)} + \frac{\sin(\delta + \phi) \cdot \sin(\phi - \beta)}{\cos(\beta - \theta)}} \right]^2$$

$\beta \rightarrow$ Angle of inclined backfill

$\phi \rightarrow$ Internal Angle of friction

$\theta \rightarrow$ Angle of face of wall with vertical

$\delta \rightarrow$ friction angle b/w wall and soil.

Generally δ is assumed as $\frac{2}{3}\phi$.

Note:-

(1) If wall is vertical ($\theta = 0$).

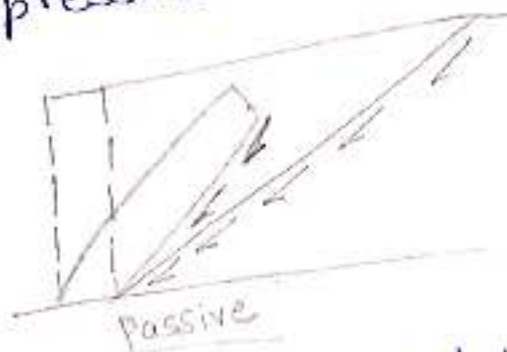
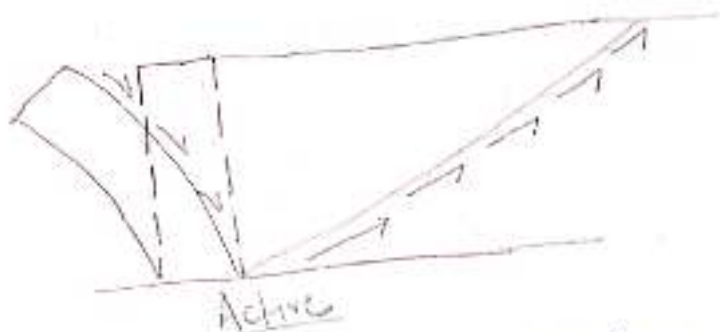
(2) If backfill is horizontal ($\beta = 0$).

(3) If wall is smooth then ($\delta = 0$).

then $(K_a)_{\text{Coulomb's}} = (K_a)_{\text{Rankine}}$

Note 2:-

Effect of wall friction
- The wall friction reduces the active pressure but increases the passive pressure.



Note 3:-

In Coulomb's Theory, failure plane is assumed to be planar but in actual it is spiral specially in passive case hence Coulomb's Theory is not preferred for passive earth pressure analysis.

Note 4:-

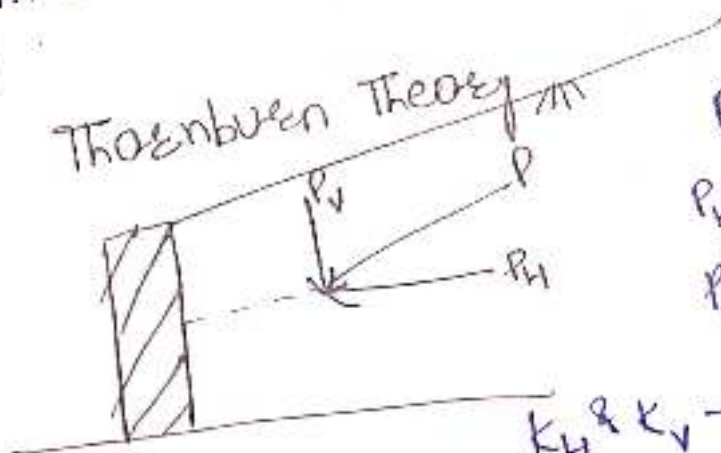
In Rankine's Theory, elemental failure is considered whereas in Coulomb's Theory wedge failure is considered.

Note 5:-

Earth pressure can also be computed using Culmann's Theory, Rebhmann's Theory, Poncet's Theory and Thornburn Theory which are graphical in nature.

Culmann's Theory is based upon Coulomb's Theory.

Thornburn Theory



$$P = \sqrt{P_H^2 + P_V^2}$$

$$P_H = \frac{1}{2} K_H H^2$$

$$P_V = \frac{1}{2} K_V H^2$$

K_H & $K_V \rightarrow$ Earth pressure constant

Sheet pile wall :-

- Sheet pile walls consist of number of sheet piles driven side by side to form a continuous vertical wall into the medium which is used to retain the earth mass.
- Sheet pile walls are generally used in water front structures, temporary construction, river training works, coffer dam, to prevent the piping failure below the dams and to prevent the failure of face of excavation.

