

# Scales

1. Basic Information
2. Types and important units
3. Plain Scales (3 Problems)
4. Diagonal Scales - information
5. Diagonal Scales (3 Problems)
6. Comparative Scales (3 Problems)
7. Vernier Scales - information
8. Vernier Scales (2 Problems)
9. Scales of Cords - construction
10. Scales of Cords (2 Problems)

# Engineering Curves – I

1. Classification
2. Conic sections -  
explanation
3. Common  
Definition
4. Ellipse – ( six methods of construction)
5. Parabola – ( Three methods of construction)
6. Hyperbola – ( Three methods of construction )
7. Methods of drawing Tangents & Normals ( four cases)



## Engineering Curves – II

1. Classification
2. Definitions
3. Involute - (five cases)
4. Cycloid
5. Trochoids – (Superior and Inferior)
6. Epic cycloid and Hypo - cycloid
7. Spiral (Two cases)
8. Helix – on cylinder & on cone
9. Methods of drawing Tangents and Normals (Three cases)

# Loci of Points

1. Definitions - Classifications
2. Basic locus cases (six problems)
3. Oscillating links (two problems)
4. Rotating Links (two problems)



# Orthographic Projections - Basics

1. Drawing – The fact about
2. Drawings - Types
3. Orthographic (Definitions and Important terms)
4. Planes - Classifications
5. Pattern of planes & views
6. Methods of orthographic projections
7. 1<sup>st</sup> angle and 3<sup>rd</sup> angle method – two illustrations

# Conversion of pictorial views in to orthographic views.

1. Explanation of various terms
2. 1st angle method - illustration
3. 3rd angle method – illustration
4. To recognize colored surfaces and to draw three Views
5. Seven illustrations (no.1 to 7) draw different orthographic views
6. Total nineteen illustrations ( no.8 to 26)



# Projection of Points and Lines

1. Projections – Information
2. Notations
3. Quadrant Structure.
4. Object in different Quadrants – Effect on position of views.
5. Projections of a Point – in 1st quadrant.
6. Lines – Objective & Types.
7. Lines perpendicular to one plane.
8. Lines inclined to one plane.
9. Lines inclined to both planes.
10. Imp. Observations for solution
11. Important Diagram & Tips.
12. Group A problems 1 to 5
13. Traces of Line ( HT & VT )
14. To locate Traces.
15. Group B problems: No. 6 to 8
16. HT-VT additional information.
17. Group B1 problems: No. 9 to 11
18. Group B1 problems: No. 9 to 1
19. Lines in profile plane
20. Group C problems: No.12 & 13
21. Applications of Lines:: Information
22. Group D: Application Problems: 14 to 23
23. Lines in Other Quadrants/( Four Problems)

# Projections of Planes:

1. About the topic:
2. Illustration of surface & side inclination.
3. Procedure to solve problem & tips:
4. Problems:1 to 5: Direct inclinations:
5. Problems:6 to 11: Indirect inclinations:
6. Freely suspended cases: Info:
7. Problems: 12 & 13
8. Determination of True Shape: Info:
9. Problems: 14 to 17



## Projections of Solids:

1. Classification of Solids:
2. Important parameters:
3. Positions with Hp & Vp: Info:
4. Pattern of Standard Solution.
5. Problem no 1,2,3,4: General cases:
6. Problem no 5 & 6 (cube & tetrahedron)
7. Problem no 7 : Freely suspended:
8. Problem no 8 : Side view case:
9. Problem no 9 : True length case:
10. Problem no 10 & 11 Composite solids:
11. Problem no 12 : Frustum & auxiliary plane:

## Section & Development

1. Applications of solids:
2. Sectioning a solid: Information:
3. Sectioning a solid: Illustration Terms:
4. Typical shapes of sections & planes:
5. Development: Information:
6. Development of diff. solids:
7. Development of Frustums:
8. Problems: Standing Prism & Cone: no. 1 & 2
9. Problems: Lying Prism & Cone: no.3 & 4
10. Problem: Composite Solid no. 5
11. Problem: Typical cases no.6 to 9



# Intersection of Surfaces:

1. Essential Information:
2. Display of Engineering Applications:
3. Solution Steps to solve Problem:
4. Case 1: Cylinder to Cylinder:
5. Case 2: Prism to Cylinder:
6. Case 3: Cone to Cylinder
7. Case 4: Prism to Prism: Axis Intersecting.
8. Case 5: Triangular Prism to Cylinder
9. Case 6: Prism to Prism: Axis Skew
10. Case 7 Prism to Cone: from top:
11. Case 8: Cylinder to Cone:

# Isometric Projections

1. Definitions and explanation
2. Important Terms
3. Types.
4. Isometric of plain shapes-1.
5. Isometric of circle
6. Isometric of a part of circle
7. Isometric of plain shapes-2
8. Isometric of solids & frustums (no.5 to 16)
9. Isometric of sphere & hemi-sphere (no.17 & 18)
10. Isometric of Section of solid.(no.19)
11. Illustrated nineteen Problem (no.20 to 38)



# SCALES

DIMENSIONS OF LARGE OBJECTS MUST BE REDUCED TO ACCOMMODATE ON STANDARD SIZE DRAWING SHEET. THIS REDUCTION CREATES A SCALE OF THAT REDUCTION RATIO, WHICH IS GENERALLY A FRACTION..

**SUCH A SCALE IS CALLED REDUCING SCALE  
AND  
THAT RATIO IS CALLED REPRESENTATIVE FACTOR.**

SIMILARLY IN CASE OF TINY OBJECTS DIMENSIONS MUST BE INCREASED FOR ABOVE PURPOSE. HENCE THIS SCALE IS CALLED **ENLARGING SCALE**.  
**HERE THE RATIO CALLED REPRESENTATIVE FACTOR IS MORE THAN UNITY.**

FOR FULL SIZE SCALE

**R.F.=1 OR ( 1:1 )**

MEANS DRAWING  
& OBJECT ARE OF  
SAME SIZE.

Other RFs are described  
as

1:10, 1:100,  
1:1000, 1:1,00,000

**USE FOLLOWING FORMULAS FOR THE CALCULATIONS IN THIS TOPIC.**

**A** REPRESENTATIVE FACTOR (R.F.) =  $\frac{\text{DIMENSION OF DRAWING}}{\text{DIMENSION OF OBJECT}}$

=  $\frac{\text{LENGTH OF DRAWING}}{\text{ACTUAL LENGTH}}$

=  $\sqrt{\frac{\text{AREA OF DRAWING}}{\text{ACTUAL AREA}}}$

=  $\sqrt[3]{\frac{\text{VOLUME AS PER DRWG.}}{\text{ACTUAL VOLUME}}}$

**B** LENGTH OF SCALE = R.F. **X** MAX. LENGTH TO BE MEASURED.

## **BE FRIENDLY WITH THESE UNITS.**

1 KILOMETRE = 10 HECTOMETRES

1 HECTOMETRE = 10 DECAMETRES

1 DECAMETRE = 10 METRES

1 METRE = 10 DECIMETRES

1 DECIMETRE = 10 CENTIMETRES

1 CENTIMETRE = 10 MILIMETRES

## **TYPES OF SCALES:**

1. PLAIN SCALES ( FOR DIMENSIONS UP TO SINGLE DECIMAL)
2. DIAGONAL SCALES ( FOR DIMENSIONS UP TO TWO DECIMALS)
3. VERNIER SCALES ( FOR DIMENSIONS UP TO TWO DECIMALS)
4. COMPARATIVE SCALES ( FOR COMPARING TWO DIFFERENT UNITS)
5. SCALE OF CORDS ( FOR MEASURING/CONSTRUCTING ANGLES)



