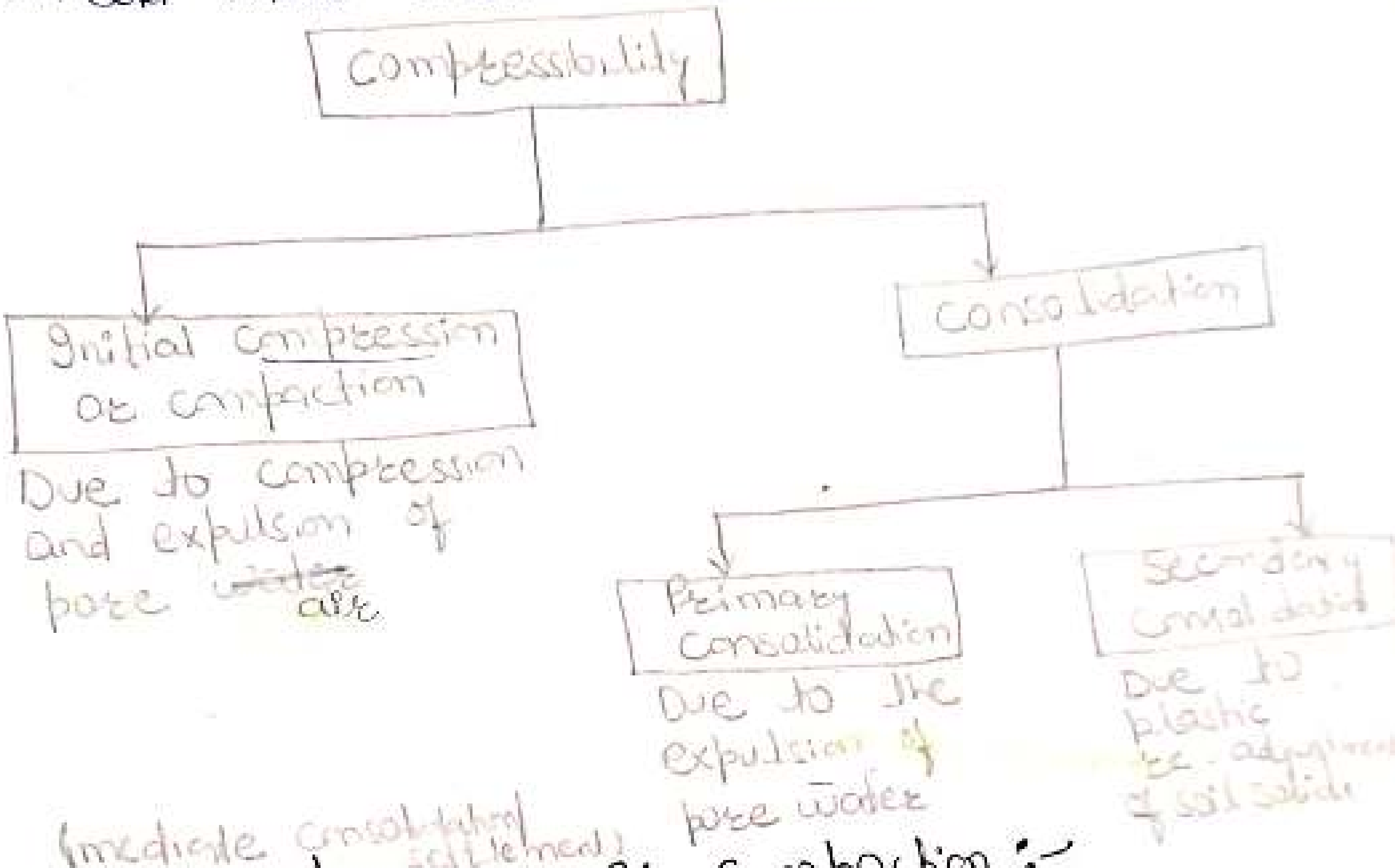
Chapter 7
Compressibility and Consolidation

Compressibility :-

- It is defined as gradual decrease in the volume of soil due to application of load on soil mass under consideration.



Initial Compression or Compaction :-

- It is an instantaneous process which occurs immediately after loading and is due to compression and expulsion of pore air.

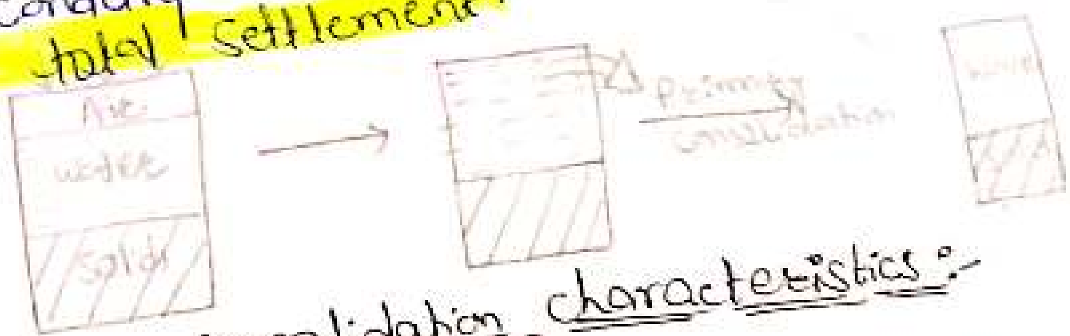
Primary Consolidation :-

- It begins when soil reaches to full saturation (after the expulsion of pore air) and remains saturated during entire process of primary (1°) consolidation.
- It occurs due to expulsion of pore water and completes when expulsion

of pore water stops.
- It is a time taking phenomenon.
- Rate of consolidation depends upon
(a) Permeability of soil
(b) length of drainage path
(c) Loading condition!

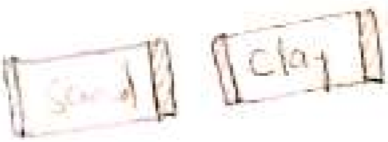
Secondary consolidation:- creep is related to

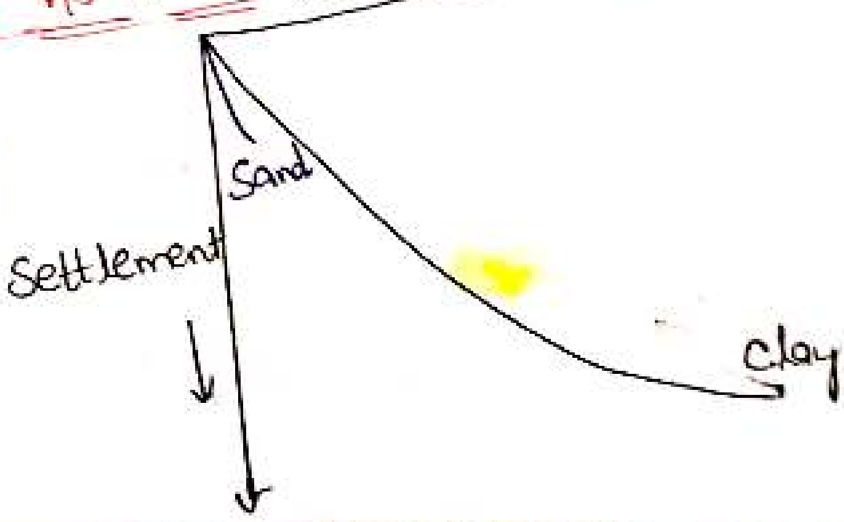
- It starts after completion of primary consolidation.
- If load is constant and volume change is recorded with passage of time, without expulsion of pore water then volume change takes place due to plastic readjustment of soil solids which is also termed as secondary (2°) consolidation.
- In coarse grained soil secondary consolidation is insignificant but in highly plastic clays secondary consolidation can be 10% to 20% of total settlement.



