UNIT-II (2nd Part)

6 Control Charts for Variables

The General Theory of Control Chart

A control chart is an important aid or statistical device used for the study and control of the repetitive processes. Control chart was developed by Dr. W.A. Shewhart and it is based upon the fact that variability does exist in all the repetitive processes.

Variability. In nature two extremely similar things are difficult to obtain. If at all we come across exactly similar things, it must be only by chance. This fact holds good for production processes as well. No production process is good enough to produce all items of products exactly alike. Most industrial and administrative situations involve a combination of materials, men and machines. Each of these elements of combination has some inherent or natural variability, the causes of which cannot be isolated plus the unnaturual variability or variability due to assignable causes which can be isolated and therefore controlled and reduced to economic minimum.

For example, suppose drilling operation is to be performed on castings. Now, what are the possible sources of variation ? First the material of which the casting is made will have some variability from unit to unit. Some units will be harder than others. If the operation is being done on a mass production by number of workers on different similar machines, the conditions of these machines may differ, some machines may be in poor condition, or improperly maintained. Hence, the second source of variation is the machine. The third source of variation, man, is the most variable of them all. His decisions and actions directly affect the extent of variability than the other sources, materials and machines. There may be differences among the skill of the workers doing the same job. The same person may act in different ways in different psychological conditions. His mental worries, diet, physical conditions and poor working environment also adds to the variability in the quality characteristics of the product.

Therefore, we conclude that there exist two kinds of variations :

1. Variation due to chance causes.
2. Variation due to assignable causes.

Variations due to assignable causes

These variations possess greater magnitude as compared to those due to chance causes and can be easily traced or detected. The power of the Shewhart control chart lies in its ability to separate out these assignable causes of quality variation (say in length, thickness, weight or diameter or a component). The variations due to assignable causes may be because of the following factors :

1. Differences among machines.

2. Differences among workers.
3. Differences among materials.
4. Differences in each of these factors over time.
5. Differences in their relationship to one another.

These variations may also be caused due to change in working conditions, mistake on the part of the operator, lack of quality mindness etc.

Chance Variations (Random Variations)

Variations due to chance causes are inevitable in any process or product. They are difficult to trace and difficult to control even under best conditions of production. Since these variations may be due to some inherent characteristic of the process or machine which functions at random. For example, a little play between nut and screw at random may lead to back-lash error and may cause a change in dimension of a machined part. The chance factors effect each component in a separate manner. It has been established that if the variations are due to chance factors alone, the observations will follow a 'normal curve'. Knowledge of the behaviour of chance variation is the foundation on which control chart analysis rests.

If after a random selection, observations are made under the same conditions and if the distribution of observation follows a standard curve (normal curve), then it is assumed that the variations are due to chance causes and no assignable causes of error are present. The conditions which produced these variations are accordingly said to be "under control". On the other hand, if the variations in the data do not conform to a pattern that might reasonably be produced by chance causes, then it is concluded that one or more assignable causes are at work. In this case conditions producing the variations are said to be "Out of control".

Definition of Control Chart

A control chart is a graphical representation of the collected information. The information may pertain to measured quality characteristics or judged quality characteristics of samples. It detects the variation in processing and warns if there is any departure from the specified tolerance limits.

In other words, control chart is a device which specifies the state of statistical control, second a device for attaining statistical control, and third, a device to judge whether statistical control has been attained. The control limits on the chart are so placed as to disclose the presence or absence of the assignable causes of quality variation. This makes possible the diagnosis and correction of many production troubles and often brings substantial improvements in product quality and reduction of spoilage and rework. Moreover, by identifying certain of the quality variations as inevitable chance variations, the control chart tells when to leave the process alone and thus prevents unnecessarily frequent adjustments that tend to increase the variability of the process rather than to decrease it.

With the help of a control chart it is possible to find out the natural capability of a production process, which permits better decisions on engineering tolerances and better comparisons between alternative designs and also between alternative production methods. Through improvement of conventional acceptance procedures, it often provides better quality assurance at lower inspection cost.

There are many types of control charts designed for different control situations, each with its own advantages and disadvantages and with its own field of application. However, all have a few characteristics in common and are interpreted in much the same manner. The control charts which are most commonly used are :

1. Control charts for measurable quality characteristics (control charts for variables). This includes $\bar{X}$ and R charts and charts for $\bar{X}$ and σ.

2. Control charts for fraction defective (P-chart).

3. Control chart for number of defects per unit (C-chart).

The control charts for variables are useful for controlling fully automatic processes, where the operator is probably responsible for three or more machines.

Control charts for fraction defective and defects per unit are attribute control charts. A fraction defective control chart discloses erratic fluctuations in the quality of inspection which may result in improvement in inspection practice and inspection standards.

Control Charts for Variables

Control charts based upon measurements of quality characteristics are called as control charts for variables. Control charts for variables are often found to be a more economical means of controlling quality than control charts based on attributes. The variable control charts that are most commonly used are average or $\bar{X}$-charts, range or R-charts and σ- or standard deviation charts.

Some Possible Objectives of the Control Charts

Control charts are based on statistical techniques. In general, control charts for variables, either $\bar{X}$ and R or $\bar{X}$ and σ charts are used for some or all of the following purposes.

1. $\bar{X}$ and R or $\bar{X}$ and σ charts are used in combination for the control process.

$\bar{X}$-chart shows the centring of the process, *i.e.* it shows the variation in the averages of samples. It is the most commonly used variables chart.

R-chart shows the uniformity or consistency of the process *i.e.* it shows the variations in the ranges of samples.

It is a chart for measure of spread.

σ-chart shows the variation of the process.

2. The control charts are used to determine whether a given process can meet the existing specifications without a fundamental change in the production process. In other words they tell whether the process is in control and if so at what dispersion.

