8 | *Control Charts for Attributes*

Practical Limitations of the Control Chart for Variables

$\bar{X}$ and R charts are powerful devices for the diagnosis of quality problems and means for routine detection of sources of trouble. But their use is limited to only a small fraction of quality characteristics, specified for manufactured product. The limitations of $\bar{X}$ and R charts are as discussed below.

1. $\bar{X}$ and R charts can be used for quality characteristics that can be measured and expressed in numbers. However, many quality characteristics can be observed only as attributes, *i.e.*, by classifying each item inspected into one of the two classes, either conforming or non-conforming to the specifications. For example, while inspecting the castings in addition to conformity to dimensions, it may be necessary to inspect other quality characteristics such as blow holes, cracks, swells, undercuts, finish, etc. each of which singly or in combination may make the casting defective. This type of data can be collected only on the basis of number of products that conforms to the specifications and the number of products failing to conform to the specifications.

2. Furthermore, $\bar{X}$ and R charts can be used only for one measurable characteristic at a time. For example, a firm may be producing a part or an assembly involving 50 different dimensions (or quality characteristics). For each dimension a separate $\bar{X}$ and R chart is necessary, however, it will be impracticable and uneconomical to have 50 such charts and hence the manufacturer may prefer to analyze the results in terms of defective or non-defective items.

3. For reason of economy even in some cases, where the direct measurement of variable quality characteristics is possible, it is common practice to classify them as good or bad on the basis of inspection by Go-No-Go gauges. In such cases $\bar{X}$ and R charts may be plotted for the most important and troublesome quality characteristic.

No dimension should be chosen for $\bar{X}$ and R chart unless there is an opportunity to save cost from reduction of spoilage, rework etc. The quality improvement resulted from the $\bar{X}$ and R chart together with the opportunity to save cost should compensate the cost of taking the measurement, keeping the charts and analyzing them.

As an alternative to $\bar{X}$ and R charts, and as a substitute when characteristic is measured only by attribute a control chart based on fraction defective P is used. (P-chart)

Fraction defective, P' may be defined as the ratio of the number of defective articles found in any inspection to the total number of articles actually inspected. Fraction defective is always expressed as a decimal fraction.

Percent defective (100 P'), *i.e.* 100 times the fraction defective. For actual calculation of the control limits fraction defective is used, however, for charting and for general presentation of results to shop personnel and management generally percent defective is used.

Comparison of $\bar{X}$ and R chart with P chart :

1. P chart is an attribute control chart, *i.e.* for quality characteristic that can be classified as either conforming to the specifications. For example, dimensions checked by Go-No.Go gauges. Whereas, $\bar{X}$ and R chart is used for quality characteristic that can be measured and expressed in numbers.

2. The cost of collecting the data for P chart is less than the cost of collecting the data for $\bar{X}$ and R chart. For example 10 shafts might be inspected with 'go-no-go'' gauge in the time required to measure a single shaft diameter with a micrometer. Secondly, P chart uses data already collected for other purpose.

3. The cost of computing and charting may also be less since P chart can be applied to any number of quality characteristics observed on one article. But separate $\bar{X}$ and R chart is required for each measured quality characteristic, which may be impracticable and uneconomical.

4. P-chart is best suited in cases where inspection is carried out with a view to classifying an article as accepted or rejected. $\bar{X}$ and R chart are best suited for critical dimensions.

5. P-chart though discloses the presence of assignable causes of variations, it is not as sensitive as $\bar{X}$ and R chart. For actual diagnosis of causes of troubles, $\bar{X}$ and R charts are best, still P chart can be used effectively in the improvement of quality.

6. The sample size is generally larger for P chart than for $\bar{X}$ and R chart. The variations in the sample size influences the control limits much more in $\bar{X}$ and R charts than in P chart.

7. The control chart for fractions defective provides management with a useful record of quality history.

Control limits (3σ limits) on P chart.