**PLAIN SCALE:-** This type of scale represents two units or a unit and its sub-division.

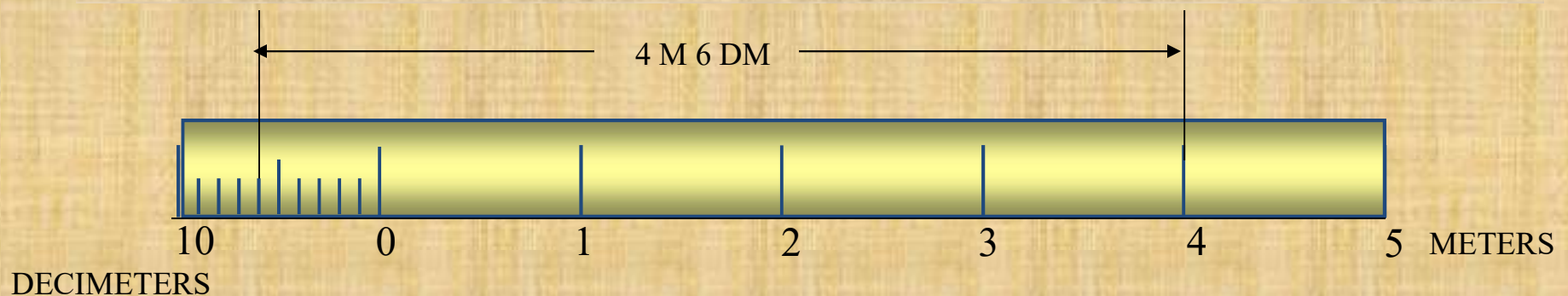
**PROBLEM NO.1:-** Draw a scale 1 cm = 1m to read decimeters, to measure maximum distance of 6 m. Show on it a distance of 4 m and 6 dm.

**CONSTRUCTION:-**  $\frac{\text{DIMENSION OF DRAWING}}{\text{DIMENSION OF OBJECT}}$   
a) Calculate R.F. =

$$\text{R.F.} = 1\text{cm} / 1\text{m} = 1/100$$

$$\begin{aligned}\text{Length of scale} &= \text{R.F.} \times \text{max. distance} \\ &= 1/100 \times 600 \text{ cm} \\ &= 6 \text{ cms}\end{aligned}$$

- b) Draw a line 6 cm long and divide it in 6 equal parts. Each part will represent larger division unit.
- c) Sub divide the first part which will represent second unit or fraction of first unit.
- d) Place ( 0 ) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. **Take height of scale 5 to 10 mm for getting a look of scale.**
- e) After construction of scale mention its RF and name of scale as shown.
- f) Show the distance 4 m 6 dm on it as shown.



$$\text{R.F.} = 1/100$$

PLANE SCALE SHOWING METERS AND DECIMETERS.

**PROBLEM NO.2:-** In a map a 36 km distance is shown by a line 45 cms long. Calculate the R.F. and construct a plain scale to read kilometers and hectometers, for max. 12 km. Show a distance of 8.3 km on it.

**CONSTRUCTION:-**

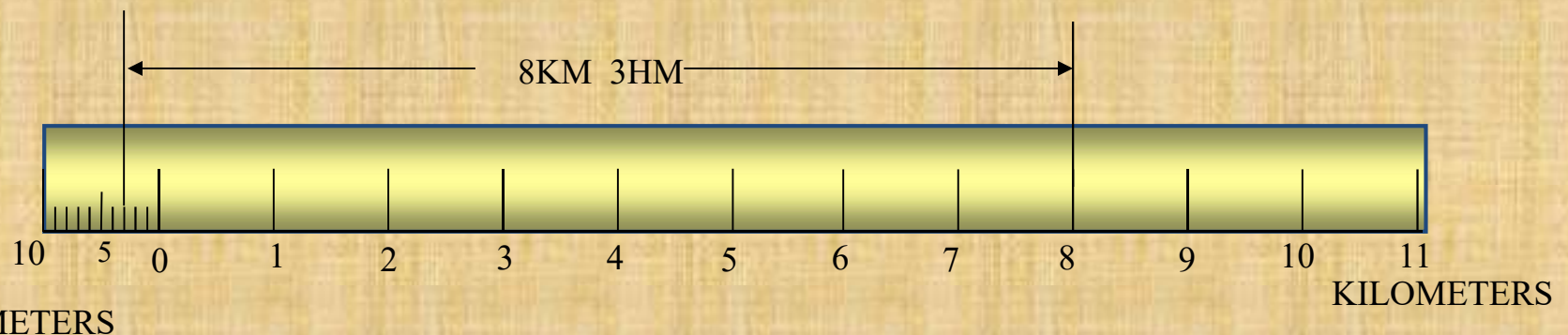
a) Calculate R.F.

$$\text{R.F.} = 45 \text{ cm} / 36 \text{ km} = 45 / 36 \cdot 1000 \cdot 100 = 1 / 80,000$$

$$\begin{aligned} \text{Length of scale} &= \text{R.F.} \times \text{max. distance} \\ &= 1 / 80000 \times 12 \text{ km} \\ &= 15 \text{ cm} \end{aligned}$$

**PLAIN SCALE**

- b) Draw a line 15 cm long and divide it in 12 equal parts. Each part will represent larger division unit.
- c) Sub divide the first part which will represent second unit or fraction of first unit.
- d) Place ( 0 ) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. **Take height of scale 5 to 10 mm for getting a look of scale.**
- e) After construction of scale mention it's RF and name of scale as shown.
- f) Show the distance 8.3 km on it as shown.



$$\text{R.F.} = 1 / 80,000$$

PLANE SCALE SHOWING KILOMETERS AND HECTOMETERS



**PROBLEM NO.3:-** The distance between two stations is 210 km. A passenger train covers this distance in 7 hours. Construct a plain scale to measure time up to a single minute. RF is 1/200,000 Indicate the distance traveled by train in 29 minutes.

**CONSTRUCTION:-**

- a) 210 km in 7 hours. Means speed of the train is 30 km per hour ( 60 minutes)

$$\begin{aligned}\text{Length of scale} &= \text{R.F.} \times \text{max. distance per hour} \\ &= 1/200,000 \times 30\text{km} \\ &= 15 \text{ cm}\end{aligned}$$

- b) 15 cm length will represent 30 km and 1 hour i.e. 60 minutes.

Draw a line 15 cm long and divide it in 6 equal parts. Each part will represent 5 km and 10 minutes.

- c) Sub divide the first part in 10 equal parts, which will represent second unit or fraction of first unit.

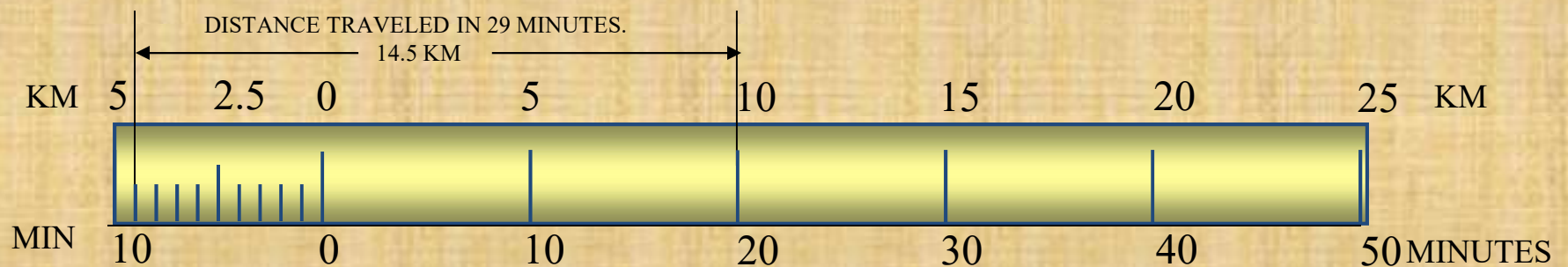
Each smaller part will represent distance traveled in one minute.