3. To secure information to be used in establishing or changing production procedures. Such changes may be either elimination of assignable causes of variation that may be called for whenever the control chart makes it clear that specifications cannot be met with present methods.

For example, where both upper and lower values are specified for a quality characteristic, as in the case of dimensional tolerances. If the basic variability of the process is so great that it is impossible to make all the products within the specification limits, and when the specification cannot be changed then the alternatives will be (a) To make a fundamental change in the production process that will reduce the basic variability or (b) To suffer and sort out the good (non-defective) products from the bad (defective products).

4. To secure information when it is necessary to widen the tolerances. Sometimes the control chart shows so much basic variability that some product is sure to be made outside the tolerances, a review of the situation may show that the tolerances are tighter than necessary for the functioning of the product. Therefore, the appropriate action will be to change the specifications to widen the tolerances for the sake of economy.

5. To secure information to be used in establishing or changing inspection procedure or acceptance procedures or both.

6. To provide a basis for current decisions on acceptance or rejection of manufactured or purchased product. It is possible to reduce inspection costs by using the control charts for variables for acceptance.

7. To provide a basis for current decisions during production as to when to hunt for causes of variation and take action so as to correct them, and when to leave a process alone.

8. To familiarize personnel with the use of the control charts.

Relationship between $\bar{X}'$, σ' and the Values of $\bar{X}$

Suppose, four subgroups each consisting of 5 items are taken from the universe, each subgroup will have its average $\bar{X}$. Let, $\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4$ be the average of the first, second, third and fourth subgroups respectively.

Then, $$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \bar{X}_4}{4}$$

where, the symbol $\bar{\bar{X}}$ denotes the average of the averages.

If many random samples of any given size are taken from a universe, the averages ($\bar{X}$ values) of the samples will themselves form a frequency distribution, having its own central tendency and dispersion or spread.

If the universe is normal, statistical theory tells that the expected frequency distribution of the $\bar{X}$ values will also be normal. Even though the samples are drawn from non-normal universe the distribution of the averages of samples tends to be approximately normal.

The average $\bar{\bar{X}}$ of such a frequency distribution of $\bar{X}$ values apparently tends to be nearer to $\bar{X}'$, the average of the universe. It is observed that larger the samples taken, more likely, the average of the samples will be close to the average of the universe. The spread of the frequency distribution depends on the spread of the universe as well as upon the sample size n, the larger the value of n, the less will be the spread of the $\bar{X}$ values.

Therefore, statistical theory tells that in the long run,

(a) $\bar{\bar{X}} = \bar{X}'$ i.e. the averages of the $\bar{X}$ values will be the same as $\bar{X}'$ the average of the universe.

(b) $\sigma\bar{X} = \frac{\sigma'}{\sqrt{n}}$, where $\sigma\bar{X}$ is the standard deviation of the expected frequency distribution of the average and σ' is the standard deviation of the universe.

Relationship between σ' and $\bar{\sigma}$

Similar to $\bar{X}$ values σ values also differ from one subgroup to the next. In the long run, the standard deviations of samples of any size from the universe will follow a chance pattern.

The relationship between $\bar{\sigma}$ and σ' for a particular sample size is represented by the ratio $C_2 = \frac{\bar{\sigma}}{\sigma'}$

where, $\bar{\sigma}$ = the average of the standard deviations of the samples of any given size and,

σ' = the standard deviation of the universe from which the samples are taken.

The values of the factor 'C_2' are given in Table *B* of Appendix for different sample sizes. For example the value of C_2 for a sample of 5 items = 0.8407 (from Table *B*).

It is observed that, even with the use of this C_2 factor the standard deviation of small subgroup does not give reliable information about σ', the standard deviation of the universe. However, larger the number of subgroups used for calculating $\bar{\sigma}$, the greater should be the confidence in the estimate from $\bar{\sigma}$ of the unknown standard deviation of the universe.

Relationship between σ' and $\bar{R}$

According to statistical theory the ratio between the average range $\bar{R}$ and the standard deviation of the universe σ' is given by, $d_2 = \frac{\bar{R}}{\sigma'}$, for a particular sample size n. For example for a subgroup of 5 the 'd_2' factor for Table *B* (Appendix) = 2.326.

One practical use of this d_2 factor is to provide an alternative method of estimating σ' [standard deviation of an unknown universe for a series of samples or subgroups]. For example, for the 100 subgroups of 5 items, $\bar{R}$ = 25.70. The d_2 factor from Table B = 2.326. Hence, the estimate of σ' from $\bar{R}$ will be

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{25.70}{2.326} = 11.049.$$

A comparison of the two estimates of σ' shows a close agreement between the estimate based on σ and the one based on $\bar{R}$.

From practical point of view it is much easier to compute R for a subgroup than to compute σ. However, in some cases where the measurements themselves are costly (for example destructive tests of variable items) and it is necessary that the inferences from a limited number of tests be as reliable as possible, the extra cost of calculating standard deviation of subgroups is justified.

Choice of Variable

The variable chosen for $\bar{X}$ and R control chart should be such that it can be measured and expressed in numbers such as dimension, hardness number, tensile strength, weight, volume etc. In the introduction of the control chart technique to any organization, the choice of the right variable is often troublesome. There may be quite a large number of specified dimensions on many parts of the product. Obviously, only few of them which will result in real saving in cost should be selected for control chart purpose.

In other words, the variable selected should be one that is likely to reduce cost, *e.g.* a quality characteristic that is responsible for high rejection or rework and where the spoilage and rework costs are high.

Basis of Subgrouping

The information given by the control chart depends on the basis used for selection of subgroups, therefore the careful determination of subgroup is very important in the setting up of a control chart. The following factors should be considered while selecting a subgroup.