The day's production (or other lot) of any manufactured article or part can be thought of as a sample from a larger quantity with some unknown fraction defective. This unknown universe fraction defective depends upon a complex set of causes influencing the production and inspection operations. As a matter of chance the fraction defective in the sample may vary considerably. As long as the universe fraction defective remains unchanged, the relative frequencies of various sample fractions defectives may be expected to follow the binominal law. This is the basis of establish 3-sigma limits on control charts for p.

For binomial distribution the mean value of the total number of defectives in a sample of n is np' and the standard deviation is $\sqrt{n\,p'\,q'}$ or $\sqrt{n\,p'\,(1-p')}$

as already explained in chapter 4. If the fraction defective is defined as the ratio of the number of defectives to the total number of items in the sample n, then the mean value of the fraction defective is 'p' and the standard deviation

$\sigma_p' = \sqrt{\dfrac{p'(1-p')}{n}}$. The limits should be placed far enough from the expected average value so that a point outside the limits indicates that the universe has changed, or that a very unlikely event has happened. Industrial practice in use of the p chart generally bases control limits on 3-sigma.

Therefore, the control limits for p chart are defined as :

$$UCL_p = p' + 3\sigma_p' = p' + 3\sqrt{\frac{p'(1-p')}{n}}$$

$$LCL_p = p' - 3\sqrt{\frac{p'(1-p')}{n}}$$

Purpose of the p chart. Because of the lower inspection and maintenance costs of p charts, they usually have a greater area of economical applications than do the control charts for variables. A control chart for fraction defective may have any one or all of the following purposes :

1. To discover the average proportion of defective articles submitted for inspection, over a period of time.

2. To bring to the attention of the management, any changes in average quality level.

3. To discover, identify and correct causes of bad quality.

4. To discover, identify and correct the erratic causes of quality improvement.

5. To suggest where it is necessary to use $\bar{X}$ and R charts to diagnose quality problems.

6. In a sampling inspection of large lots of purchased articles.

Selection of quality characteristic. Different defects have unequal influence on costs. Some may be corrected by simple inexpensive rework. While others may need costly rework, or even scrapping. Hence it may be economical to concentrate attention by means of separate control charts on these defects which are responsible for greater costs, (i.e. for critical or major defects).

Sample Size. In case of p charts, the sample size must be fairly large, so that the normal distribution will approximate to a sufficient degree. Some authorities say that, if p is small, n should be large enough that there is a good chance of obtaining defective units in the sample, otherwise the occurrence of one defective would through the chart out of control. To illustrate, suppose the standard is set at $100p' = 0.50$. The sample size n required to yield an average of just one defective per sample would be 200, that is,

$$n = \frac{np'}{p'} = \frac{1.0}{0.005} = 200.$$

Therefore, sample size chosen should be such as to contain at least one defective.

A rule of thumb followed in most industrial applications is that the sample size should not be less than 100. If the sample size is too large, an accuracy greater than necessary will be achieved at an excessive cost. If the sample size is too small, inadequate or unreliable information may result. One writer has suggested that the sample size should be large enough to get one or more defective items per sample at least 90 out of 100 times.

Frequency of sampling. With regard to the frequency of sampling, judgment must be used. If the time span between samples is great, a shift in the population proportion may go undetected for some time and the cost of this should be considered. On the other hand, frequent sampling will increase the cost of sampling and maintaining the control chart. In general, if control problems are being encountered at a particular work station, the analyst will take more frequent samples than he will at a trouble free station.

Selection of subgroups. In all control charts, subgroups selected should be such that, the chances of variation within the subgroup should be minimum. Similar to the $\bar{X}$ and R charts, in the control chart for fraction defection the most natural basis for selecting subgroup is the order in which production takes place.

Generally, subgroups selected should consist of items produced in a day or the products produced in one production order. Sometime a control chart showing daily per cent defective may be supplemented by charts showing weekly and monthly figures. The daily chart may be used as a basis for current action on the manufacturing process by production supervisors, methods analyst, and operators : the weekly chart may be used by manufacturing executives such as departmental heads ; the monthly chart may be used in quality reports to top management.