Type of sheet pile wall :-

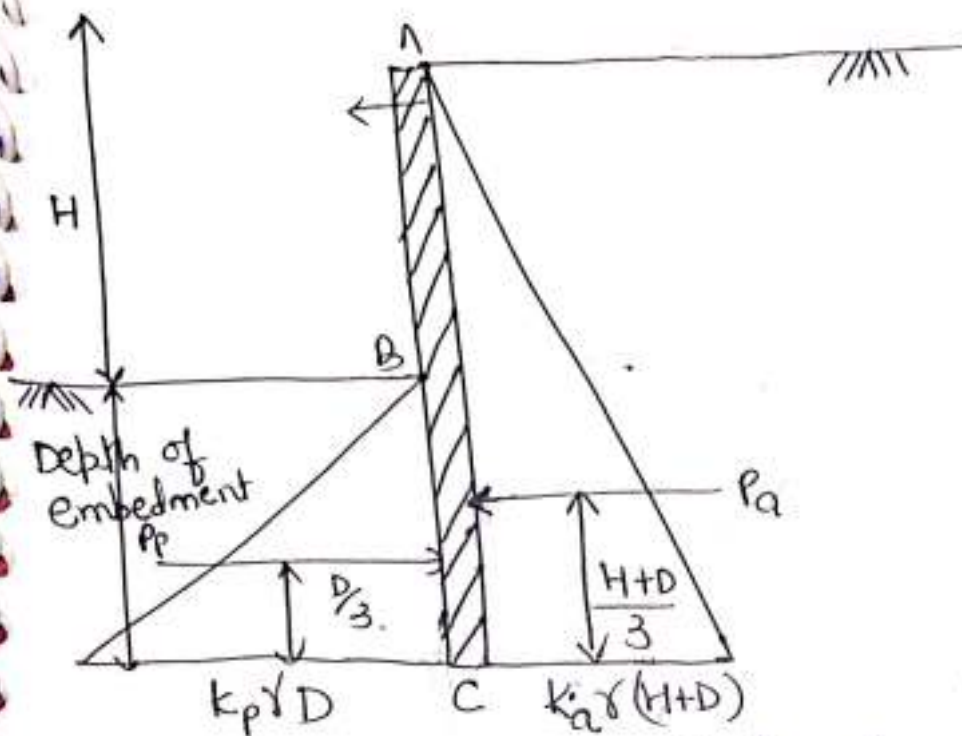
(1) Cantilever sheet pile wall :-

- In Actual, Rotation of sheet pile wall takes place about pivot point O hence sheet pile wall is subjected to -

	Right side	Left side
Active stage	ABO	OC
Passive stage	OC	OB

Case 1 :- For cohesionless soil

For simplicity in calculations sheet pile wall is assumed to rotate about base point C hence sheet pile wall is subjected to active pressure in right side ABC and also subjected to passive pressure in left side BC.



Minimum depth of embedment

$$\sum M_C = 0$$

$$P_p \cdot \frac{D}{3} - P_a \cdot \left(\frac{H+D}{3} \right) = 0$$

$$\Rightarrow \frac{1}{2} k_p \gamma D^2 \cdot \frac{D}{3} - \frac{1}{2} k_a \gamma (H+D)^2 \cdot \left(\frac{H+D}{3} \right) = 0$$

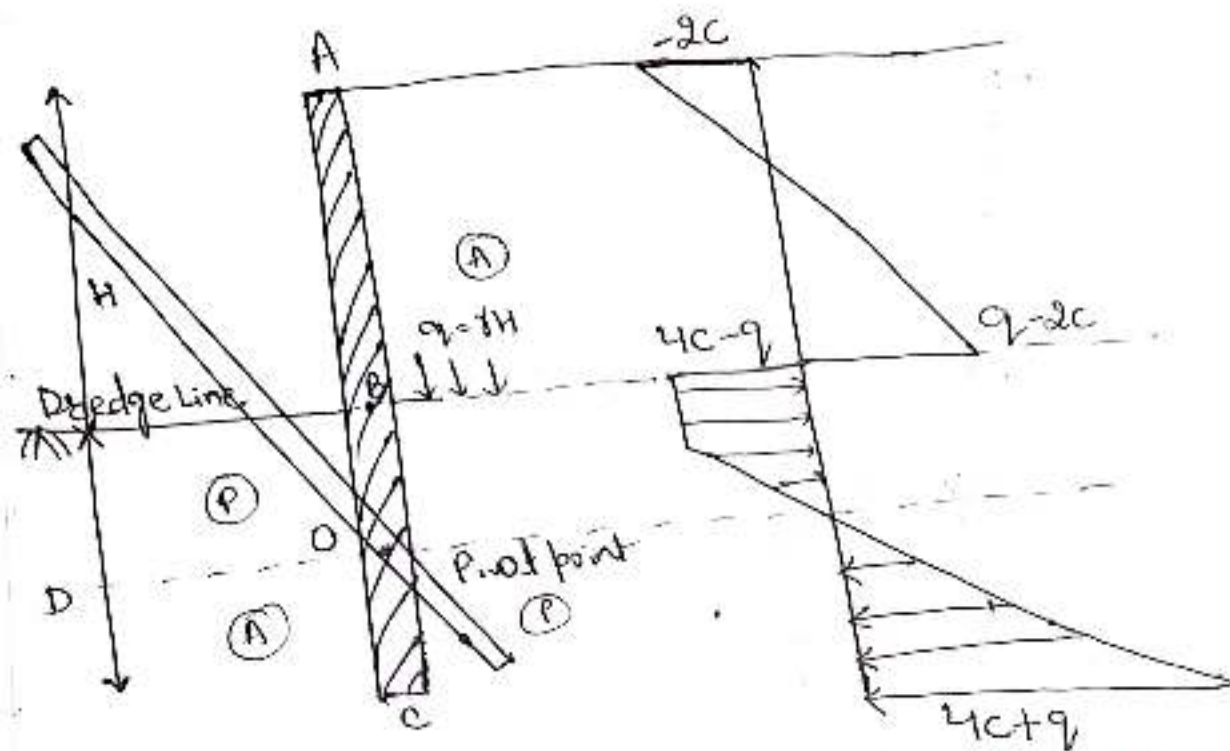
Solve for D_{min} .