1. Each subgroup should be as homogeneous as possible.
2. There should be maximum opportunity for variation from one subgroup to another.
3. Samples should not be taken at exactly equal intervals of time.

Particularly if the primary purpose of keeping the charts is to detect shifts in the process average, all items included in the sample should be drawn from the same population. Secondly, one subgroup should consist of items produced as nearly as possible at one time, *i.e.* from one population and the next subgroup should consist of all items produced at a single later time, *i.e.* from second population. Since, if one sample is drawn from one population and a second sample from second population, the probability of detecting a difference between the two populations is high. This permits a minimum chance of variation within a subgroup when all the items of a subgroup are drawn at random from a single population.

Similarly, when all items of one subgroup are taken from one population and all items of another subgroup are taken from second population, then it gives a maximum chance of variation from one subgroup to another.

It should also be remembered that the samples should not be taken at equal time intervals. For instance, the ten o'clock sample might be taken one day at 10.10 and the next day at 9.45. It is better if the operators do not know in advance just which items are to be selected as the sample for inspection.

However, sometimes the scheme of subgrouping may need to be modified because (*i*) There may be practical difficulties in taking homogeneous samples or (*ii*) If the purpose of the control chart is to provide a basis for acceptance. In such cases as far as possible each subgroup should be representative of all the production over a given period of time, the next subgroup should consist of products intended to be representative of all the production of approximately the same quantity of product in a later period and so forth.

Size and Frequency of Subgroups

Size of Subgroup (Sample Size)

To provide maximum homogeneity within subgroup, the size of subgroup (sample size) should be as small as possible. However, four or five is the most commonly accepted subgroup size, on statistical grounds. The distribution of $\bar{X}$ is nearly normal for subgroups of four or more even though the samples are taken from a normal universe ; this fact is helpful in interpretation of control chart limits.

Secondly, if subgroup size of five is used there is ease of computation of the average, which can be obtained by multiplying the sum by two and moving the decimal point one place to the left.

Larger subgroups such as 10 or 20 are sometimes advantageous when it is desired to make the control chart sensitive to small variation in the process average. The large sample will cause the limits of a control chart to be closer to control line on the chart (control limits will become narrower) and it becomes easy to detect small variations. This is because the standard deviation of P, $\bar{X}$ or R varies inversely with $\sqrt{n}$. Hence the larger the sample size, the smaller the standard deviation, and the closer 3σ limits will be to the central line on the chart.

However, if the cost of measurement is quite high then it may be necessary to use smaller sample size of two or three.

Frequency of Sampling

There are two possible ways :

(*a*) To take larger samples at less frequent intervals or

(*b*) Smaller samples at more frequent intervals.

The selection will be governed by the cost of taking and analyzing measurements and also the benefits to be derived from action based on control chart.

In general, the frequency of sampling depends on just how well the operation is going. In a typical firm, at the time the control charts have been established, the analyst may take a sample every hour. However, if the process remains in control for 2 or 3 days, the frequency of sampling may be reduced to one every 3 or 4 hours. If the process continues in control for a period of a few weeks, the frequency may be reduced still further. On the other hand, if difficulties are encountered in keeping the process in control, more frequent samples are taken. For example, during a troublesome period on a short-cycle operation, the analyst may want to take a sample every $\frac{1}{4}$ hour to discover an out of control condition as soon as possible, since he has reason to believe that the possibility of this condition occurring is high.

Hence, frequency of subgroups should be more at the initial stages and could be reduced when the function of the control chart is only to maintain the process control over current production. The frequency of taking a subgroup may be expressed either in terms of time such as once an hour, or as a proportion of the items produced, such as 5 out of 100.

Control Limits

For plotting control charts generally $\pm 3\sigma$ limits are selected and they are termed as control limits. They present a band within which the dimensions of the components are expected to fall. With 3σ limits, since 99.7 percent of the samples from a given population will fall within these limits the remaining 0.3 percent will fall outside the limits. This means that, in the long run, 3 samples out of every 1,000 will fall outside the $\pm 3\sigma$ limits even if no change takes place in the population average. Since three out of thousand is a very small risk, $\pm 3\sigma$ limits have been found to give good practical results.

So long as the sample average is within 3σ limits it is assumed that any variation between the sample average and the desired population average is due to chance causes, that is, no assignable causes of variation are present. However, as soon as the sample average varies from the desired population average by 3σ or more of the mean, it is assumed that the variation is due to assignable causes and that a shift has taken place in the population average. Actually, if a sample average falls exactly at one of the 3σ points, it is usally assumed that no change has taken place but it is absolutely essential to take another sample soon after to verify this assumption.

When it is found that a shift has taken place, the next step is to find the assignable causes. This calls for investigating the production equipment, materials, and the operator's methods. For example, it may be found that the operator is not setting the machine correctly, or that the machine has deteriorated and lost its accuracy. Whatever the cause, it must be found and eliminated so that future production is not affected adversely.

Chance of Making an Error

As already explained, with 3σ limits in the long run, 3 samples out of every

1,000 will fall outside the 3σ limits even if no change takes place in the population average. As a result, there will be occasion when we shall be looking for an assignable cause of variation when none exists, because no shift has taken place. This condition is described by writers in statistics as a Type I error. We assume a change when actually it has not taken place, and consequently we spent time and money on a needless investigation. If 2σ limits are selected as control limits, there would be an appreciably larger number of occasions when we would be looking for a non-existent assignable cause of variation than there would be with 3σ limits. In fact the probability of making a Type I error would now be 0.045 instead of 0.03 with 3σ limits.

On the other hand, if we conclude that the universe has not changed when it really has changed, this conclusion is described as Type II error. In brief, wider the limits, the greater the probability of making Type II error and lesser the probability of making Type I error.