Primary consolidation characteristics:-

i.) Settlement v/s Time:-





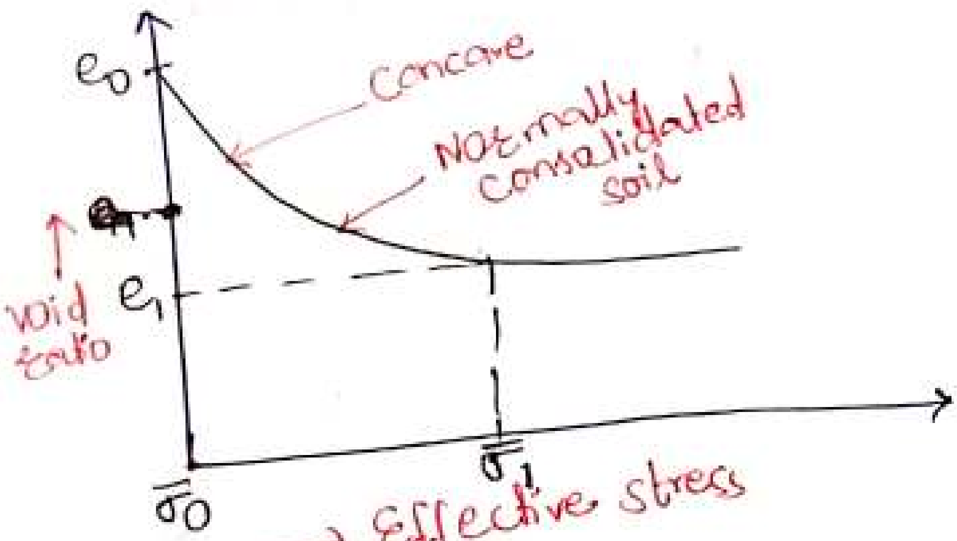
- The rate of consolidation in sand is more due to its high permeability but total settlement of clay is much greater than that of sand, at same loading rate and having same thickness.
- The settlement in clay is higher because of its high void ratio or porosity thus for practical purposes settlement of sand can be neglected in comparison to clay.

(2.) Void Ratio v/s Effective stress (Normal Consolidation curve or normal compression curve) :-

- Normally consolidated soils are those which are subjected to first time in the history to the presently applied effective stress.

(Present $\geq$ past).

Name, symbol, formula → t/s



Slope of e v/s $\bar{\sigma}$ curve $= \dfrac{e_0 - e_1}{\bar{\sigma}_0 - \bar{\sigma}_1}$

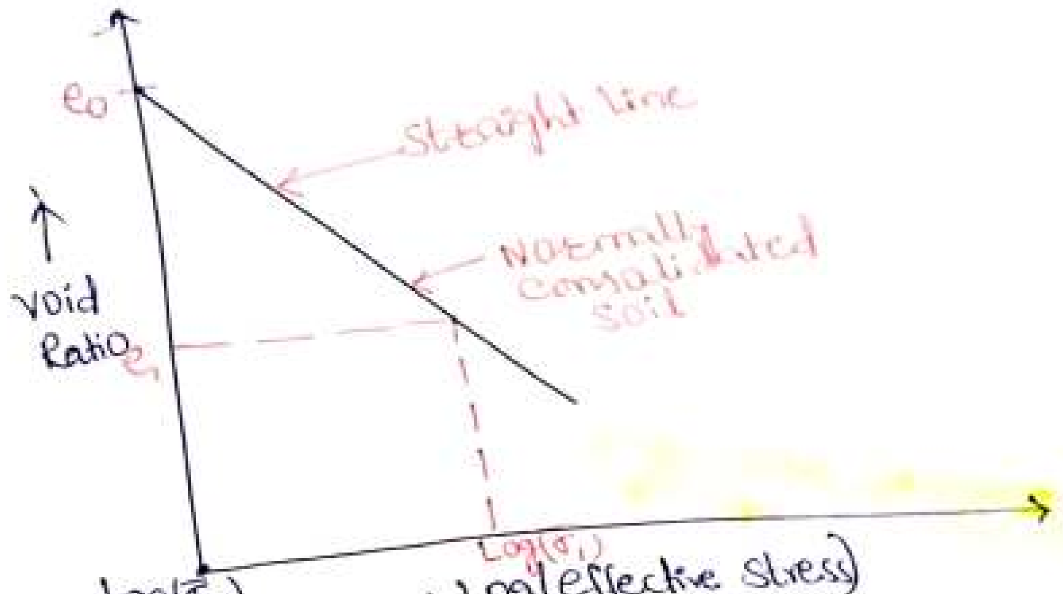
Coefficient of compressibility $= \left| \dfrac{e_0 - e_1}{\bar{\sigma}_0 - \bar{\sigma}_1} \right|$

$$\boxed{a_v = \frac{\Delta e}{\Delta \bar{\sigma}}}$$

Scanned with CamScanner

- a_v is not used widely, because it is not constant for whole curve. But when curve is plotted in semi-log graph paper straight line is obtained.

3. <u>Void Ratio v/s log-effective stress</u>:-



Slope of e v/s $\log(\bar{\sigma})$ curve $= \dfrac{e_0 - e_1}{\log(\bar{\sigma}_0) - \log(\bar{\sigma}_1)}$

$\left\{\begin{array}{c}\text{coefficient of compression} \\ \text{or} \\ \text{compression Index}\end{array}\right\} = \left|\dfrac{e_0 - e_1}{\log\bar{\sigma}_0 - \log\bar{\sigma}_1}\right|$

$$C_c = \frac{e_0 - e_1}{\log\bar{\sigma}_1 - \log\bar{\sigma}_0}$$

$$\boxed{C_c = \frac{\Delta e}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}}$$

Last choice

Empirical relation for determination of C_c :-

(A.) For undisturbed clay with medium sensitivity :-

Imp. $\boxed{C_c = 0.009 (w_L - 10)}$ $w_L \rightarrow$ L.L in %

(B.) For remoulded ~~so~~ clay with low sensitivity :-

$\boxed{C_c = 0.007 (w_L - 7)}$

(C.) For organic clay

$C_c = 0.115\, w_n$

$w_n \rightarrow$ Natural water content in %

(D.) When initial void Ratio (e_0) is known.