Where the subgroup consists of daily or weekly production, the subgroup size is almost certain to vary. In this case three possible solutions to the problem are :

1. Compute control limits for every subgroup and show these fluctuating limits on the p chart.

2. Estimate the average subgroup size, and compute one set of limits for this average and draw them on control chart. This method is approximate and is appropriate only when the subgroup sizes are not too variable. Points near the limits may have to be re-examined in accordance with (1).

3. Draw several sets of control limits on the chart corresponding to different subgroup sizes. This method is also approximate and is actually across between (1) and (2), again points falling near the limits should be re-examined in accordance with (1).

Choice between 'p' chart and 'np' chart

Whenever subgroup size is variable control chart for fraction defective (p chart) is used. However, if subgroup size is contant the chart for actual number of defectives, known as np chart is used. When subgroup size is constant, the np

chart is preferred over p chart for the following reasons : (1) np chart saves one calculation for each subgroup, the division of number of defectives by sub-group size to get fraction defective p. (2) Some people may understand the np chart more readily. However, to avoid confusion it would be better to use p chart even for constant subgroup.

Control limits for np chart. The average fraction defective $\bar{p}$ is used as the best available estimate of p' :

$$\bar{p} = \frac{\Sigma np}{\Sigma n}$$

$$UCL_{(np)} = n\bar{p} + 3\sigma_{np} = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{(np)} = n\bar{p} - 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

Essential steps in the control chart. The steps in setting the control chart for fraction defective are as follows :

1. Record the data for each subgroups on number inspected and number of defectives. Any occurrence that might be clues to an explanation of points out of control or to changes in the quality level should be noted on the data sheet as supplementary remarks.

2. Compute p (fraction defective) for each subgroup.

$$p = \frac{\text{Number of defectives in subgroup}}{\text{Number inspected in subgroup}} = \frac{np}{n}$$

3. Compute $\bar{p}$, the average fraction defective.

$$\bar{p} = \frac{\text{Total number of defectives during a period}}{\text{Total number inspected during period}}$$

whenever practicable, it is necessary to have data for at least 25 subgroups before computing $\bar{p}$ and establishing trial control limits.

4. Compute trial control limits for each subgroup. These are :

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Central line $= \bar{p}$

5. Plot each point as obtained, plot trial control limits as soon as calculated. Points which are falling outside control limits are identified, (if any).

In case of $\bar{X}$ and R chart it may not be necessary to draw lines connecting the points which represent the successive subgroups. But in case of the p chart a line connecting the points is usually helpful in interpretation of the chart. Such a line assists in the interpretation of trends.

Continuing the control chart. The p chart is used, not only for finding out the presence or absence of assignable causes of variation but also for judging whether the quality level is maintained at same desired fraction defective. If some points fall outside the control limits or if there are extreme runs above or below the central line, there may be two reasons :

1. The assignable causes of variation may be present.

2. The quality level is different from the assumed standard p'.

If there are no extreme runs or no points outside the control limits, no assignable cause are present. The value of p' may be then selected as a standard average fraction defective for future control.

In most of the cases, the control chart for the preliminary period will show a condition somewhat between the two extremes of perfect statistical control and complete absence of control. That is, there will be few points outside the control limits. In such cases, the points outside the control limits are eliminated and the value of p and control limits are re-calculated. At this stage, if all points are within control limits, the new value of $\bar{p}$ is taken as the standard fraction defective for future control.

Low spots on the control chart, (i.e., points below the lower control limit) call for different kind of attention from that given to the points above the upper control limits (high spots). They sometimes point to faulty inspection and may indicate the necessity of providing better inspection standards or securing better inspectors. It may be worth examining to find out why quality for one subgroup was so much better than the standard ; this will help to bring about more permanent quality improvement.