Because of the circumstances explained above, it became necessary to find a balance between these two types of errors. Each involves a cost, and the problem was to find that combination which would minimize the total cost. Studies revealed that, on the average use of 3σ limits would represent the most economical alternative. This may not be true in every case, and therefore other limits are sometimes employed. But in general, 3σ limits are widely used in the industry.

Starting the Control Charts

Making and recording measurements. The information given by control chart is influenced by variations in quality as well as variations in measurement. Any measuring system will have its own inherent variability which should not be increased due to assignable causes such as error in reading or recording.

Calculation Procedure

1. *Calculate the average $\overline{X}$ and range R for each subgroup.* A good number of samples of items manufactured are collected at random, at different intervals of time and their quality characteristics (say diameter, thickness, weight, length etc.) are measured. For each sample mean value and the range is calculated. For example, if a sample contain 5 items whose dimensions are X_1, X_2, X_3, X_4 and X_5, the sample average

$$\overline{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

The range is computed by subtracting the lowest value from the highest value. [Range, R = Highest value – Smallest value].

2. *Calculate the grand average $\overline{\overline{X}}$ and average range $\overline{R}$.* After calculating the average and range of each sub-group the next step is to find $\overline{\overline{X}}$ and $\overline{R}$ where $\overline{\overline{X}}$ is the average of the $\overline{X}$ values for each sub-group. This is the sum of $\overline{X}$ values divided by the number of sub-groups.

i.e.,

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{N}$$

where, $\overline{\overline{X}}$ = average of averages

and N = Number of sub-groups.

Similarly, the average $\overline{R}$ is the sum of the ranges of the sub-groups divided by the number of sub-groups.

i.e.,

$$\overline{R} = \frac{\Sigma R}{N}.$$

3. *Calculation of 3 sigma limits on control chart for $\overline{X}$ chart.* Tables B, C, D and E of Appendix may be used to obtain the relevant factors like A, A_1, A_2, D_1, D_2, D_3 and D_4 for a particular sample size, according to the method used.

If Table B is to be used, the next step is to estimate σ'. From Table B find the value of the factor d_2 for a particular sample size. Then

$$\sigma' = \frac{\overline{R}}{d_2}.$$

Now, 3 $\sigma_{\overline{X}}$ can be calculated from the relationship

$$\sigma_{\overline{X}} = \frac{\sigma'}{\sqrt{n}}.$$

Upper Control Limit $\overline{X}$

$$= \overline{\overline{X}} + 3\,\sigma_{\overline{X}}.$$

Lower Control Limit $\overline{X}$

$$= \overline{\overline{X}} - 3\sigma_{\overline{X}}.$$

The two steps in the calculation of 3 $\sigma_{\overline{X}}$ may be consolidated as,

$$3\sigma_{\overline{X}} = \frac{3\,\overline{R}}{d_2\sqrt{n}}.$$

To shorten the calculations of control limits from $\overline{R}$, this factor $\frac{3}{d_2\sqrt{n}}$ the multiplier of $\overline{R}$ has been computed for each value of n from 2 to 20 and tabulated in Table C of Appendix. This factor is designated as A_2.

The formulas for 3-sigma control limits on charts for $\overline{X}$ then become

$$UCL\overline{X} = \overline{\overline{X}} + A_2\overline{R}$$

$$LCL\overline{X} = \overline{\overline{X}} - A_2\overline{R}$$

If control limits are to be calculated from $\overline{\sigma}$ rather than from $\overline{R}$, then

$$\overline{\sigma} = \frac{\Sigma\sigma}{N},$$

where N = number of sub-groups, using the C_2 factor from Table B to estimate σ',

$$\sigma' = \frac{\bar{\sigma}}{C_2}$$

and $$3\sigma_{\bar{X}} = \frac{3\sigma'}{\sqrt{n}}$$

or $$3\sigma_{\bar{X}} = \frac{3\bar{\sigma}}{C_2\sqrt{n}}.$$

To shorten the calculations for control limits from $\bar{\sigma}$, the factor $\frac{3}{C_2\sqrt{n}}$, the multiplier of $\bar{\sigma}$ in the above calculation has been computed for each value of n from 2 to 25, hence by 5's to 100 and tabulated in Table D of Appendix. This factor is designated as A_1. The formulas for 3-sigma control limits using factor A_1 are :

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_1\bar{\sigma}$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{\sigma}.$$

For those situations where it is desired to calculate control limits directly from known or standard values of σ' and $\bar{X}$, the factor $\frac{3}{\sqrt{n}}$ has been computed and tabulated in Table E, Appendix. This factor is designated as A. The formulas for 3-sigma control limits using this factor are

$$UCL_{\bar{X}} = \bar{X}' + A\,\sigma'$$
$$LCL_{\bar{X}} = \bar{X}' - A\,\sigma'$$

Calculate the Control Limits for R Chart

The control limits on the chart for ranges (R chart) are given by

$$UCL_R = D_4\bar{R}$$
$$LCL_R = D_3\bar{R}$$

Factors D_4 and D_3 have been given in Table C of the Appendix.

For calculating the control limits on R chart directly from the known or assumed values of σ'. Then the control limits on R chart are given by

$$UCL_R = D_2\sigma'$$
$$LCL_R = D_1\sigma'$$

where the factors D_1, D_2 can be obtained from Table E, Appendix for a particular sample size.

Plot the $\bar{X}$ and R charts. While plotting the $\bar{X}$ chart, the central line on the $\bar{X}$ chart should be drawn as a solid horizontal line at $\bar{\bar{X}}$ The upper and lower control limits for $\bar{X}$ chart should be drawn as dotted horizontal lines at the computed values.