$C_c = 0.54 (e_0 - 0.3)$

or

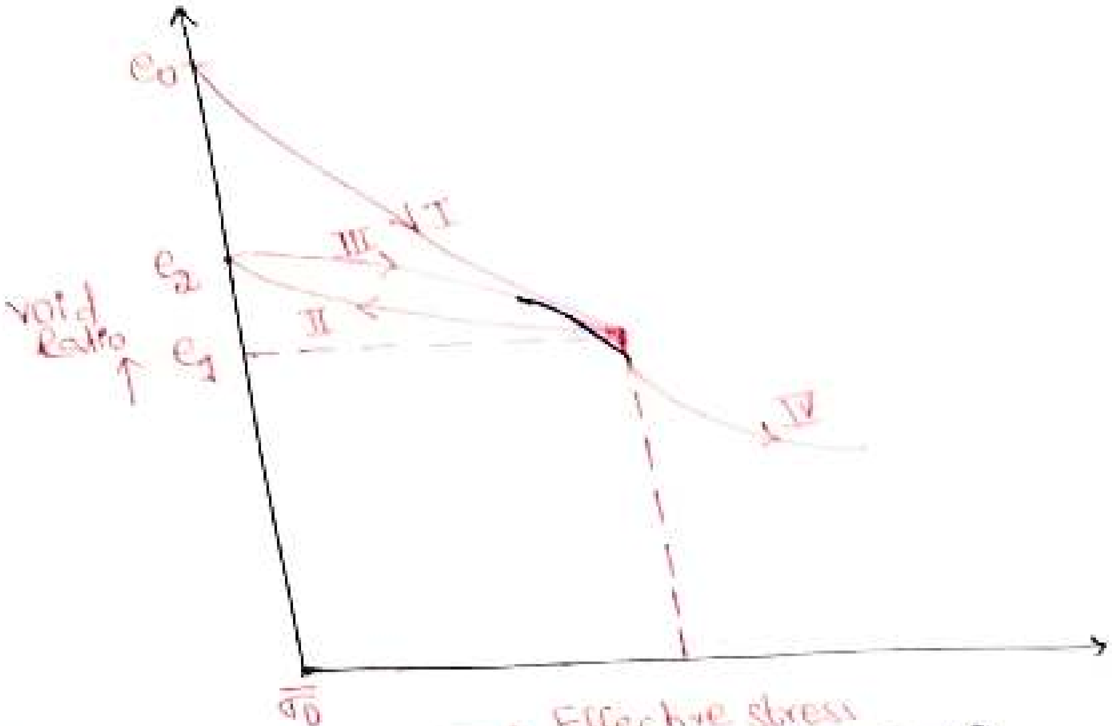
$C_c = 1.15 (e_0 - 0.35)$

4. Over consolidated soil / pre-consolidated soil :-

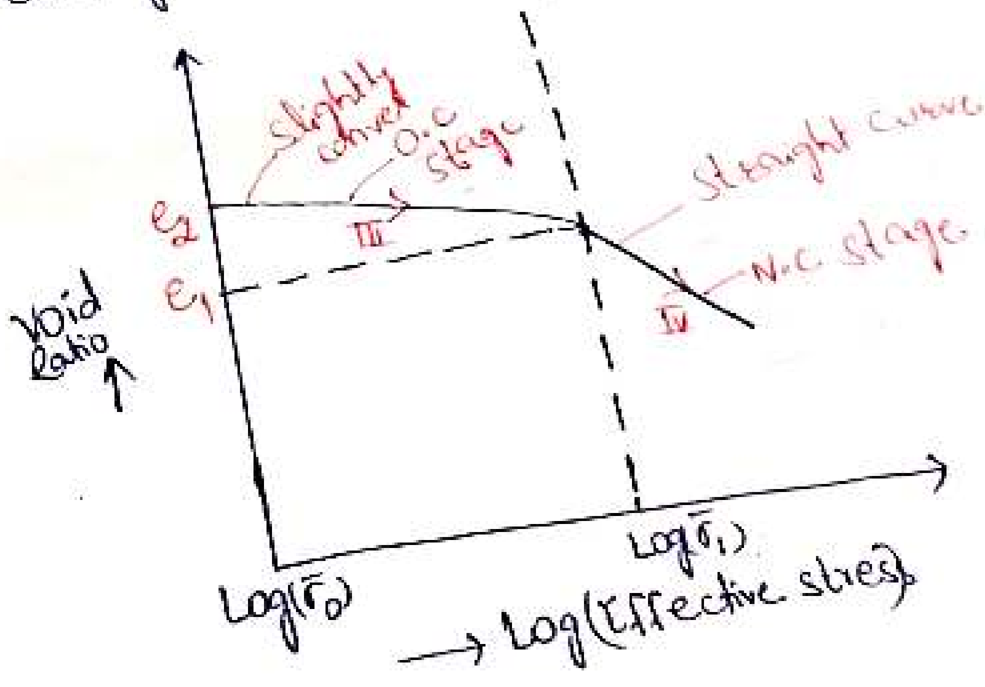
- If presently applied effective stress is less than applied effective stress in the past then soil is called to be in over-consolidated stage.

$\boxed{\text{Present} \leq \text{past}}$

- Over consolidated soil shows less volume change and behaves stable.
- Highly over-consolidated clay behaves like dense sand.



(I.) Normally Consolidated stage (concave)
or
virgin Compression curve
(II.) Rebound | swelling curve
(III.) over consolidated stage (convex)
(IV) Normally consolidated stage (Concave)



Slope of e v/s $\log \bar{\sigma}$ curve in O.C stage $= \dfrac{e_2 - e_1}{\log \bar{\sigma}_0 - \log \bar{\sigma}_1}$

Coefficient of Recompression $= \left| \dfrac{e_2 - e_1}{\log \bar{\sigma}_0 - \log \bar{\sigma}_1} \right|$

$$C_R = \frac{e_2 - e_1}{\log \bar{\sigma}_1 - \log \bar{\sigma}_0}$$

$$\boxed{C_R = \frac{\Delta e}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}}$$

Note :-
- Generally C_R is $\frac{1}{5}$ to $\frac{1}{10}$ times of C_c.

Over Consolidation Ratio (O.C.R) :-

$$O.C.R = \frac{\text{Maximum applied eff. stress in Past}}{\text{Maximum applied eff. stress in present}}$$

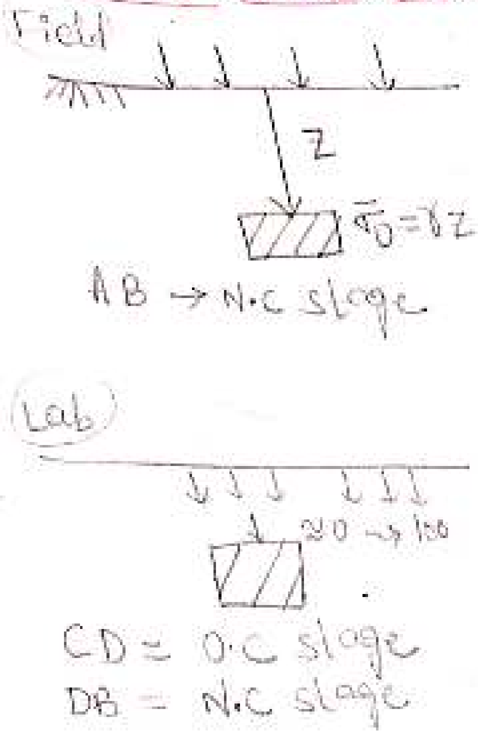
O.C.R > 1   O.C stage
O.C.R = 1   N.C stage
O.C.R < 1   under ~~over~~ consolidated stage

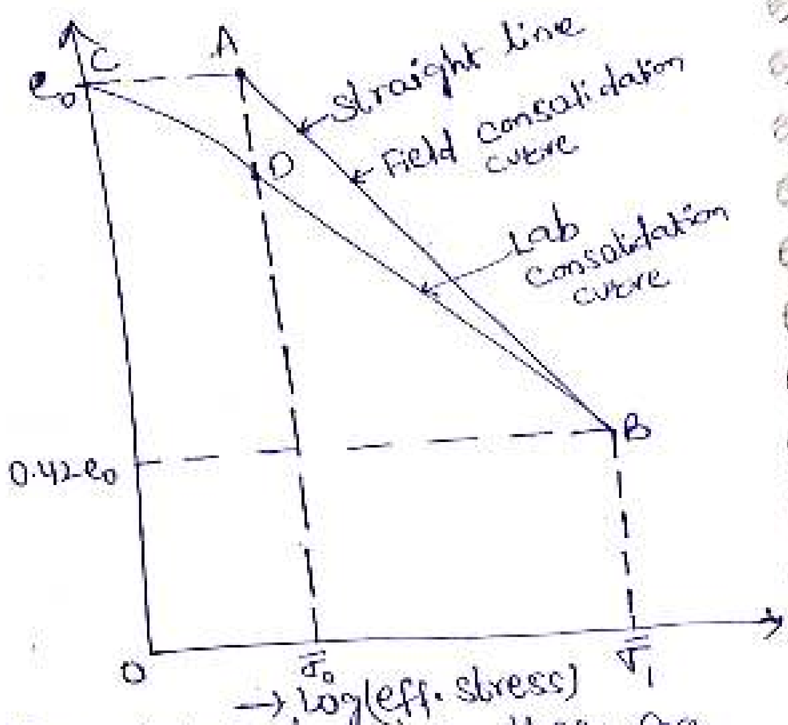
e.g. under construction building

Note :-
Melting of glaciers, removal of construction load, rising of water table and desication of clay are some example of over consolidation.