Periodic Review and Revision of p'. The standard fraction defective p' should be reviewed from time to time. Where many p charts are being used periodical review should be carried out to revise the standard fraction defective ; at some fixed regular interval. For example, if a sub-group consists of one day's production, it may be satisfactory to have a 2 months' review period, with half the charts reviewed each month. Where sub-group consists of production orders or of lots submitted for inspection, then the periodical review may be carried out once every 20 sub-groups, or once every 40 sub-group etc.

Whenever there is an evidence of decrease in average per cent defective it may be because of improvement in the quality rather than relaxed inspection. In such cases the fraction defection p' may be revised downwards, to have a better quality level. On the other hand, the quality control engineer should be reluctant to revise p' upward. The value of p' should not be increased merely on the basis of a poorer quality level that seems to have resulted from reduced attention to quality on the part of production personnel.

Sensitivity of the chart. When the sample size is small, the fraction defective does not give clear indication about the quality of the product from which

a sample is taken. The larger the sample size, the more closely it is expected to give correct information about the quality level of the product. Consider for example, samples of various sizes drawn from product that is 2% defective. A sample of 5 will generally be either 0 or 20% defective (since it may contain either 0 or 1 defective). A sample of 20 will generally be from 0 to 10% defective. A sample of 100 will generally be 0 to 6% defective. A sample of 2,000 will generally be from 1.06 to 2.54% defective. This ''generally' applies to 3-sigma limits.

For this reason smaller the subgroup size, the less sensitive is the p chart to changes in quality level and less satisfactory indicator of assignable causes of variation. The higher the fraction defective (up to 50%), the better the results obtainable from small samples as indicator of lack of control.

Control Charts for Defects

Difference between a defect and defective. An item is said to be defective if it fails to conform to the specifications in any of the characteristics. Each characteristics that does not meet the specifications is a defect. An item is defective if it contains at less one defect. For example, if a casting contains undesirable hard spots, blow holes etc., the casting is defective and the hard spots, blow holes etc. which makes the casting defective are the defects.

The np chart, applies to the number of defectives in subgroups of constant size. Whereas C chart applies to the number of defects in a subgroup of constant size. In most of the cases, each subgroup for C chart consists of a single article and the variable C consists of the number of defects observed in one article. However, it is not necessary that the subgroup for the C chart be a single article, it is essential only that the subgroup size be constant in the sense that the different subgroups have substantially equal opportunity for the occurrence of defects.

Basis for Control Limits on C chart. Control limits on C chart are based on Poisson distribution. Therefore, two conditions must be satisfied. The first condition specifies that the area of opportunity for occurrence of defects should be fairly constant from period to period. The expression may be in terms of defects per unit being employed. For example, while inspecting the imperfections of a cloth it is necessary to take some units area say 100 square metres and count the number of imperfections per unit (*i.e.* per 100 square metres). Another example, may be number of paint imperfections per square metre area of painted surface. However, C chart need not be restricted to a single type of defect ; but may be applicable for the total of many different kinds of defects observed on any unit.

Second condition specifies that opportunities for defects are large, while the chances of a defect occurring in any one spot are small. For example, consider a case in which the product is a large unit, say a radio, which can have defects at number of points although any one point has only few defects.

Area of Application. The control chart for defects generally called C chart, has much more restricted field of usefulness as compared to $\bar{X}$ and R charts and

p charts. However, there are certain manufacturing and inspection situations in which the C chart is definitely needed. Some representative types of defects to which C chart may be applied are as follows :

1. Number of surface defects, in a role of coated paper or a sheet of photographic film etc.

2. Number of defective rivets in an aircraft wing.

3. Number of surface defects observed in a galvanized sheet or a painted, plated or enameled surface of a given area.

4. Number of breakdowns at a weak spots in insulation in a given length of insulated wire subjected to specified test voltage.

5. Number of small air holes in glass bottles.

6. Number of imperfections observed in a cloth of unit area.

7. Number of defects such as blow holes, cracks, undercuts etc in a casting or a welded piece.

8. Total number of defects of all types in complex assemblies such as tractor sub-assemblies, radio receiving sets, sewing machines etc.