Similarly, for R chart the central line should be drawn as a solid horizontal line at $\bar{R}$ The upper control limit should be drawn as dotted horizontal line at the computed value of UCL_R. If the sub-group size is seven or more, the lower

control limit should be drawn as dotted horizontal line at LCL_R. However, it should be noted that if the sub-group size is six or less, the lower control limit for R is zero.

Plot the averages of the sub-groups in $\bar{X}$ chart, in the order collected and the ranges in R chart which should be below the $\bar{X}$ chart so that the sub-groups correspond to one another in both the charts. Points outside the control limits are indicated with cross on $\bar{X}$ chart [(×)], and the points outside the limits on R chart by a circle [⊙].

Drawing Preliminary Conclusions from Control Charts

Lack of control is indicated by points falling outside the control limits on either $\bar{X}$ or R chart. It means that some assignable causes of variation are present, it is not a constant cause system.

When all the points fall inside the control limits ; we say that the process is in control. It really means for all practical purposes it acts as if no assignable causes of variation are present. Based on how many points fall outside the control limits, 1 out of 35 points or 2 out of 100 points can also be tolerated, and the process is said to be in control.

Even though all points fall within the control limits, in order to detect shifts in the process average in manufacturing, it is customary to use various practical working rules as stated below. These rules depend only on the extreme runs.

(*a*) Whenever a run of 7 consecutive points is on one side of the control line.

(*b*) Whenever in 11 successive points on the control chart, at least 10 are on the same side of the central line.

(*c*) Whenever in 14 successive points on the control chart, at least 12 are on the same side of the central line.

(*d*) Whenever in 17 successive points on the control chart, at least 14 are on the same side of the central line.

(*e*) Whenever in 20 successive points on the control chart, at least 16 are on the same side of the central line.

Sequence of points on one side suggests the need to review the position of the central line on the chart.

Interpretation of Processes in Control. With evidence from the control chart that a process is in control, we are in a position to judge what is necessary to permit the manufacture of product that meets the specifications for the quality characteristic charted. The control chart data gives us estimates of :

1. The centring of the process ($\bar{X}'$ may be estimated from $\bar{\bar{X}}$).
2. The dispersion of the process (σ' may be estimated as $\bar{R}/d_2$).

Some Control Chart Patterns

(*a*) *Chance pattern of variation.* $\bar{X}$ chart for chance pattern of variation is shown in Fig. 6.1. The important characteristics are :

(*i*) Most of the points are near centre line $\bar{X}$

(*ii*) Very few points are near control limits.

1. Action to remove assignable causes of variation when out of control condition has been detected.

2. Action to establish the process average.

3. Action to establish the process dispersion.

Once the process is brought into control with satisfactory average and dispersion, an important purpose of the control chart is to help continue this happy state of affairs. This involves :

(a) To leave the process alone as long as it stays in control, and

(b) To hunt for and remove the assignable causes of variation where the control chart shows lack of control.

Whenever an aimed at value $\bar{X}$ is used for the central line on the control chart, efforts should be made to maintain that value in the manufacturing process itself.

Action to reduce the process dispersion may necessitate for fundamental change in machines or methods. The information given by the control chart about the natural tolerances that will be held by the various machines or production methods may make it possible to fit the process dispersion to the job in hand. Operations for which the close tolerances are necessary may be assigned to those machines that will hold the close tolerances, and operations on which the wide tolerances are satisfactory may be assigned to those machines that will hold only wide tolerances.

Process Capability Analysis

Control limits versus specification limits :

When the production activity is being planned, an effort will be made to provide a combination of equipment, materials and manpower such that the output will meet specifications. Specification data are provided on the blue print. For example, the blue print may state the required dimension for the shaft diameter is 35.50 ± 0.25 mm. This gives an upper specification limit of 35.75 mm. and lower specification limit as 35.25 mm. Any shaft produced outside this limit would have to be scrapped and reworked. Therefore, every attempt will be made to set up the job in such a way that these limits will be met, if possible, by all the shafts.

We know that the dispersion of the individual items is described in terms of the standard deviation σ', while the dispersion of sample averages drawn from the same population is described in terms of the standard error of the mean $\sigma_{\bar{X}}$. For a given population the standard error of the mean is always less than the standard deviation of the individual items

since, $$\sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}}.$$

Consequently, the individual items have a greater dispersion than that of the sample average.

In brief, the average diameter of a sample of shafts may be within specification limits, while some of the shafts included in the sample may not be within the specification limits. Therefore, even though the process seems to be in control, it is necessary to see whether the process is capable of producing the parts within the specified limits. This can be done by carrying out the process capability analysis as described below.

Process Capability

Process capability may be defined as the "minimum spread of a specific measurement variation which will include 99.7% of the measurements from the given process". In other words, process capability = $6\sigma'$ since, $6\sigma'$ is taken as a measure of the spread of the process, which is also called natural tolerance. Process capability study is carried out to measure the ability of the process to meet the specified tolerances.

By this study, it becomes possible to know the percentage of the products which will be produced within $\pm 3\sigma$ limits on either side of the mean $\bar{X}$

A process capability analysis consists of :

1. Measuring the process capability to find out whether the process is inherently capable of meeting the specified tolerance limits.

2. Discovering why a process 'capable' is failing to meet specifications.

Methods of Calculating Process Capability

1. *Standard deviation method.* By this method process capability study may be made by gathering the required data—at least 50 observations and preferably 100 or more if possible—and computing the standard deviation of this data by using the relation already described in chapter 4.

2. *The average range method.* The average range method discussed in details below is preferred for process capability analysis for the following reasons :

(a) It is easier to calculate—no square root is involved.

(b) Trends occurring in the study or other abnormal conditions can be detected.

(c) When the average range method is used the capability study can serve as a base—period analysis.