drought

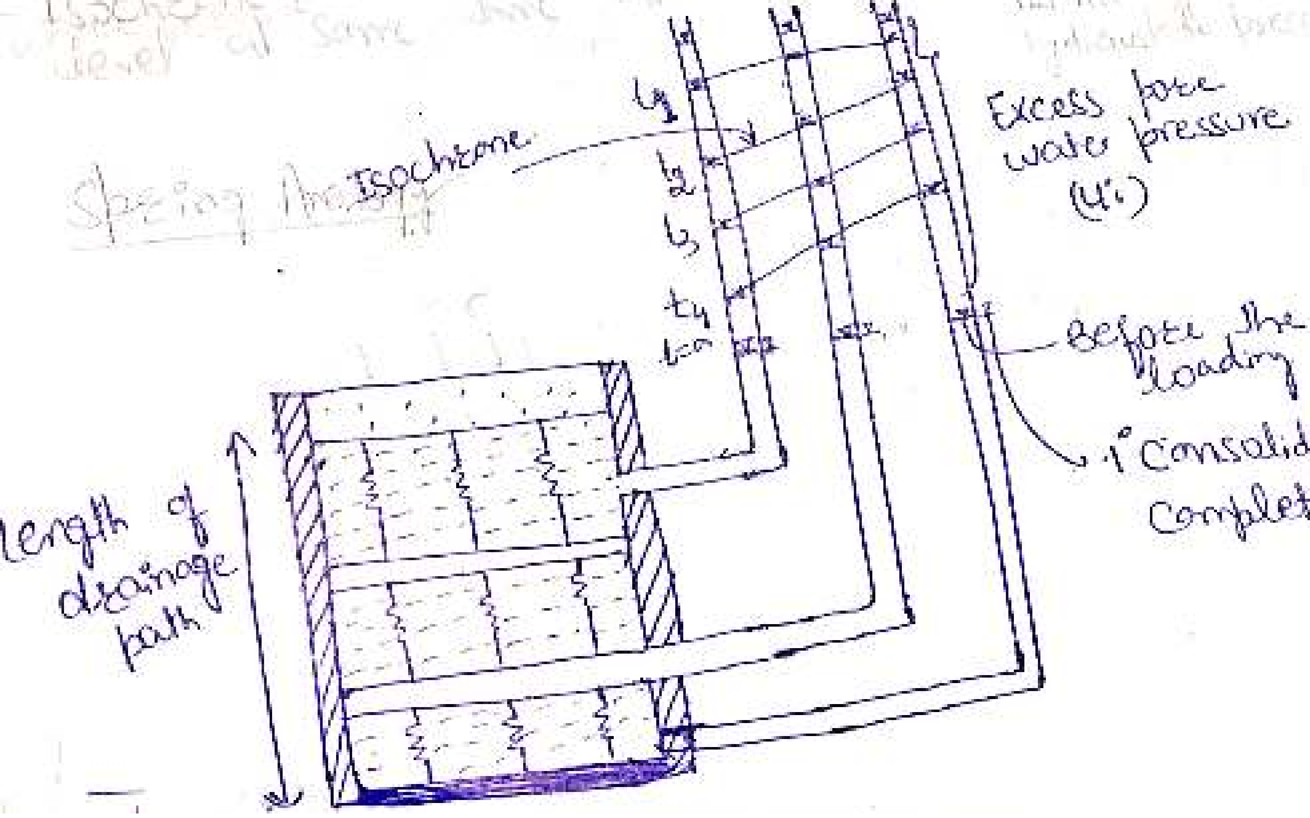
(5) Field Consolidation Curve :-





- Samples are slightly disturbed when these are taken out from the ground.
- The disturbance causes a slight decrease in the slope of lab consolidation curve.

Terzaghi's one dimensional Consolidation Theory :-



(1.) Application of total stress ($\Delta\sigma$)

(2.) At $t = 0$ [Excess pore water pressure will develop]

$$\Delta\sigma = \cancel{\Delta\bar{\sigma}}^{\,0} + u_i$$

(3.) At time 't' [Excess pore pressure decreases, eff. stress increases and water expelled out]

$$\Delta\sigma = \Delta\bar{\sigma}\uparrow + u_i\downarrow$$

(4.) At $t = \infty$ [Excess pore pressure reduces to zero, 1° consolidation completed]

$$\Delta\sigma = \Delta\bar{\sigma} + \cancel{u_i}^{\,0}$$

Assumptions of Terazghi Theory :-

(i.) Soil is homogenous and isotropic.

(ii.) Darcy's Law is valid.

(iii.) The consolidation is 1 dimensional it means flow of expelling water is unidirectional (in vertical) and there is no change in area (volume change is due to change in depth only).

$$\frac{\Delta V}{V_0} = \frac{\Delta H}{H_0} \cdot \frac{\cancel{A}}{\cancel{A}}$$

$$\boxed{\frac{\Delta V}{V_0} = \frac{\Delta H}{H_0}}$$

(iv.) Soil is fully saturated and remains saturated during entire process of consolidation it means initial compression is not considered.

(v.) Strain coming on soil mass is negligible or small it means soil solids are incompressible hence secondary consolidation is not considered.

(vi.) Hydrodynamic lag is considered whereas plastic lag is ignored however plastic lag is found to be exist. It means time required for plastic readjustment is ignored it means Secondary (2°) consolidation is not considered.

(vii.) Coefficient of consolidation and coefficient of compressibility are constant.

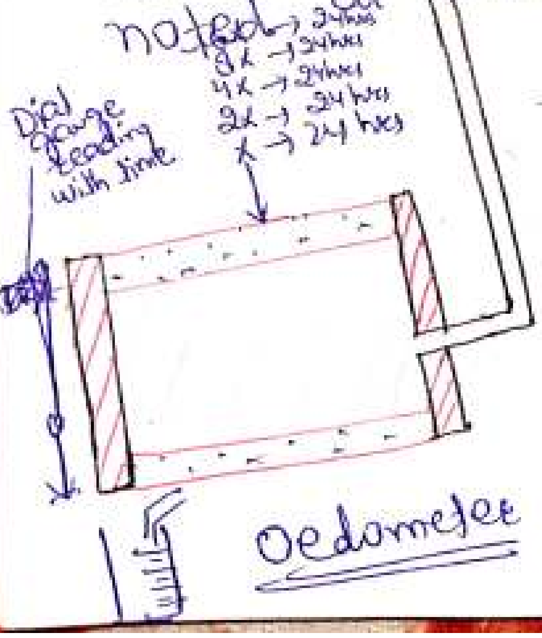
- In order to develop field condition in the Laboratory Consolidation test is being performed in Consolidometer or Oedometer.

There are two types of consolidometer:-

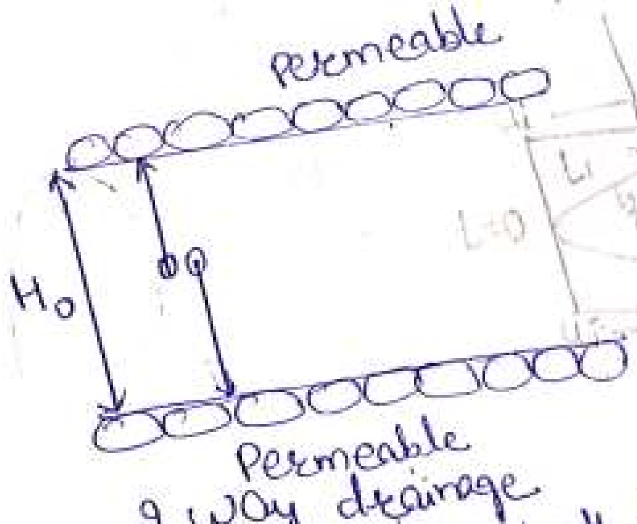
i) Fixed ring cell (only top plate is allowed to move).

ii) Floating ring cell (both plates are allowed to move).

In consolidation test, pressure on the specimen in each stage is doubled (so that significant change in void ratio can be obtained in soil type) and settlement of given sample of soil is noted at different time.