$\Rightarrow$ If F.O.S is given

$$F.O.S = \frac{M_e}{M_o} = \frac{P_p \cdot \frac{D}{3}}{P_a \cdot \left(\frac{H+D}{3} \right)} = \frac{\frac{1}{2} k_p \gamma D^2 \cdot \frac{D}{3}}{\frac{1}{2} k_a \gamma (H+D)^2 \cdot \left(\frac{H+D}{3} \right)}$$

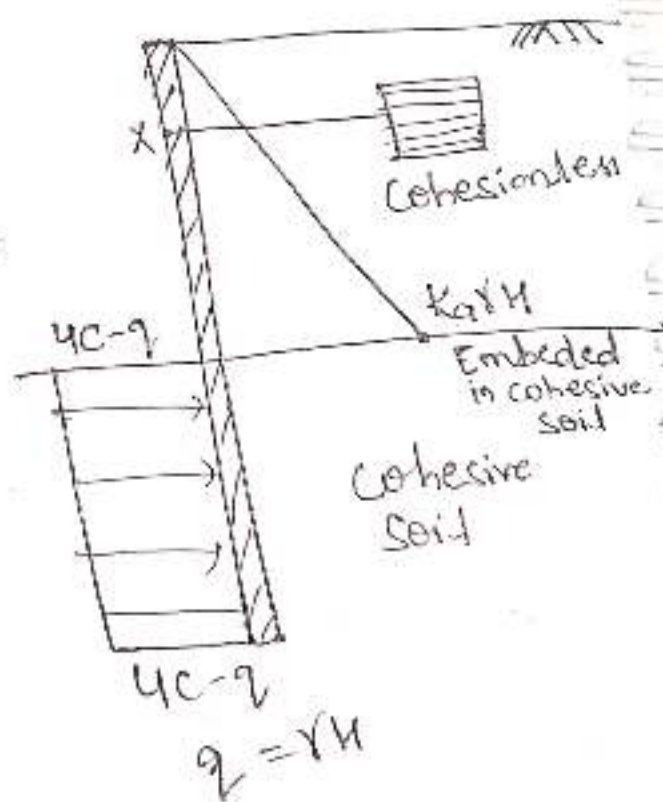
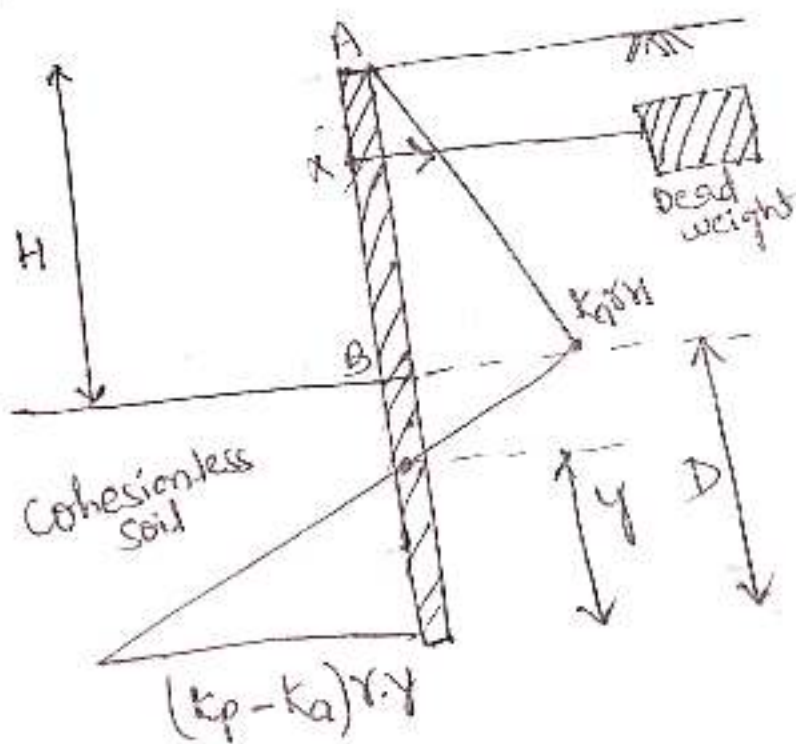
Solve for D_{safe} .