In industry, no product can be absolutely perfect and flaws are bound to occur, though rarely. Even though a complex product possesses few defects, it may serve the function for which it is meant. It may have minimum number of defects which can be tolerated. Under conventional per cent defective system, charts would almost show 100 per cent rejection which could make them of little use for control of quality. In such situations our aim is to control and keep these rare occurrences of defects at a minimum possible.

The C-chart technique helps to keep these defects per unit at the lowest level.

Calculations of control limits on C chart. The standard deviation of the Poisson is

$$\sigma_c = \sqrt{np'} = \sqrt{C'}.$$

Thus, 3σ limits on a C chart are
$$UCL_C = C' + 3\sqrt{C'}$$
$$LCL_C = C' - 3\sqrt{C'}$$
and   centre line = $\bar{C}$

where, $\bar{C} = \dfrac{\text{Number of defects in all samples}}{\text{Total number of samples}}$

Since, the Poisson is not a symmetrical distribtion, the upper and lower 3-sigma limits do not correspond to equal probabilities of a point on the control chart falling outside the limits even though there has been no change in the universe. Because of this reason sometimes probability limits are used on C chart. Generally 0.995 and 0.005 probability limits are used in such case.

The position of limits corresponding either to these probabilities or any other desired probabilities may readily be determined from Table G, Appendix.

Slight departure of the actual distribution from the true Poisson usually will cause the standard deviation to be slightly greater than $\sqrt{C}$. Limits based on $3\sqrt{C}$ may really be at a little less than 3σ. This fact in itself generally does not justify discarding $3\sqrt{C}$ or $3\sqrt{C'}$ as a basis for calculating limits. In some situations to which C chart is applied, such as records of number of defects observed in inspection of complex assemblies, this use of limits tighter than 3 sigma may actually be desirable. Of course, the decision will be governed by the cost considerations.

Method of Application. A comparison is made between the number of defects in the sample and the average of the desired distribution of defects per sample, with the help of the control chart for defects.

The central line of C chart represents the desired average number of defects per sample $\bar{C}$. After constructing the control chart, the analyst will take periodic samples, count the number of defects a particular sample contains, and plots this number on the control chart.

If some points fall above the upper control limits, it means there is an increase in the average number of defects per sample. In such case, the factors of production would be examined with a view to finding and eliminating the assignable cause of variation. On the other other hand, in the event of suspected decrease in the average number of defects per sample, the factors of production would be examined with a view to finding the assignable causes of variation so that the causes could be incorporated as a permanent part of the production process, where this was done the control chart would have to be revised to reflect permanent change in the population.

Evaluating the level of control. At all times, management must decide whether the process is in control at a satisfactory level. The decision will be governed by cost criteria. Reducing the average defects will improve the quality, which has certain monetary advantages, but it may bring about an increase in the manufacturing costs. But if the resultant decision calls for a reduction in the average number of defects per sample, changes must be made in the production process to bring about the reduction. Once the possible chances have been made appropriate control chart would be established and the resultant average number of defects per sample now be considered to be normal.

Conditions favourable to the economic use of the control chart for defects. The C chart may be used to advantage in different types of situation, as follows :

1. It may be applied to a count of defects all of which must be eliminated following 100 per cent inspection. In this respect, it is used primarily for reducing cost of rework incident to correcting the defects.

2. The chart serves to keep management and producton supervisors informed about the quality level, and as a basis for executive pressure to improve the general

quality level and eliminate out of control points. Some times, it may suggest the lack of definite inspection standards. For example, inspection of sub-assemblies and assemblies of complex products.

3. Where certain number of defects per unit are tolerable, even though it is desired to hold their number to minimum, the C chart may be applied to periodic samples of production. In this case the chief objective is to maintain or improve the quality of outgoing products, leading to fewer rejections by customer's inspection and for better customer relations.

4. It is applied for special short studies of the variation of quality of a particular product or manufacturing operation.