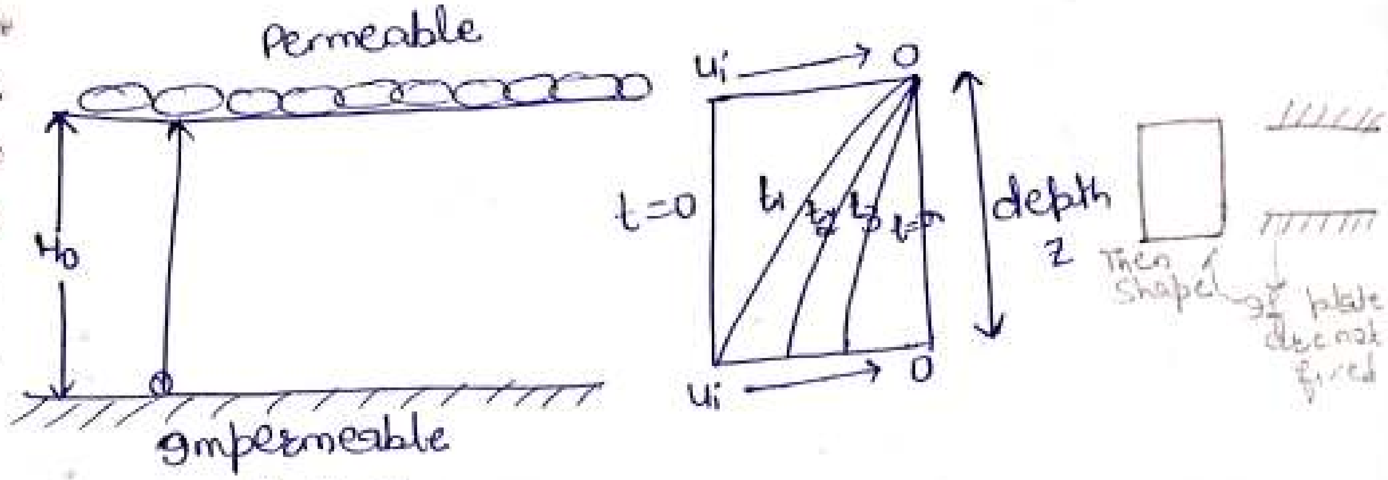


Oedometer



2 way drainage

Length of drainage path $d = \frac{H_0}{2}$



1 way drainage

Length of drainage path $\boxed{d = H_0}$

Terazghi's 1-D Consolidation Equation:-

**

$$\boxed{\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}}$$

where $\frac{\partial u}{\partial t}$ = Rate of change of excess pore H_2O pressure, it also represents the rate of consolidation.

C_v = coefficient of consolidation

$k = C_v \cdot m_v \cdot \gamma_w$

**

$$\boxed{C_v = \frac{k}{m_v \cdot \gamma_w}}$$

$k \rightarrow$ coefficient of permeability

$m_v \rightarrow$ coefficient of volume compressibility or modulus of volume change

$$m_v = \frac{-\frac{\Delta V}{V_0}}{\Delta \sigma}$$

$$m_v = \frac{-\frac{\Delta e}{1+e_0}}{\Delta \sigma} = \left(-\frac{\Delta e}{\Delta \sigma}\right) \times \frac{1}{1+e_0} = \frac{a_v}{1+e_0}$$

$$\boxed{\therefore m_v = \frac{a_v}{1+e_0}}$$

Where $a_v \rightarrow$ Coefficient of compressibility $= \left|\frac{\Delta e}{\Delta \sigma}\right|$
$e_0 \rightarrow$ Initial void ratio

Note :-

	Coefficient of	Symbol	formula	unit
1.	Compressibility	a_v	$\frac{\Delta e}{\Delta \sigma}$	$\frac{m^2}{KN}$
2.	Compression	C_c	$\frac{\Delta e}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$	unitless
3.	Recompression	C_e	$\frac{\Delta e}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$	unitless
4.	Consolidation	C_v	$\frac{K}{m_v \cdot \gamma_\omega}$	$\frac{m/s}{\frac{m^2}{KN} \cdot \frac{KN}{m^3}} = \frac{m^2}{sec.}$
5.	Volume Compressibility	m_v	$\frac{a_v}{1+e_0}$	$\frac{m^2}{KN}$

Solutions of Terzaghi's Solution :-

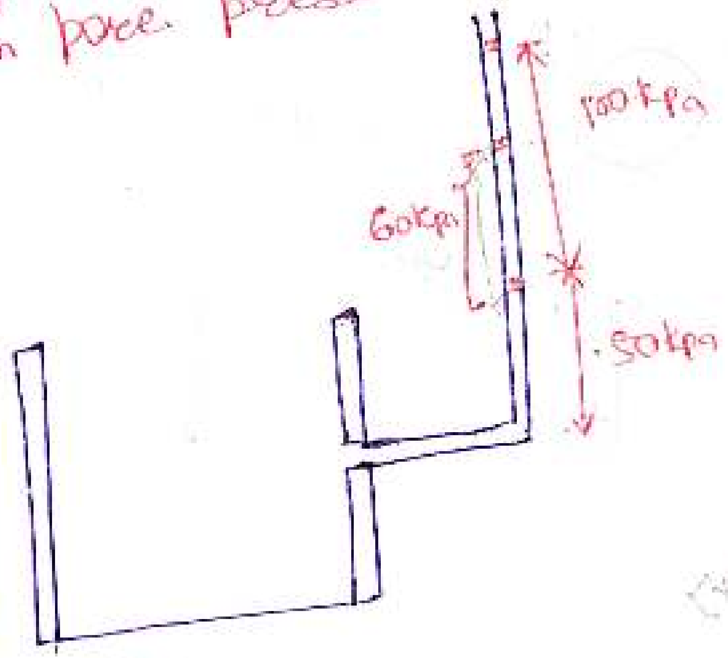
(1) Degree (Ratio) of Consolidation (%U) :-

- It is the fraction of ultimate consolidation completed at any time t during the consolidation.

- It is defined only for primary consolidation

	At $t=0$ ($\%U=0$)	At $t=\infty$ ($\%U=100\%$)	At any time 't'	
a.) when excess pore water pressure is known	u_i	0	u	$\%U = \frac{u_i - u}{u_i} \times 100$
b) when settlement is known	0	ΔH	Δh	$\%U = \frac{\Delta h}{\Delta H} \times 100$
c.) when void ratio is known	e_0	e_{100}	e	$\%U = \frac{e_0 - e}{e_0 - e_{100}} \times 100$

Q:-
Hydrostatic pressure = 50 kpa
Loading 100 kpa
After 1 year %U = 40%
then pore pressure after 1 year



Ans. 110 kpa

2) Time factor :-

→ It is the parameter which relates to the degree of consolidation and time required for that consolidation.

$$\boxed{T_v = C_v \cdot \frac{t_{(time)}}{d^2}}$$

where C_v – coefficient of consolidation

$$C_v = \frac{k}{m_v \cdot \gamma_w} = \frac{k}{\frac{a_v}{1+e_0} \cdot \gamma_w} = \frac{k}{\frac{\Delta e}{\Delta \sigma} \cdot \left(\frac{\gamma_w}{1+e_0}\right)} = \frac{k \Delta \sigma}{\frac{\Delta e}{1+e_0} \cdot \gamma_w}$$

d → length of drainage path

for 2 way drainage $\boxed{d = \frac{H_0}{2}}$

for 1 way drainage $\boxed{d = H_0}$

t → time at which time factor is required.

T_v = Time Factor

$T_v = \frac{\pi}{4} U^2 = \frac{\pi}{4}\left(\frac{\%U}{100}\right)^2$ $\%U \le 60\%$

$T_v = 1.781 - 0.9332 \log_{10}(100 - \%U)$ $\%U > 60\%$

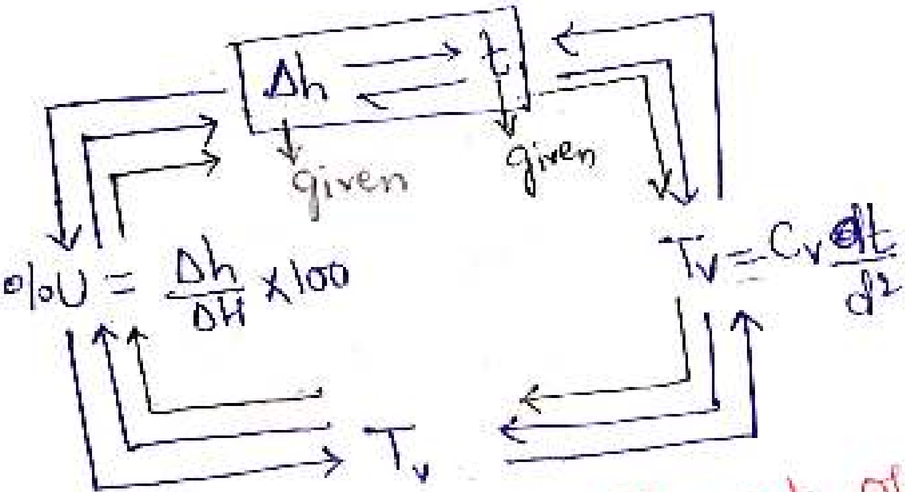
%U	50%	90%	60%	100%
T_v	0.196	0.848	0.283	∞

Theoretically for 100% consolidation infinite time is required but primary consolidation is assumed to be completed when degree of consolidation at least ≥ 90%.