Case B: In cohesive soil / clay ($\phi = 0$) $K_a = K_p = 1$
Assume the rotation about pivot point O.

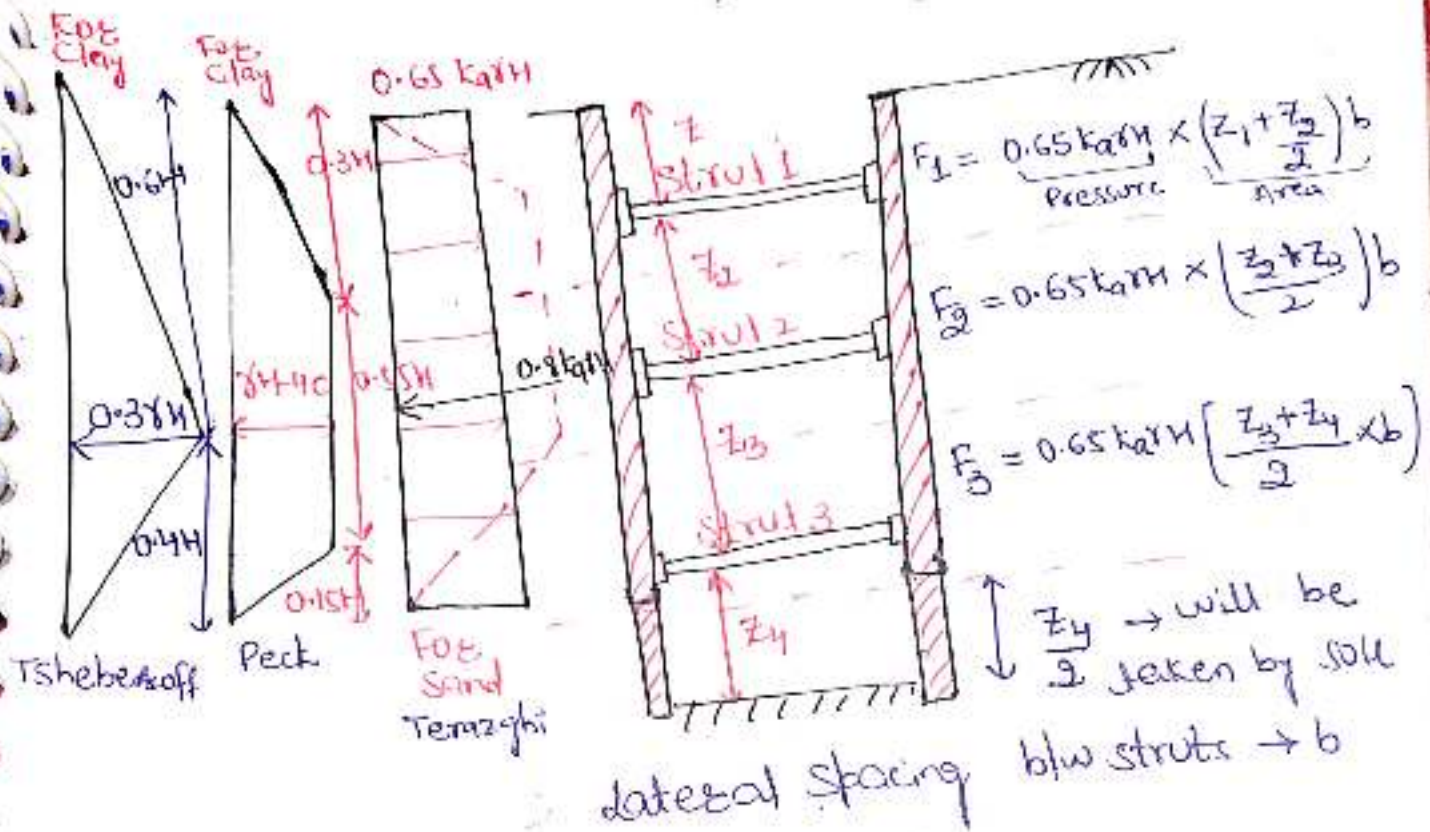


2) Anchored Sheet pile (Anchored bulk Head) :-

For Information only



(3) BRACED SHEETING (Determination of strut load):-



Note:-
F.O.S. Against heaving in Braced cut