5. It is used for sampling inspection of acceptance procedures based on defects per unit.

u Chart. When the subgroup size varies from sample to sample, it is necessary to use u charts. The control limits on u chart will however vary.

In other words, if C is the total number of defects found in any sample and n is the number of inspection units in a sample, we set up a control chart on which we would plot a quantity.

$$u' = \frac{C}{n} = \frac{\text{Number of defects in a sample}}{\text{Number of units in a sample}}$$

such a chart is called u-chart.

As already stated, limit lines on such chart will vary from sample to sample. The larger the number of units in a sample, the narrower the limits.

The formulas for control limits on u chart are :

$$UCL_u = u' + 3\sqrt{\frac{u'}{n}}$$

Central line $= u'$

$$LCL_u = u' - 3\sqrt{\frac{u'}{n}}.$$

Adaptation of the C chart in Quality Rating (Demerit of control chart). The product may contain number of different kinds of defects. For example, a casting may have number of cracks porosity, hard spots etc. For such product all types of defects are not of equal importance. Some defects are more serious than others. In such cases, it is advantageous to weigh these defects according to some scale that measures their seriousness.

According to their seriousness the defects can be classified as

Class A defects. Very serious (critical).

—These defects will render product totally unfit for service, makes the product useless, unsalable.

—Will cause operating failure of the product in series which cannot be readily corrected, e.g. open induction coil, transmitter without carbon, etc.

—Liable to cause personnel injury or damage to the property.

Class B defects. Serious (Major).

—These defects will probably, but not surely cause operating failure of the unit in service.

—Will surely cause adjustment failure, operation below standard etc. (But less serious than class A).

—Will surely cause increased maintenance or decreased life.

Class C defects. Moderately Serious.

—These defects will probably cause operating failure of the unit in service.

—Likely to cause trouble of nature less serious than operating failure.

—Likely to cause increased maintenance or decreased life.

—Major defects of appearance, finish or workmanship.

Class D defects. Not serious (Minor).

—These will not cause operating failure of the unit in service.

—It includes minor defects of appearance, finish or workmanship.

The next step is to assign weightage or demerits to each of class defects according to their relative seriousness. These will ordinarily be somewhat arbitrary, but are supposed to reflect the economic consequences of failure to correct each type of defect. The standard deviation is given by,

$$\sigma_{cw} = \sqrt{\sum_i^\gamma w_i^2 . C_i}$$

where ω_i is the weight assigned and C_i is the expected number of defects per sample and r is the number of classes of defects. One system prevalent in industries is to assign 10 points for class A, 5 points for class B, 2 points for class C and 1 point for class D defects.

With some prduct it may be required to have a single index of its prduct quality. Such a index may be obtained from use of demerit per unit data as follows :

$$\text{Quality Index} = \frac{\text{Observed demerits per unit}}{\text{Expected demerits per unit}}$$

Application of Defects per Unit Chart for Average number inspected in an Assembly Department

Where inspection will be conducted in the assembly line, a C chart is maintained at each section. It shows the number of units checked daily, and number of errors discovered by type. The charts are analyzed for concentration and inspection efforts are centred on the attributes that fail most frequently.

CONTROL CHARTS FOR ATTRIBUTES

The quality control department maintains trend chart for each for section. These charts are constructed by counting all possible errors per time period on equipments at each inspection station and dividing this quantity into the quality average for the same period. This gives an index of performances of the section. It makes possible to compare one foreman's quality effort to another's.

A report is sent up to top management each week. This report shows the quality level of each job and contains a listing of the production and inspection foreman in order of quality produced during the previous week.

The true value of the control chart, however, lies in its standard and decision limit for control of current quality performance. The base-period analysis is performed strictly for the purpose of estimating past performance so that achievable standard and natural decision limits may be established.

Comparison between Attribute Charts and Variable Charts

Choosing a particular type of chart is a question of balancing the cost of collecting and analyzing the type of data required to plot the chart against usefulness of the conclusions that can be drawn from the chart.