3. *Single range method.* A rough estimate of process capability may be obtained by this method. Take a certain number of observations and then find the difference between the largest and smallest readings. Based on certain confidence level we can predict the percentage of the products (depending upon the number of observations) that will lie within the observed range of the sample. This method is valuable when used in conjunction with the average range method.

Basis of Process Capability Study

When a process is functioning under a chance cause system the distribution of the individual products coming from it will tend to be normal, and if we set the tolerance at $\pm 3\sigma$ for the distribution, 99.7% of the products will fall within these tolerance limits. Thus the study is aimed in determining the standard deviation of the individual measurements of products when the process is in control.

The process capability analysis consists of the following elements :

1. The specification tolerance.
2. The determination of whether the process average is "centred" mid way between the tolerance limits.
3. Measurement of inherent (piece to piece) variability of the process.
4. Measurement of actual variability over a period of time.
5. Causes of differences between inherent and actual variability.

An effective way of making the analysis is by means of a control chart and a frequency distribution.

When making the study it is important to minimize the effect of factors such as unnatural material variation, process adjustment etc. Hence homogeneous material should be used, no process adjustments should be made during the study, trained operators should be allowed to perform the work. A number of samples are then taken over a period of time. Each sample consists of consecutively made pieces.

The analysis is done in the following manner :

1. Calculate the average $\overline{X}$ and range R of each sample.
2. Calculate the grand average $\overline{\overline{X}}$. This measures the centring of the process.
3. Calculate control limits and plot $\overline{X}$ and R charts.

This measures the stability of the process, *i.e.* the extent to which it changes with time.

4. Calculate the process capability $6\sigma' = 6\left(\frac{\overline{R}}{d_2}\right)$.

This measures the piece to piece variability of the process.

Possible Relationship of a Process in Control to Upper and Lower Specification Limits

When a controlled process must meet two specification limits on individual values, upper specification limit and lower specification limits, the possible situations may be grouped into three general classes as described below.

(1) $(X_{max} - X_{min}) > 6\sigma'$

where X_{max} = Upper specification limit

X_{min} = Lower specification limit.

In this case the spread of the process ($6\sigma'$) is considerably less than the difference between the upper specification limit and lower specification limit. The first situation is shown in Fig. 6.6.

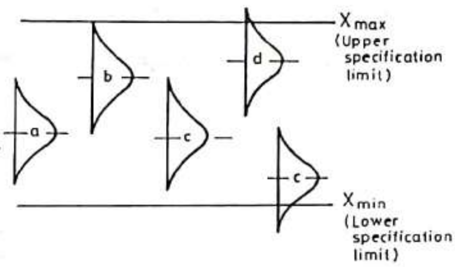


Fig. 6.6. $(X_{max} - X_{min}) > 6\sigma'$.

The frequency curves *a, b, c, d* and *e* show various positions in which the process might be centred.

Conclusion. (*i*) With any position *a, b* or *c*, practically all the products manufactured will meet specifications as long as the process stays in control.

(*ii*) It may be considered economically advisable to permit $\overline{X}$ to go out of control if it does not go too far, *i.e.* the distribution may be allowed to move between positions *b* and *c*. This may avoid the cost of frequent machine setups and the delays due to hunting for assignable causes of variation that will not be responsible for unsatisfactory product.

(*iii*) If $\left(\frac{X_{max} - X_{min}}{6\sigma'}\right)$ ratio is considerably large, frequency of control chart may be reduced.

(*iv*) If there is an economic advantage to be gained by tightening the specification limits, it may be considered.

With the process in position *d*, some product will fall above the upper specification limit, in position *c* some product will fall below the lower specification limit. In both cases, it is absolutely necessary to change the centring of the process, bringing it closer to position *a*.

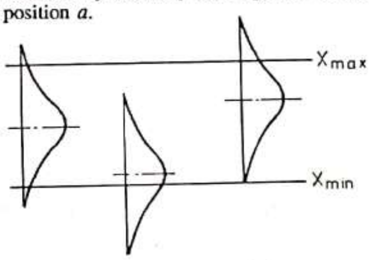


Fig. 6.7. $(X_{max} - X_{min}) < 6\sigma'$.

(2) $(X_{max} - X_{min}) < 6\sigma'$. In this case the spread of the process ($6\sigma'$) is

appreciably greater than the difference between the specification limits as shown in Fig. 6.7.

Conclusion. In this type of situation defective parts will always be there, therefore, the remedy will be

(*i*) Increase the tolerance.

(*ii*) Reduce the dispersion, by making fundamental changes in the production methods, machines used.

(*iii*) Suffer and sort out the defectives, if it is economical than making the fundamental changes.

(*iv*) It is still important to maintain the centring of the process.

(3) $(X_{max} - X_{min}) = 6\sigma'$. In this situation the spread of the process is approximately equal to the difference between upper and lower specification limits as shown in Fig. 6.8.

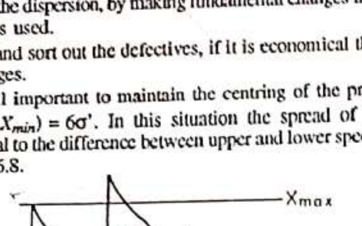


Fig. 6.8. $(X_{max} - X_{min}) = 6\sigma'$.

Conclusion. (*i*) In this case it is necessary to take steps to retain the centring of the process. Here the process is exactly centred, a little change in centring will cause many components to fall outside the specification limits. This usually calls for continuous use of the control charts for $\bar{X}$ and R with subgroups at frequent intervals and immediate attention to points out of control.

(*ii*) It is advisable to increase tolerances if they are tighter than is really necessary.

(*iii*) Reduce dispersion if it is economical.