- d) Place ( 0 ) at the end of first unit. Number the units on right side of Zero and subdivisions on left-hand side of Zero. **Take height of scale 5 to 10 mm for getting a proper look of scale.**

- e) Show km on upper side and time in minutes on lower side of the scale as shown.

After construction of scale mention it's RF and name of scale as shown.

- f) Show the distance traveled in 29 minutes, which is 14.5 km, on it as shown.



$$\text{R.F.} = 1/100$$

PLANE SCALE SHOWING METERS AND DECIMETERS.

**PLAIN SCALE**

We have seen that the plain scales give only two dimensions, such as a unit and it's subunit or it's fraction.

The diagonal scales give us three successive dimensions that is a unit, a subunit and a subdivision of a subunit.

## DIAGONAL SCALE

The principle of construction of a diagonal scale is as follows.

Let the XY in figure be a subunit.

From Y draw a perpendicular YZ to a suitable height.

Join XZ. Divide YZ in to 10 equal parts.

Draw parallel lines to XY from all these divisions and number them as shown.

From geometry we know that similar triangles have their like sides proportional.

Consider two similar triangles XYZ and 7' 7Z,  
we have  $7Z / YZ = 7'7 / XY$  (each part being one unit)

Means  $7'7 = 7 / 10 \times XY = 0.7 XY$

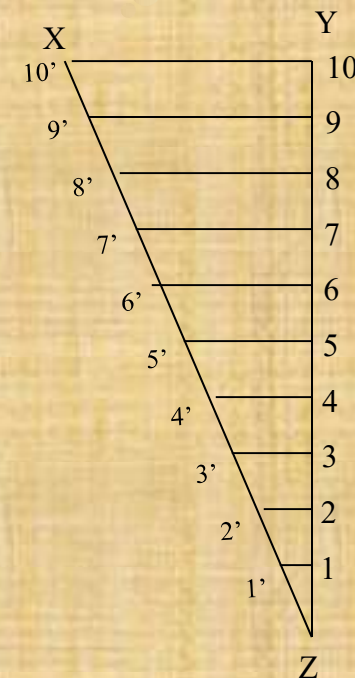
∴

Similarly

$$1' - 1 = 0.1 XY$$

$$2' - 2 = 0.2 XY$$

Thus, it is very clear that, the sides of small triangles, which are parallel to divided lines, become progressively shorter in length by 0.1 XY.



The solved examples ON NEXT PAGES will make the principles of diagonal scales clear.

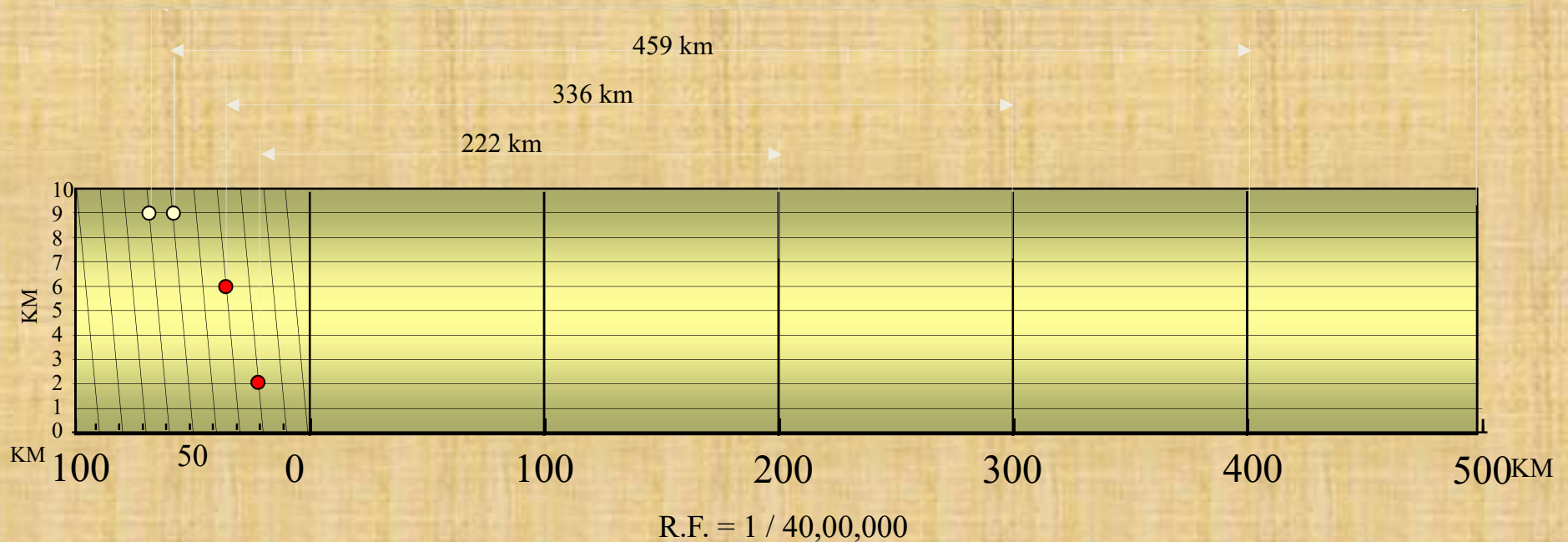


PROBLEM NO. 4 : The distance between Delhi and Agra is 200 km.  
In a railway map it is represented by a line 5 cm long. Find it's R.F.  
Draw a diagonal scale to show single km. And maximum 600 km.  
Indicate on it following distances. 1) 222 km 2) 336 km 3) 459 km 4) 569 km

## DIAGONAL SCALE

SOLUTION STEPS:  $RF = 5 \text{ cm} / 200 \text{ km} = 1 / 40,00,000$   
Length of scale =  $1 / 40,00,000 \times 600 \times 10^5 = 15 \text{ cm}$

Draw a line 15 cm long. It will represent 600 km. Divide it in six equal parts. (each will represent 100 km.)  
Divide first division in ten equal parts. Each will represent 10 km. Draw a line upward from left end and mark 10 parts on it of any distance. Name those parts 0 to 10 as shown. Join 9<sup>th</sup> sub-division of horizontal scale with 10<sup>th</sup> division of the vertical divisions. Then draw parallel lines to this line from remaining sub divisions and complete diagonal scale.



DIAGONAL SCALE SHOWING KILOMETERS.

**PROBLEM NO.5:** A rectangular plot of land measuring 1.28 hectares is represented on a map by a similar rectangle of 8 sq. cm. Calculate RF of the scale. Draw a diagonal scale to read single meter. Show a distance of 438 m on it.

## DIAGONAL SCALE

**SOLUTION :**

1 hector = 10, 000 sq. meters

1.28 hectares =  $1.28 \times 10, 000$  sq. meters  
 $= 1.28 \times 10^4 \times 10^4$  sq. cm

8 sq. cm area on map represents  
 $= 1.28 \times 10^4 \times 10^4$  sq. cm on land

1 cm sq. on map represents  
 $= 1.28 \times 10^4 \times 10^4 / 8$  sq cm on land

1 cm on map represent

$$= \sqrt{1.28 \times 10^4 \times 10^4 / 8} \text{ cm}$$

$$= 4, 000 \text{ cm}$$

1 cm on drawing represent 4, 000 cm, Means RF =  $1 / 4000$   
 Assuming length of scale 15 cm, it will represent 600 m.

Draw a line 15 cm long.

It will represent 600 m. Divide it in six equal parts.