Soil properties change $\Rightarrow$ C_v change
Drainage path change $\Rightarrow$ d change
% U change $\Rightarrow$ T_v change.

Rule :-

जब भी Settlement हो, time हो
then T_v & cycle.



Determination of coefficient of consolidation (C_v) :-

- It depends upon type of soil and loading condition.
- Generally it had been observed that liquid limit increases coefficient of consolidation decreases.

$w_L \uparrow \rightarrow$ Compressibility $\uparrow \rightarrow a_v \uparrow \rightarrow m_v \uparrow \rightarrow C_v \downarrow$

$\therefore C_v = \frac{k}{m_v \gamma_w}$

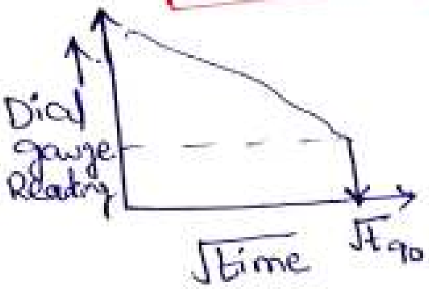
$\uparrow C_c = 0.009 (w_L \uparrow - 10)$

There are two methods to determine C_v :-

(A) Square Root of time fitting method :-

Given by Taylor,

$\boxed{\%U = 90\%}$



$$(T_v)_{90} = C_v \frac{t}{d^2}$$

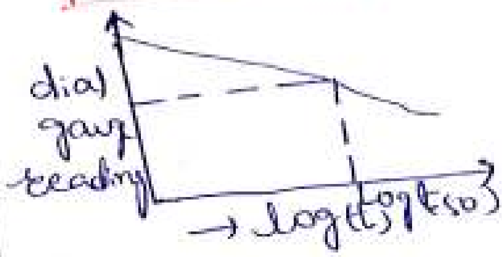
$$0.848 = C_v \cdot \frac{t_{90}}{d^2}$$

$$\boxed{C_v = \frac{0.848 \times d^2}{t_{90}}}$$

(B) Logarithmic of time fitting method :-

Given by Casagrande,

$\boxed{\%U = 50\%}$



$$(T_v)_{50} = C_v \frac{t}{d^2}$$

$$0.196 = C_v \cdot \frac{t_{50}}{d^2}$$

$$\boxed{C_v = \frac{0.196 \times d^2}{t_{50}}}$$

Square-Root of time fitting method is better.

Settlement Analysis :-

- The total settlement is the sum of initial / immediate settlement, primary consolidation settlement and secondary consolidation settlement.

$$\boxed{S = S_i + \underset{1^\circ}{S_c} + \underset{2^\circ}{S_s}}$$

→ I.S CODE GUIDELINES :-

I Total Permissible settlement	for steel structure	for R.C.C structure
for isolated footing on sand	50mm	50 mm
for isolated footing on clay	50	75 mm
for raft footing on sand	75	75
for raft footing on clay	100	100

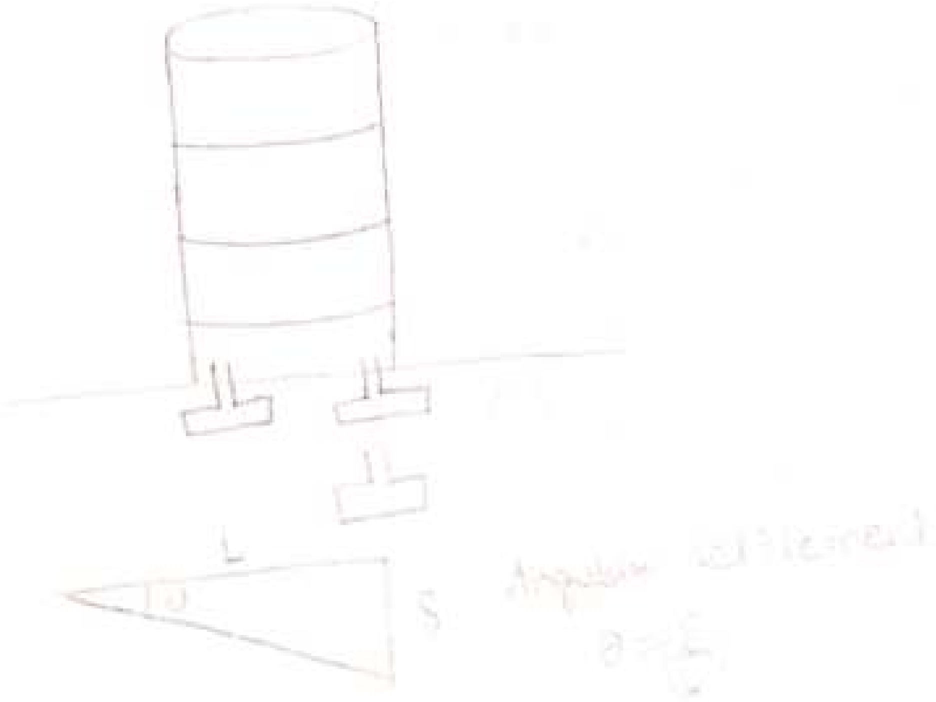
II. Permissible Angular settlement

	For steel structure	For RCC structure
For isolated footing	$\frac{1}{300}$	$\frac{1}{666}$
For Raft footing	$\frac{1}{300}$	$\frac{1}{500}$

III. Permissible differential settlement (δ) :-

$$\boxed{\delta = \text{Angular settlement} \times L}$$

$L \rightarrow$ Distance b/w two footings.



Determination of initial / Immediate Settlement

- Cone penetration test data are used to determine the initial settlement due to expulsion of pore air.

$$S_i = \frac{H_0 \bar{\sigma}_0}{1.5 q_c} \log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

q_c = Cone penetration resistance value

H_0 = Thickness of compressible layer

$\bar{\sigma}_0$ = Initial Effective stress at the centre of compressible layer

$\Delta\bar{\sigma}$ = Change in effective stress at the centre of compressible layer.

Elastic Settlement :-

- A Small elastic settlement occurs due to squeezing of particles.

$$\boxed{S_i = \frac{qB(1-\mu^2)}{E_s} \cdot I_t}$$

Where

q = Uniform pressure applied on the footing

B = width of footing (least lateral Dimension)

μ = Poissons ratio of soil (0.35 to 0.5)

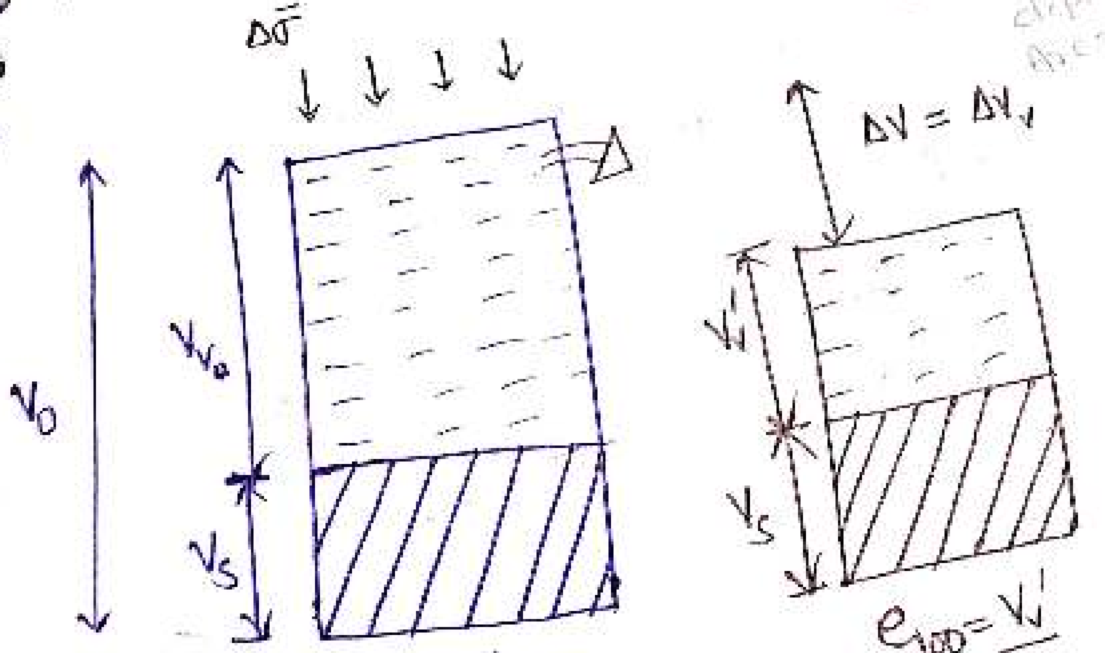
E_s = Young's Modulus of soil.