Use of process capability Data in Manufacturing Planning :

The information obtained from process capability is of great importance in solving quality problems. For example,

— The design engineer, knowing the capability of the process and the available equipment, has a more rational basis while setting the specifications.

— The planning engineer can assign the jobs with more rigid tolerances to the most capable machines and that with wider tolerance to the less precise machines.

— The tool designer can spot the places where tooling improvements must be made to maintain the process capability.

— The capability information helps the foreman to decide which machine may require overhaul.

— The machine set up man learns which machine requires the most attention to set up and which one needs only normal care.

— The machine operator and inspector can decide which machines need the closest watching in production.

— The purchasing agent has a means to compare the actual performance of equipment with the manufacturer's claims.

In manufacturing planning some problems may arise from the relationship of machine capability to product tolerance. The following alternative may be useful to solve such problems.

1. If the machine capability is inadequate to meet the tolerances :

(*a*) Try to shift the job to another machine with more adequate capability.

(*b*) Try to improve the machine capability, sometimes the machine may need overhauling or the tooling may need to be reviewed.

(*c*) Try to get review of the tolerances. The availability of specific information showing what tolerances can be achieved may soften the engineer's attitude on widening the tolerance.

(*d*) Sort off the good product from the bad, if it is economical.

2. If the machine capability is equal to the tolerance. This usually should be treated as in (1) above, since it means that tools must be set exactly at the nominal and gives no allowance for tool wear.

3. If the machine capability is adequate to meet the tolerance :

(*a*) If the machine capability is of the order of 2/3rds to 3/4ths of the tolerance or less, it is the acceptable situation.

(*b*) If the machine capability is less than one-half of the tolerance, consider reducing the tolerance. The ability to tighten the tolerance on one part in an assembly may permit loosening a difficult tolerance on another part. Closer guarantee of tolerances may also help to improve the sale.

(*c*) 100% inspection is not necessary and a sampling procedure should be used.

Why does a capable manufacturing process give defects ?

In practice there could be defects even though the process is capable of meeting the specified tolerances. Reasons for such defects may include :

1. *Poor Centering*. The manufacturing process is poorly centered (refer Fig. 6.6 (d, e). The setting should be changed to midpoint of tolerance to avoid defects.

2. *Trend in the manufacturing process*. (refer Fig. 6.3). This can be acceptable if only a few units are to be made and the process is set to an appropriate level. Even though the process is in control in the present situation the process may produce defective articles in the long run if it is continued to manufacture large number of units. The effect can be reduced by periodic checking and resetting.

3. *Shift*. (refer Fig. 6.4). Sudden shift in the process can occur in conjunction with a change of material operator etc. In such cases, measures such as checking and adjustment are needed, based on experience.

4. *Mixed Lot*. A lot which consists of units not manufactured under exactly the same conditions is a mixed lot. In such cases the cause of difference should be traced out and removed.

Problem 1. Control charts for $\overline{X}$ and R are maintained on certain dimensions of a manufactured part, measured in mm. The subgroup size is 4. The values of $\overline{X}$ and R are computed for each subgroup. After 20 subgroups $\sum\overline{X} = 412.83$ and $\sum R = 3.39$. Compute the values of 3 sigma limits for the $\overline{X}$ and R charts and estimate the value of σ' on the assumption that the process is in statistical control.

Sol.
$$\overline{\overline{X}} = \frac{\sum\overline{X}}{N}$$

where N = number of subgroups

Therefore,
$$\overline{\overline{X}} = \frac{412.83}{20} = 20.6415.$$

$$\overline{R} = \frac{\sum R}{N}$$

i.e.
$$\overline{R} = \frac{3.39}{20} = 0.169.$$

σ' = population standard deviation

$$= \frac{\overline{R}}{d_2}$$

$$= \frac{0.169}{2.059} = 0.082 \text{ [for subgroup of 4 factor } d_2 = 2.059]$$

$$3\sigma_{\overline{X}} = \frac{3\sigma'}{\sqrt{n}} = \frac{3 \times 0.082}{\sqrt{4}} = 0.123.$$

For $\bar{\bar{X}}$ chart :

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}}$$
$$= 20.6415 + 0.123 = 20.7645$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}}$$
$$= 20.6415 - 0.123 = 20.5185.$$

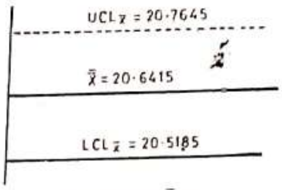


Fig. 6.12 (a). $\bar{X}$ chart.

For R Chart :

$$UCL_R = D_4 . \bar{R}$$
$$= 2.28 \times 0.169$$
[for subgroup of 4 factor $D_4 = 2.28$ from table]
$$= 0.3853.$$
$$LCL_R = D_3\bar{R}$$
$= 0 \times 0.169$ [for subgroup of 4, $D_3 = 0$] = 0.

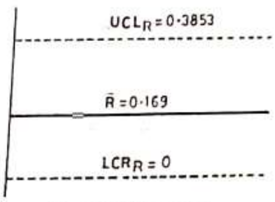


Fig. 6.13 (b). R chart.

Problem 2. In a capability study of a lathe used in turning a shaft to a diameter of 23.75 ± 0.1 mm a sample of 6 consecutive pieces was taken each day for 8 days. The diameters of these shafts are as given below :

1st day	*2nd day*	*3rd day*	*4th day*	*5th day*	*6th day*	*7th day*	*8th day*
23.77	23.80	23.77	23.79	23.75	23.78	23.76	23.76
23.80	23.78	23.78	23.76	23.78	23.76	23.78	23.79
23.78	23.76	23.77	23.79	23.78	23.73	23.75	23.77
23.73	23.70	23.77	23.74	23.77	23.76	23.76	23.72
23.76	23.81	23.80	23.82	23.76	23.74	23.81	23.78
23.75	23.77	23.74	23.76	23.79	23.78	23.80	23.78

Construct the $\bar{X}$ and R chart and find out the process capability for the machine.