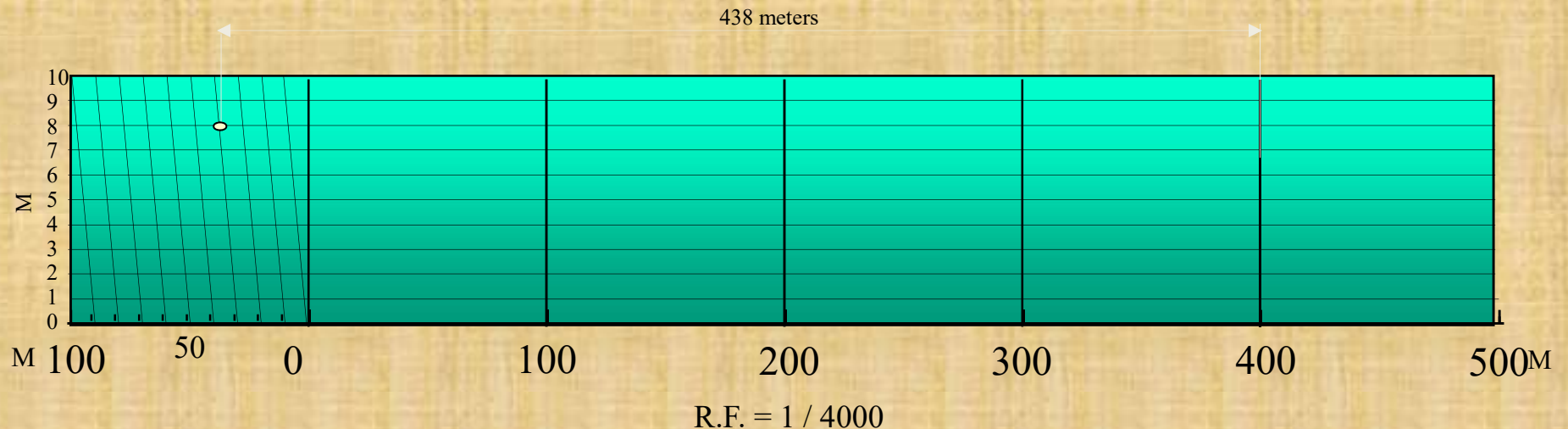
( each will represent 100 m.)

Divide first division in ten equal parts. Each will represent 10 m.

Draw a line upward from left end and mark 10 parts on it of any distance.

Name those parts 0 to 10 as shown. Join 9<sup>th</sup> sub-division of horizontal scale with 10<sup>th</sup> division of the vertical divisions.

Then draw parallel lines to this line from remaining sub divisions and complete diagonal scale.



DIAGONAL SCALE SHOWING METERS.



PROBLEM NO.6:. Draw a diagonal scale of R.F. 1: 2.5, showing centimeters and millimeters and long enough to measure up to 20 centimeters.

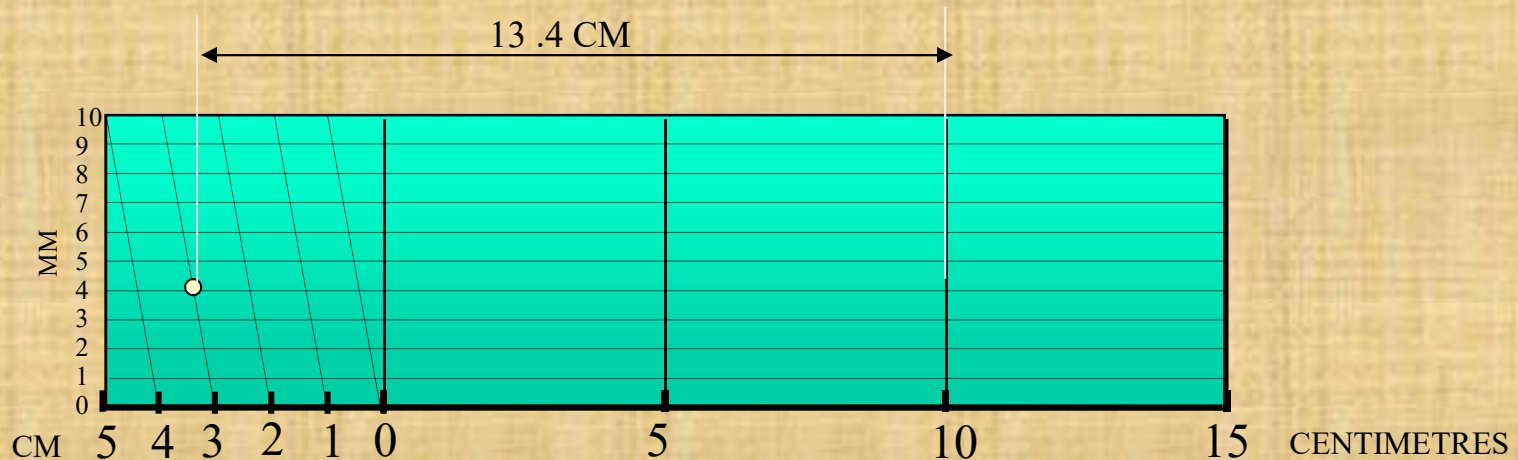
## DIAGONAL SCALE

### SOLUTION STEPS:

$$\text{R.F.} = 1 / 2.5$$

$$\begin{aligned}\text{Length of scale} &= 1 / 2.5 \times 20 \text{ cm.} \\ &= 8 \text{ cm.}\end{aligned}$$

1. Draw a line 8 cm long and divide it into 4 equal parts.  
(Each part will represent a length of 5 cm.)
2. Divide the first part into 5 equal divisions.  
(Each will show 1 cm.)
3. At the left hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.
4. Complete the scale as explained in previous problems.  
Show the distance 13.4 cm on it.



$$\text{R.F.} = 1 / 2.5$$

DIAGONAL SCALE SHOWING CENTIMETERS.

## COMPARATIVE SCALES:

These are the Scales having same R.F.  
but graduated to read different units.

These scales may be Plain scales or Diagonal scales  
and may be constructed separately or one above the other.

### SOLUTION STEPS:

#### Scale of Miles:

40 miles are represented = 8 cm

: 80 miles = 16 cm

R.F. =  $8 / 40 \times 1609 \times 1000 \times 100$

=  $1 / 8,04,500$

#### Scale of Km:

Length of scale

=  $1 / 8,04,500 \times 120 \times 1000 \times 100$

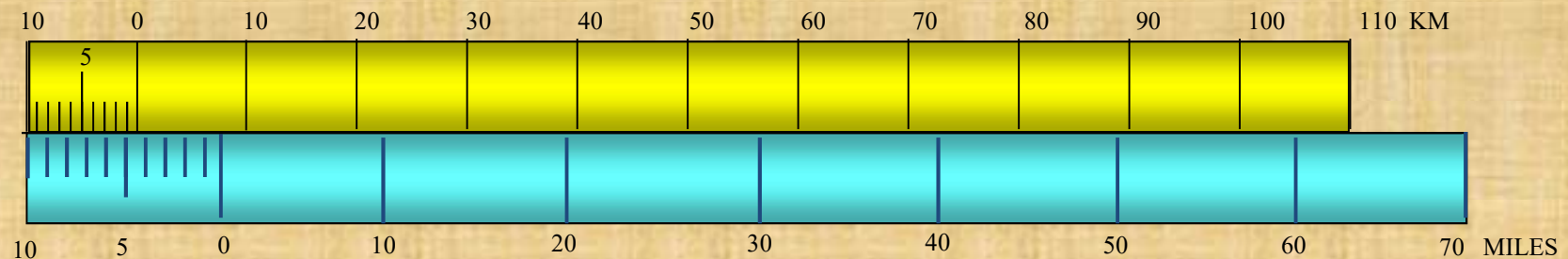
= 14.90 cm

### CONSTRUCTION:

Take a line 16 cm long and divide it into 8 parts. Each will represent 10 miles.  
Subdivide the first part and each sub-division will measure single mile.