I_t = Influence factor or shear factor which depends upon L/B Ratio.

	Flexible footing			Rigid footing
	Centre	Corner	Avg	
for circular footing	1.00	0.64	0.85	0.80
for square footing	1.12	0.56	0.95	0.82
for Rectangular				
$\frac{L}{B} = 1.5$	1.36	0.68	1.20	1.09
$\frac{L}{B} = 2$	1.52	0.76	1.31	1.22

Determination of primary consolidation Settlement :-

A) when change in void Ratio is known



$e_0 = \frac{V_{v_0}}{V_s}$ $\qquad$ $e_{100} = \frac{V_v'}{V_s}$

$$\frac{\Delta V}{V_0} = \frac{\Delta H \cdot A}{H_0 \cdot A} = \frac{\Delta H}{H_0} \quad \text{—(i)} \quad \left\{ \begin{array}{l} \text{Terzaghi's 1-D} \\ \text{consolidation} \end{array} \right\}$$

$$\frac{\Delta V}{V_0} = \frac{\Delta V_v}{V_s + V_{v0}} = \frac{\Delta V_v}{V_s\left[1 + \frac{V_{v0}}{V_s}\right]} = \frac{\left(\frac{\Delta V_v}{V_s}\right)}{1 + \frac{V_{v0}}{V_s}}$$

$$\frac{\Delta V}{V_0} = \frac{\Delta e}{1+e_0} \quad \text{—— (ii)}$$

$\because \frac{\Delta V_v}{V_s} = \frac{V_{v0} - V_v'}{V_s} = e_0 - e_{100} = \Delta e$

$\because \frac{V_{v0}}{V_s} = e_0$

From eq(i) and eq(ii), w/g

$$\boxed{\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}}$$

$\Delta H \to$ ultimate consolidation settlement

$H_0 \to$ Initial Thickness

$e_0 \to$ Initial void ratio

$\Delta e \to e_0 - e_{100}$ = change in void ratio.

$$\Delta H = H_0 \cdot \frac{\Delta e}{1+e_0}$$

(B) **When coefficient of volume compressibility (m_v) is known:-**

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0} = \frac{\Delta e}{1+e_0} \times \frac{\Delta\bar{\sigma}}{\Delta\bar{\sigma}}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{\Delta\bar{\sigma}} \times \frac{\Delta\bar{\sigma}}{1+e_0} = \frac{a_v}{1+e_0} \times \Delta\bar{\sigma} = m_v \cdot \Delta\bar{\sigma}$$

$$\boxed{\Delta H = H_0 m_v \Delta\bar{\sigma}}$$ ** --> Used when loading given.

(C) **When coefficient of compression (C_c) is known:-**

$$\boxed{\bar{\sigma}_1 = \bar{\sigma}_0 + \Delta\bar{\sigma}}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0} = \frac{\Delta e}{1+e_0} \times \frac{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)} \times \frac{\log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)}{1+e_0}$$

$$\frac{\Delta H}{H_0} = \frac{C_c}{1+e_0} \log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)$$

$$\Delta H = H_0 \cdot \frac{C_c}{1+e_0} \log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right)$$

$$\boxed{\Delta H = \frac{H_0 C_c}{1+e_0} \log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$$

Note:- (1)

If soil is in N.C stage then use C_c.

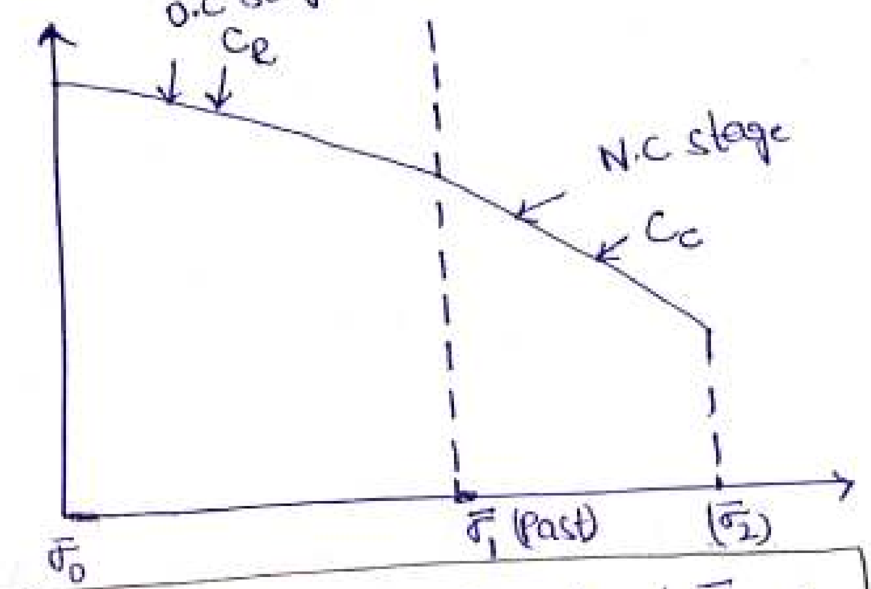
If soil is in OC stage then use C_e.

1.) Past ($\bar{\sigma}_1$)

2.) Present ($\bar{\sigma}_0$)

3) change ($\Delta\bar{\sigma}$)

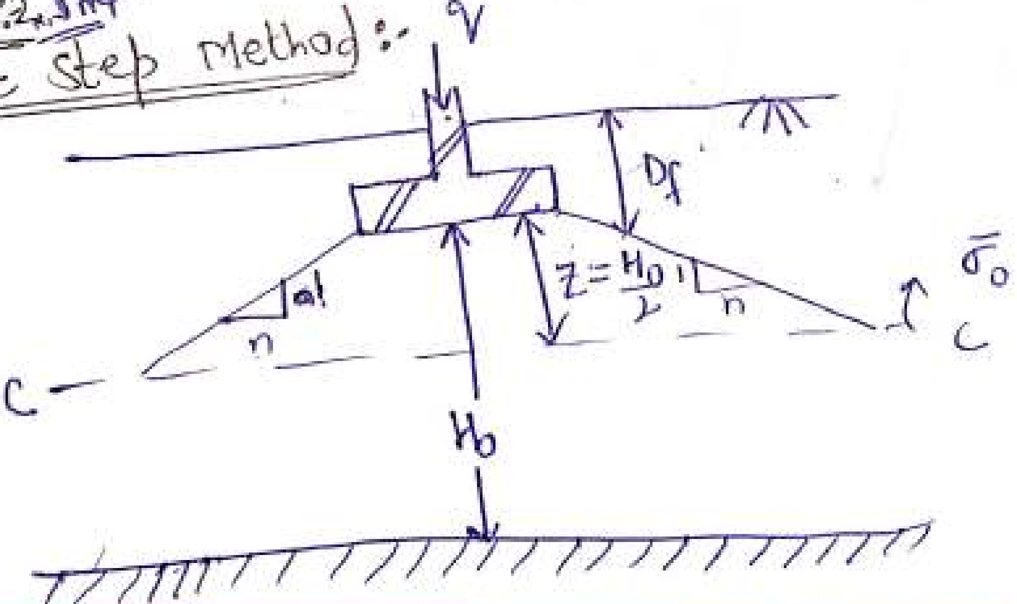
4) final ($\bar{\sigma}_2$)



$$\boxed{\Delta H = \frac{H_0 C_e}{1+e_0} \log\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0}\right) + \frac{H_0 C_c}{1+e_0} \log\left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1}\right)}$$

Note:- 2. Imp.