Sol. Average diameter for the first day

$$\bar{X}_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}{6}$$
$$= \frac{23.77 + 23.80 + 23.78 + 23.73 + 23.76 + 23.75}{6}$$
$$= 23.765.$$

Similarly, the averages for each day are calculated and the results are tabulated as below :

$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$	$\bar{X}_4$	$\bar{X}_5$	$\bar{X}_6$	$\bar{X}_7$	$\bar{X}_8$
23765	23.77	23.7716	23.7767	23.7717	23.7583	23.7767	23.7667

Now,
$$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{N}$$
$$= \frac{190.1567}{8} = 23.7696.$$

Ranges :

R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
0.07	0.11	0.06	0.08	0.04	0.05	0.06	0.07

$$\bar{R} = \frac{\Sigma R}{N} = 0.0675.$$

For $\bar{X}$ chart :

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$
$$= 23.7696 + 0.48 \times 0.0675$$
[$A_2 = 0.48$ for subgroup of from Table Appendix]
$$= 23.7696 + 0.0324 = 23.802.$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$
$$= 23.7696 - 0.0324 = 23.7322.$$

For R chart :

$$UCL_R = D_4\bar{R}$$
$$= 2 \times 0.0675 = 0.1350.$$
$$LCL_R = D_3\bar{R}$$
$= 0$ [$D_3 = 0$ for subgroup of 6 or less]

Process capability :

$$6\sigma' = 6 \times \frac{\bar{R}}{d_2}$$
$$= \frac{6 \times 0.0675}{2.534} = 0.15982$$
[for subgroup of 6, $d_2 = 2.534$ from Table Appendix]

$X_{max} - X_{min} = 0.2$ mm from data.

Therefore, $(X_{max} - X_{min}) > 6\sigma'$.

Conclusion. All manufactured products will meet specifications as long as the process stays in control.

Problem 3. The following table shows the averages and ranges of the spindle diameters in millimetres for 30 subgroups of 5 items each.

$\bar{X}$	R	$\bar{X}$	R	$\bar{X}$	R
45.020	0.375	45.600	0.275	45.26	0.150
44.950	0.450	45.020	0.175	45.650	0.200
45.480	0.450	45.320	0.200	45.620	0.400
45.320	0.150	45.560	0.425	45.480	0.225
45.280	0.200	45.140	0.250	45.380	0.125
45.820	0.250	45.620	0.375	45.660	0.350
45.580	0.275	45.800	0.475	45.460	0.225
45.400	0.475	45.500	0.200	45.640	0.375
45.660	0.475	45.780	0.275	45.390	0.650
45.680	0.275	45.640	0.225	45.290	0.350

For the first 20 samples set up an $\bar{X}$ chart and an R chart. Plot the next 10 samples on these charts to see if the process continues "under control" both as to average and range. Also find the process capability.

Sol.

$$\bar{\bar{X}} = \frac{\Sigma\bar{X}}{N} = \frac{909.170}{20} = 45.4585$$

$$\bar{R} = \frac{\Sigma R}{N} = \frac{6.250}{20} = 0.3125.$$

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{0.3125}{2.326} = 0.13435$$

$$\sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}} = \frac{0.13435}{\sqrt{5}} = 0.06009$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}} \qquad UCL_R = D_4\bar{R}$$

$$= 45.4585 + 3 \times 0.06009 \qquad = 2.11 \times 0.3125$$

$$= 45.4585 + 0.1803 \qquad = 0.6594$$

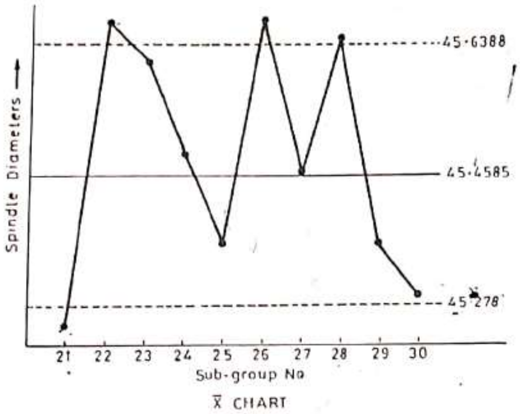
$$= 45.6388 \qquad LCL_R = D_3\bar{R} = 0$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\,\sigma_{\bar{X}}$$

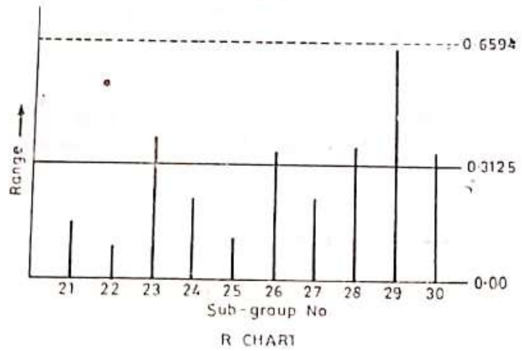
$$= 45.4585 - 0.1803 = 45.2782.$$

Process capability $= 6\sigma' = 6 \times 0.13425$

$= 0.80550$



$\bar{X}$ Chart



R Chart

Fig. 6.14.

Problem 4. A sub-group of 5 items each are taken from a manufacturing process at a regular interval. A certain quality characteristic is measured and $\bar{X}$ and R values computed. After 25 subgroups it is found that $\Sigma\bar{X} = 357.50$ and $\Sigma R = 8.80$. If the specification limits are 14.40 ± 0.40 ; and if the process is in