### CONSTRUCTION:

On the top line of the scale of miles cut off a distance of 14.90 cm and divide  
it into 12 equal parts. Each part will represent 10 km.  
Subdivide the first part into 10 equal parts. Each subdivision will show single km.



R.F. =  $1 / 804500$

COMPARATIVE SCALE SHOWING MILES AND KILOMETERS



## COMPARATIVE SCALE:

### EXAMPLE NO. 8 :

A motor car is running at a speed of 60 kph.  
On a scale of  $RF = 1 / 4,00,000$  show the distance traveled by car in 47 minutes.

### SOLUTION STEPS:

#### *Scale of km.*

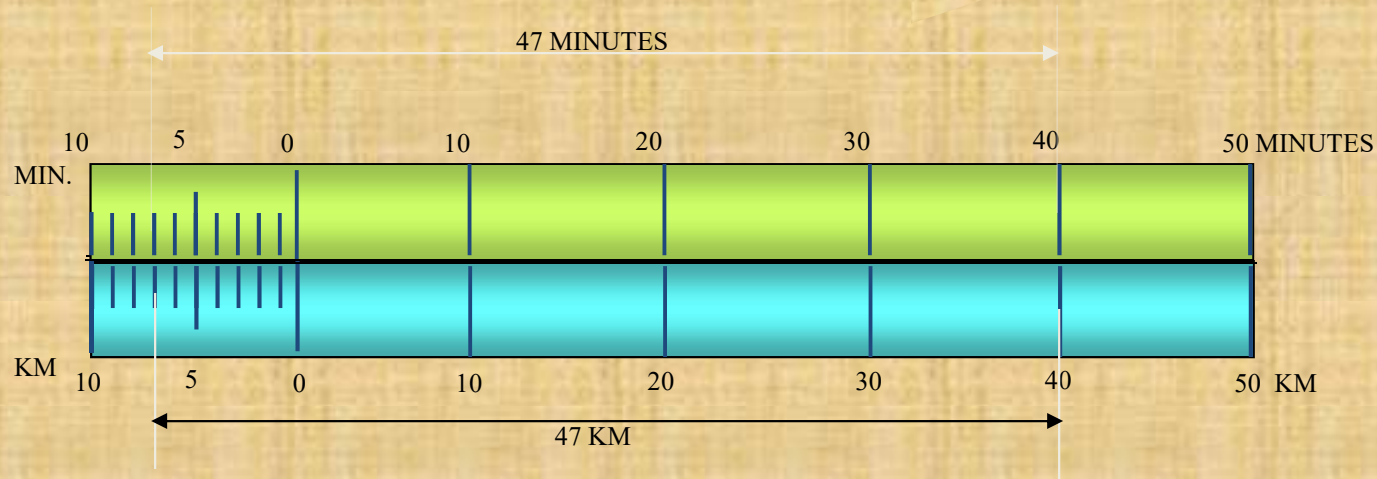
$$\begin{aligned}\text{length of scale} &= RF \times 60 \text{ km} \\ &= 1 / 4,00,000 \times 60 \times 10^5 \\ &= 15 \text{ cm.}\end{aligned}$$

### CONSTRUCTION:

Draw a line 15 cm long and divide it in 6 equal parts.  
( each part will represent 10 km.)  
Subdivide 1<sup>st</sup> part in 10 equal subdivisions.  
( each will represent 1 km.)

#### *Time Scale:*

Same 15 cm line will represent 60 minutes.  
Construct the scale similar to distance scale.  
It will show minimum 1 minute & max. 60min.



$$R.F. = 1 / 4,00,000$$

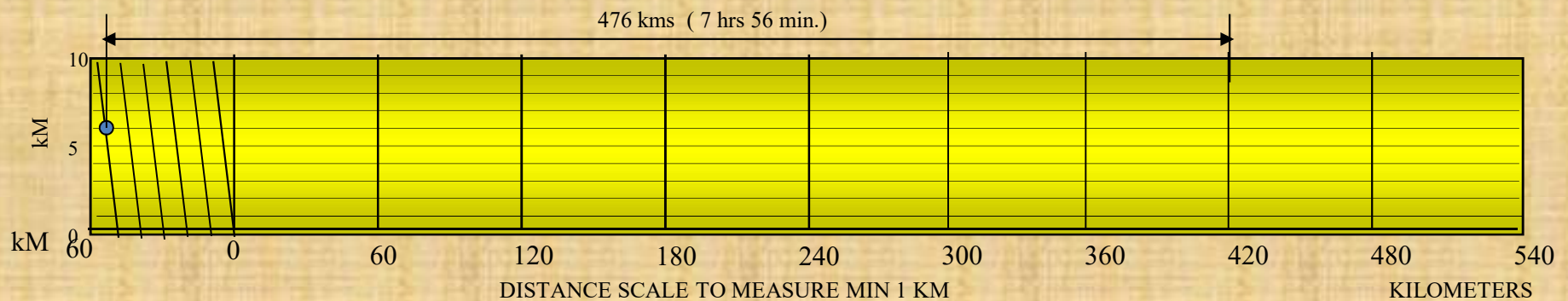
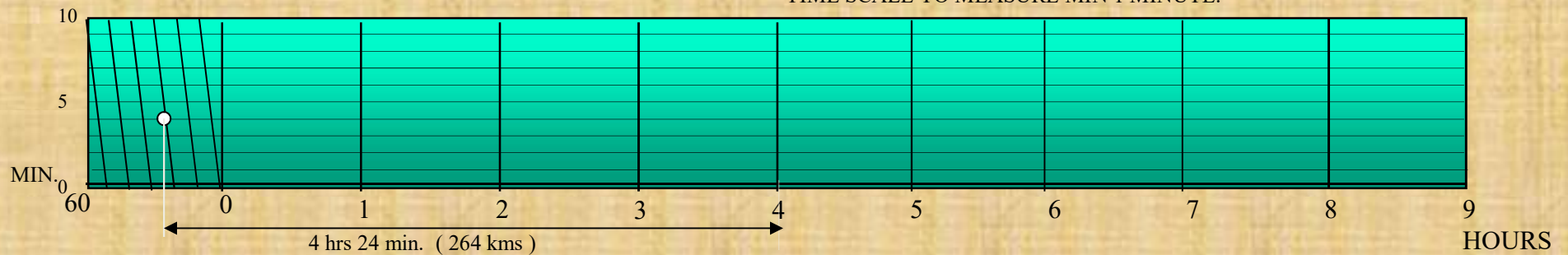
COMPARATIVE SCALE SHOWING MINUTES AND KILOMETERS

### EXAMPLE NO. 9 :

A car is traveling at a speed of 60 km per hour. A 4 cm long line represents the distance traveled by the car in two hours. Construct a suitable comparative scale up to 10 hours. The scale should be able to read the distance traveled in one minute. Show the time required to cover 476 km and also distance in 4 hours and 24 minutes.

### COMPARATIVE SCALE

TIME SCALE TO MEASURE MIN 1 MINUTE.





## Vernier Scales:

These scales, like diagonal scales, are used to read to a very small unit with great accuracy. It consists of two parts – a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions.

As it would be difficult to sub-divide the minor divisions in ordinary way, it is done with the help of the vernier. The graduations on vernier are derived from those on the primary scale.