Four step Method :-



Step1 :- H_0 [Thickness of compressible layer]

Step2 :- $\bar{\sigma}_0$ at c/c of compressible layer

Step3 :- Load distribution in 1:n

$$\Delta\bar{\sigma} = \frac{\text{force}}{\text{Area}} = \frac{q(B \cdot L)}{(B+2nz)(L+2nz)} = \frac{Q}{(B+2nz)(L+2nz)}$$

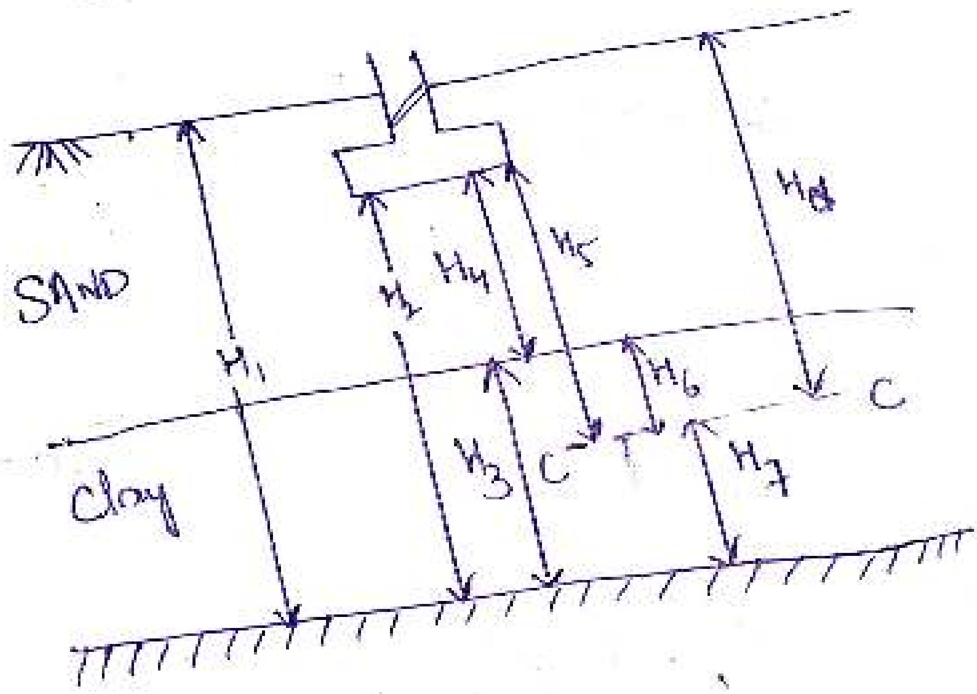
Note :-
If load distribution is not given then assume (2V : 1H)

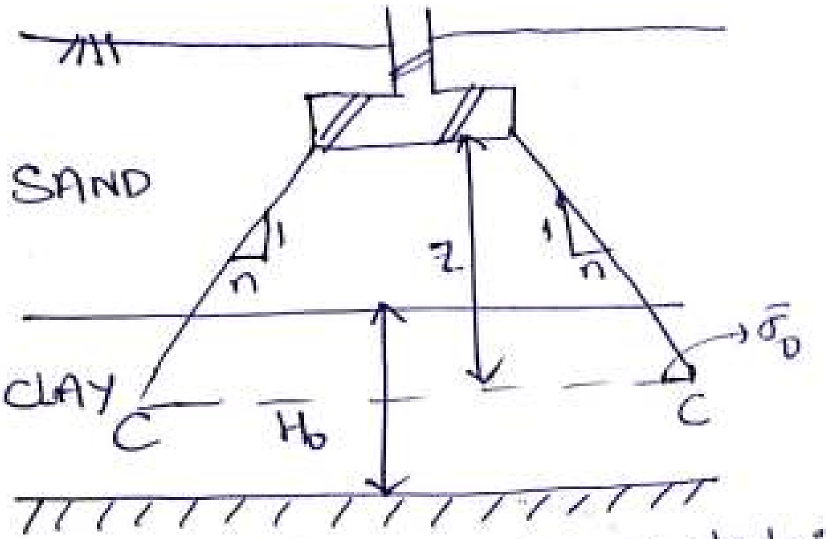
Step4 :- ultimate consolidation settlement.

$$\boxed{\Delta H = \frac{H_0 C_c}{1+e_0} \log\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$$

If hard strata is not given then assume 2B.

Notes:- Determine the settlement of clay layer :-





Note4 :- Surcharge will not distribute

$\Delta\bar{\sigma} = q$ (Surcharge)

Note5 :- Ultimate consolidation settlement depends Upon

Rate → L

- Change in effective stress ($\Delta\bar{\sigma}$)
- Thickness of compressible layer (H_0).
- Compressibility of soil (m_v, C_c, a_v)

Note6 :-

Consolidation settlement will remain same in two-way or one-way drainage condition (if soil and loading condition are same).

Determination of secondary consolidation :-

- It occurs due to plastic readjustment of soil solids in highly plastic clays and in organic clays. It is a time depending process.

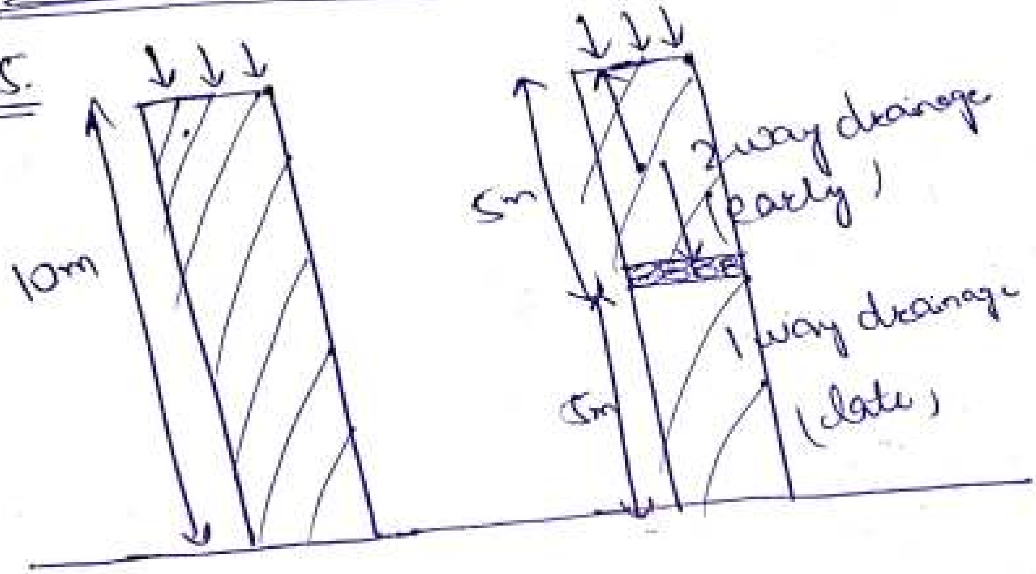
$$S_s = \frac{C_s H_{100}}{1 + e_{100}} \log \frac{t}{t_{100}}$$

H_{100} = thickness of compressible layer after 1° consolidation.

e_{100} = void ratio of soil after 1° consolidation
t_{100} = time req. by soil for 1° consolidation ($\simeq t_{90}$)
t = time at which 2° consolidation is required.
C_s = Secondary compression Index, obtained from lab after 1° consolidation by taking slope of e v/s log(time) curve.

Workbook

Q.15.



$$T_v = \frac{C_v t}{d^2} = \frac{C_v \cdot (36 \text{ months})}{(10)^2} \quad \text{—— (i)}$$

$$T_v = C_v \cdot \frac{(t_2)_y}{(d_{lower})^2} = C_v \cdot \frac{(t_2)_y}{(5)^2} \quad \text{—— (ii)}$$

From eqn (i) and eqn (ii), u/g

$$t_2 = \frac{36}{(10)^2} \times (5)^2 = 9 \text{ months}$$