	Variable Charts	Attribute Charts
1.	Example : $\bar{X}$, R, σ charts.	P, np, C, u charts.
2.	Type of data required : variables data (Measured values of characteristics.)	Attribute data (using Go-No-Go gauges).
3.	Field of application Control of individual characteristics.	Control of proportion of defectives or number of defects or number of defects per unit.
4.	Advantages (a) Provides maximum utilization of information available from data. (b) Provides details information on process average and variation for control of individual dimensions.	(a) Data required are often already available from inspection records. (b) Easily understood by all persons. Since, it is more simple as compared to $\bar{X}$ and R chart. (c) It provides over all picture of quality history.
5.	Disadvantages (a) They are not easily understood unless training is provided. (b) Can cause confusion between control limits and specification limits. (c) Cannot be used with go-no-go type gauge inspection.	(a) They do not provide detailed information for control of individual characteristic. (b) They do not recognize different degree of defectiveness. (Weightage of defects).

Generally compromise is made, it is usual to start with p chart and only for those cases shown out of control on p chart, $\bar{X}$ and R chart are plotted for detailed analysis.

PROBLEMS AND SOLUTIONS

Problem 1. Following are the inspection results of magnets for nineteen observations.

Week No.	No. of magnets Inspected	No. of defective magnets	Fraction defective
1	724	48	0.066
2	763	83	0.109
3	748	70	0.094
4	748	85	0.114
5	724	45	0.062
6	727	56	0.077
7	726	48	0.066
8	719	67	0.093
9	759	37	0.049
10	745	52	0.070
11	736	47	0.064
12	739	50	0.068
13	723	47	0.065
14	748	57	0.076
15	770	51	0.066
16	756	71	0.094
17	719	53	0.074
18	757	34	0.045
19	760	29	0.038
Total	14,091	1,030	

Calculate the average fraction defective and 3 sigma control limits, construct the control chart and state whether the process is in statistical control.

The average sample size

$$= \frac{14091}{19} = 741.63 \approx 742 \text{ say}$$

The average fraction defectives

$$= \frac{\text{Total defectives in all samples}}{\text{Total inspected in all samples}}$$

$$= \frac{1030}{14091} = 0.0731.$$

$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$
$$= 0.0731 + 3\sqrt{0.0731(1-0.00731)/742}$$
$$= 0.0731 + 0.0287 = 0.1018.$$
$$LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$$
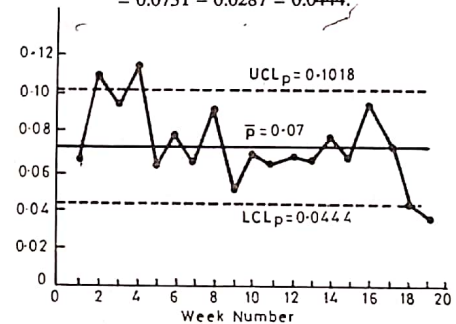$$= 0.0731 - 0.0287 = 0.0444.$$

Fig. 8.1. p Chart

Conclusion. From the resulting control chart, sample numbers 2nd and 4th goes above the upper control limits and the sample number 19th goes below the lower control limit. Therefore the process does not exhibit statistical control.

Problem 2. A certain product is given 100% inspection as it is manufactured and the resultant data are summarized by the hour. In the following table, 16 hours of data are recorded. Calculate the control limits using 3-sigma control limits and indicate the values that are out of control.

Hour	No. of units inspected	No. of defective units	Fraction defective
1	48	5	0.104
2	36	5	0.139
3	50	0	0.000
4	47	5	0.106
5	48	0	0.000
6	54	3	0.0555
7	50	0	0.000
8	42	1	0.0239
9	32	5	0.156
10	40	2	0.050
11	47	2	0.0425
12	47	4	0.085
13	46	1	0.0217
14	46	0	0.000
15	48	3	0.0625
16	39	0	0.000
Total	720	36	