Figure to the right shows a part of a plain scale in which length A-O represents 10 cm. If we divide A-O

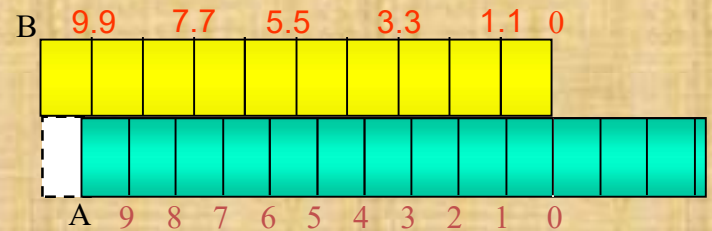
into ten equal parts, each will be of 1 cm. Now it would

not be easy to divide each of these parts into ten equal divisions to get measurements in millimeters.

Now if we take a length BO equal to  $10 + 1 = 11$  such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent  $11 / 10 = 1.1$  cm.

The difference between one part of AO and one division of BO will be equal  $1.1 - 1.0 = 0.1$  cm or 1 mm.

*This difference is called Least Count of the scale.*



Draw a Vernier scale of RF = 1 / 25 to read centimeters upto 4 meters and on it, show lengths 2.39 m and 0.91 m

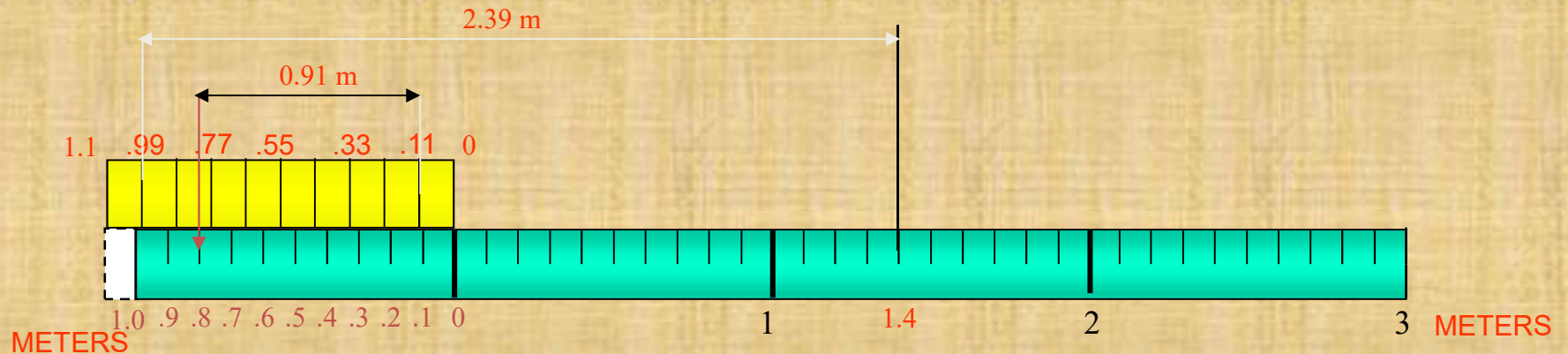
$$\begin{aligned}\text{Length of scale} &= \text{RF} \times \text{max. Distance} \\ &= 1/25 \times 4 \times 100 \\ &= 16 \text{ cm}\end{aligned}$$

Draw a line 16 cm long.  
Divide it in 4 equal parts.  
( each will represent meter )  
Sub-divide each part in 10 equal parts  
( each will represent decimeter )  
Name those properly.

Take 11 parts of Dm length and divide it in 10 equal parts.  
Each will show 0.11 m or 1.1 dm or 11 cm and construct a rectangle  
Covering these parts of Vernier.

(1) For 2.39 m : Subtract 0.99 from 2.39 i.e.  $2.39 - .99 = 1.4$  m  
The distance between 0.99 ( left of Zero) and 1.4 (right of Zero) is 2.39 m

(2) For 0.91 m : Subtract 0.11 from 0.91 i.e.  $0.91 - 0.11 = 0.80$  m  
The distance between 0.11 and 0.80 (both left side of Zero) is 0.91 m





**Example 11:** A map of size 500cm X 50cm wide represents an area of 6250 sq.Kms.  
Construct a vernier scale to measure kilometers, hectometers and decameters  
and long enough to measure upto 7 km. Indicate on it a) 5.33 km b) 59 decameters.

## Vernier Scale

**SOLUTION:**

$$\begin{aligned} \text{RF} &= \sqrt{\frac{\text{AREA OF DRAWING}}{\text{ACTUAL AREA}}} \\ &= \sqrt{\frac{500 \times 50 \text{ cm sq.}}{6250 \text{ km sq.}}} \\ &= 2 / 10^5 \end{aligned}$$

Length of  
scale = RF X max. Distance  
=  $2 / 10^5 \times 7 \text{ kms}$   
= 14 cm

**CONSTRUCTION: ( Main scale )**

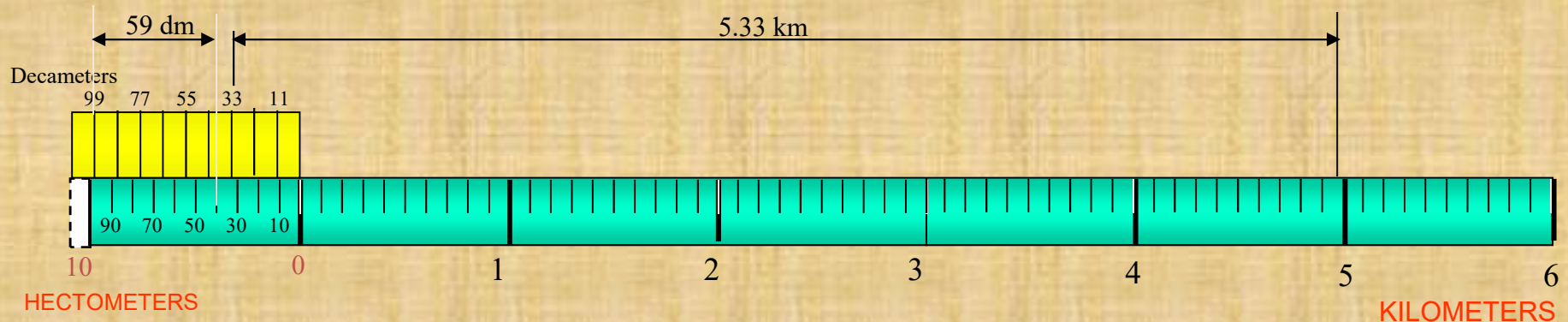
Draw a line 14 cm long.  
Divide it in 7 equal parts.  
( each will represent km )  
Sub-divide each part in 10 equal parts.  
( each will represent hectometer )  
Name those properly.

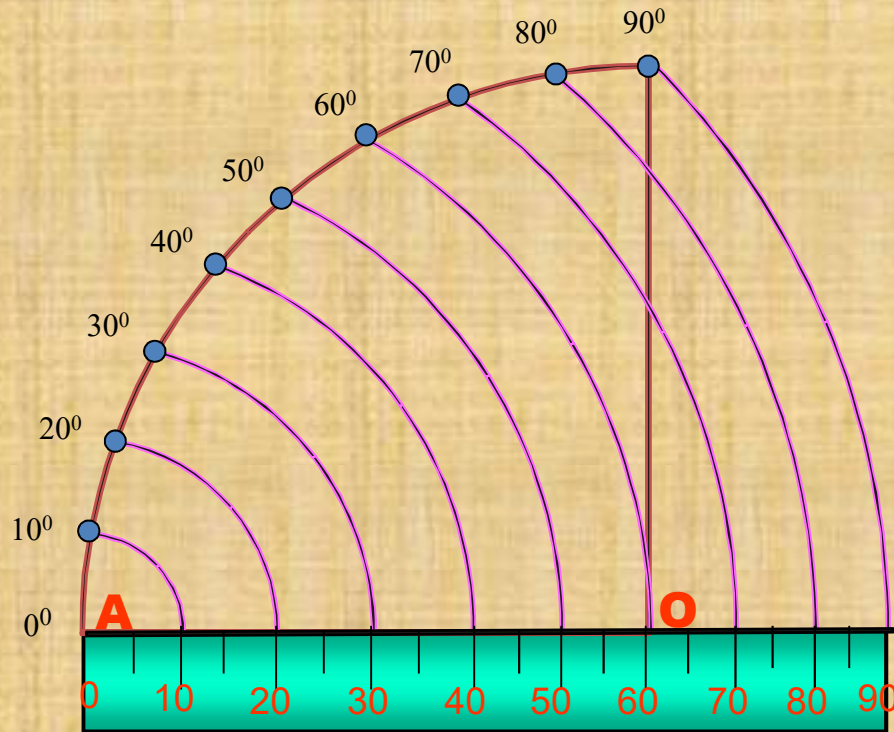
**CONSTRUCTION: ( Vernier )**

Take 11 parts of hectometer part length  
and divide it in 10 equal parts.  
Each will show 1.1 hm m or 11 dm and  
Covering in a rectangle complete scale.

**TO MEASURE GIVEN LENGTHS:**

a) For 5.33 km :  
Subtract 0.33 from 5.33  
i.e.  $5.33 - 0.33 = 5.00$   
The distance between 33 dm  
( left of Zero ) and  
5.00 (right of Zero) is 5.33 k m  
(b) For 59 dm :  
Subtract 0.99 from 0.59  
i.e.  $0.59 - 0.99 = - 0.4 \text{ km}$   
( - ve sign means left of Zero )  
The distance between 99 dm and  
- .4 km is 59 dm  
(both left side of Zero)





## SCALE OF CORDS

### CONSTRUCTION:

1. DRAW SECTOR OF A CIRCLE OF  $90^\circ$  WITH 'OA' RADIUS.  
( 'OA' ANY CONVINIENT DISTANCE )
2. DIVIDE THIS ANGLE IN NINE EQUAL PARTS OF  $10^\circ$  EACH.
3. NAME AS SHOWN FROM END 'A' UPWARDS.
4. FROM 'A' AS CENTER, WITH CORDS OF EACH ANGLE AS RADIUS  
DRAW ARCS DOWNWARDS UP TO 'AO' LINE OR IT'S EXTENSION  
AND FORM A SCALE WITH PROPER LABELING AS SHOWN.

AS CORD LENGTHS ARE USED TO MEASURE & CONSTRUCT  
DIFERENT ANGLES IT IS CALLED SCALE OF CORDS.



## PROBLEM 12: Construct any triangle and measure it's angles by using scale of cords.

### CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then construct a triangle of given sides. ( You are supposed to measure angles x, y and z)

#### To measure angle at x:

Take O-A distance in compass from cords scale and mark it on lower side of triangle as shown from corner x. Name O & A as shown. Then O as center, O-A radius draw an arc upto upper adjacent side. Name the point B.

Take A-B cord in compass and place on scale of cords from Zero.

It will give value of angle at x

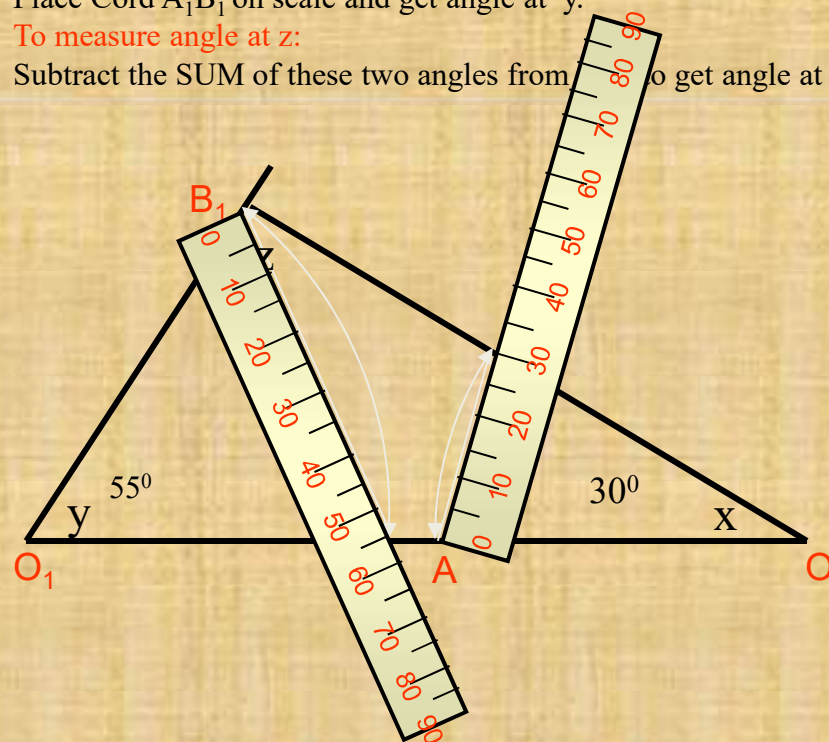
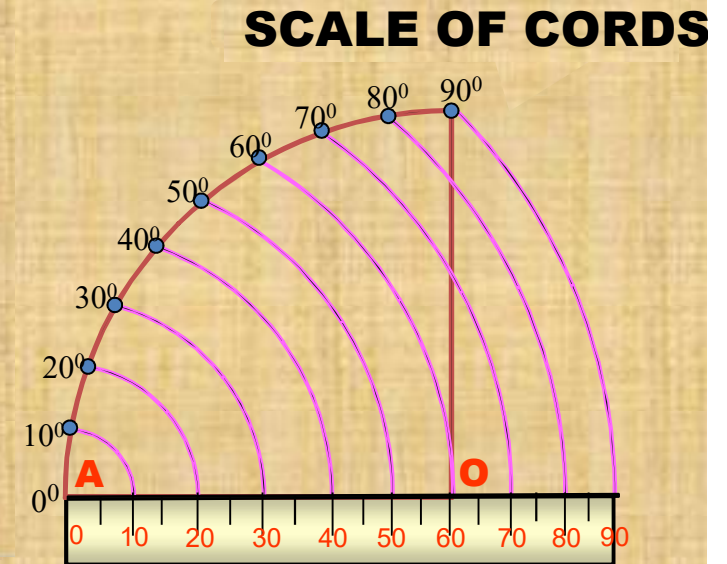
#### To measure angle at y:

Repeat same process from  $O_1$ . Draw arc with radius  $O_1A_1$ .

Place Cord  $A_1B_1$  on scale and get angle at y.

#### To measure angle at z:

Subtract the SUM of these two angles from 180 to get angle at z.



$$\text{Angle at } z = 180 - (55 + 30) = 95^\circ$$

## PROBLEM 12: Construct $25^\circ$ and $115^\circ$ angles with a horizontal line, by using scale of cords

### CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then Draw a horizontal line. Mark point O on it.

To construct  $25^\circ$  angle at O.

Take O-A distance in compass from cords scale and mark it on the line drawn, from O. Name O & A as shown. Then O as center, O-A radius draw an arc upward..

Take cord length of  $25^\circ$  angle from scale of cords in compass and from A cut the arc at point B. Join B with O. The angle AOB is thus  $25^\circ$

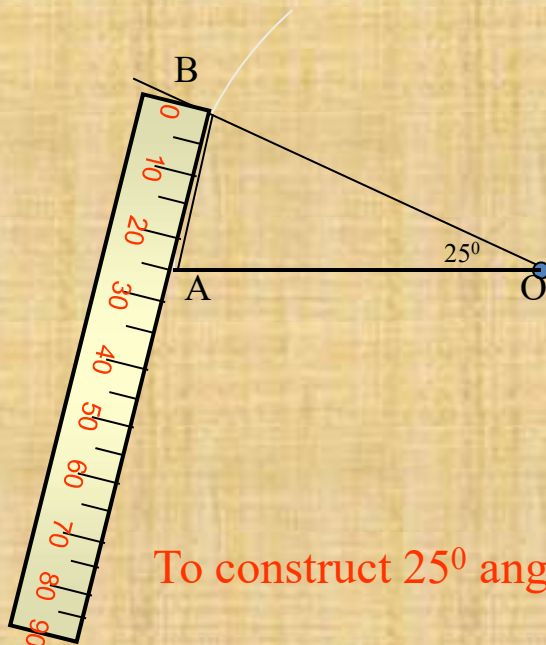
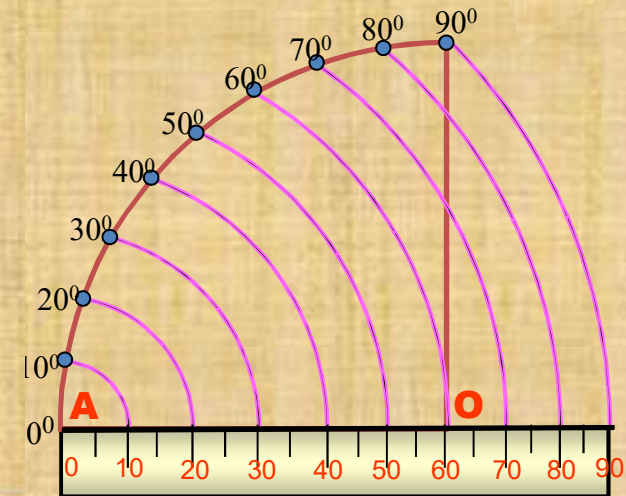
To construct  $115^\circ$  angle at O.

*This scale can measure or construct angles upto  $90^\circ$  only directly.*

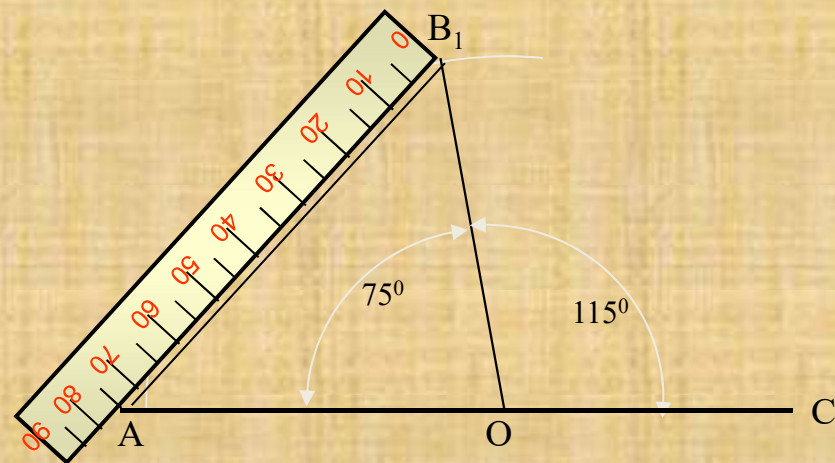
Hence Subtract  $115^\circ$  from  $180^\circ$ . We get  $75^\circ$  angle, which can be constructed with this scale.

Extend previous arc of OA radius and taking cord length of  $75^\circ$  in compass cut this arc at  $B_1$  with A as center. Join  $B_1$  with O. Now angle  $AOB_1$  is  $75^\circ$  and angle  $COB_1$  is  $115^\circ$ .

### SCALE OF CORDS



To construct  $25^\circ$  angle at O.



To construct  $115^\circ$  angle at O.



## PROBLEM 12: Construct $25^\circ$ and $115^\circ$ angles with a horizontal line, by using scale of cords

### CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then Draw a horizontal line. Mark point O on it.

To construct  $25^\circ$  angle at O.

Take O-A distance in compass from cords scale and mark it on the line drawn, from O. Name O & A as shown. Then O as center, O-A radius draw an arc upward..

Take cord length of  $25^\circ$  angle from scale of cords in compass and from A cut the arc at point B. Join B with O. The angle AOB is thus  $25^\circ$

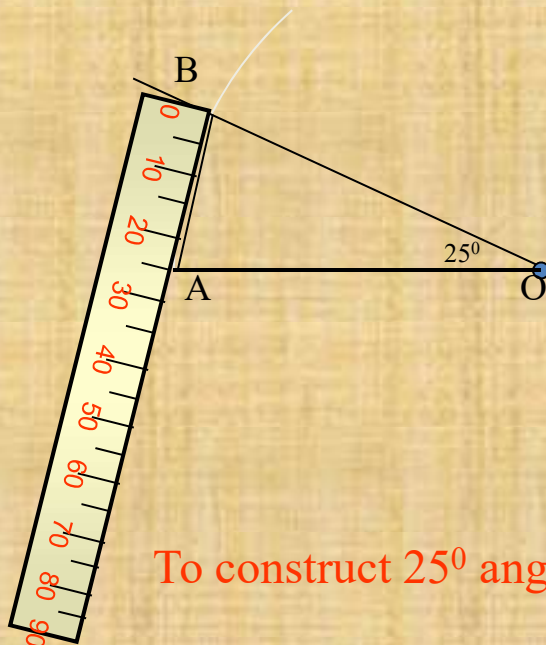
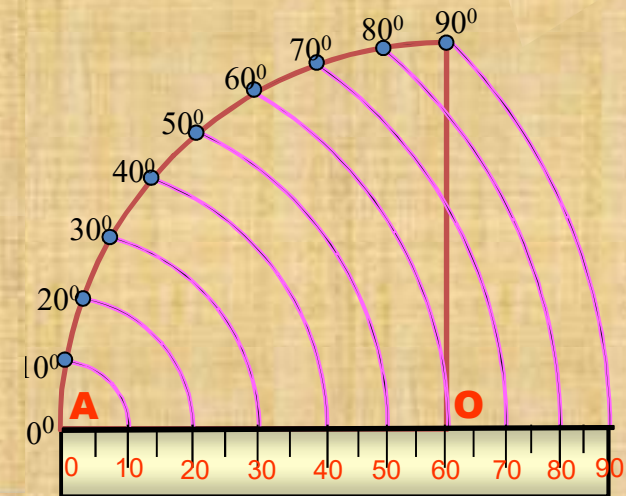
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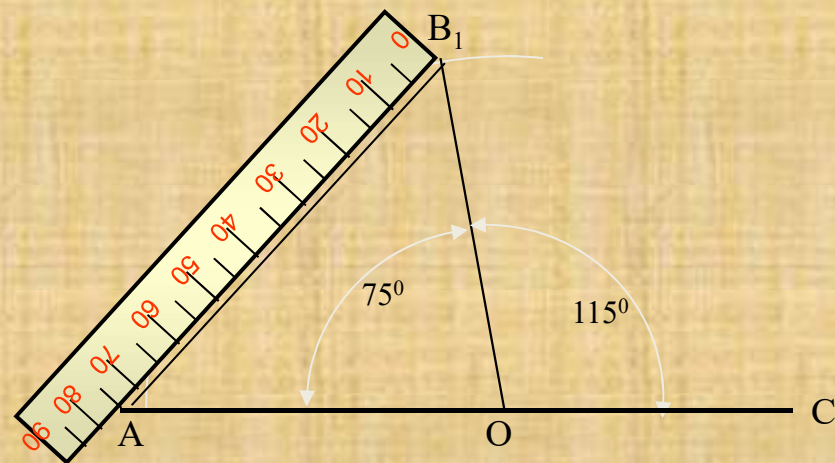
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### SCALE OF CORDS



To construct  $25^\circ$  angle at O.

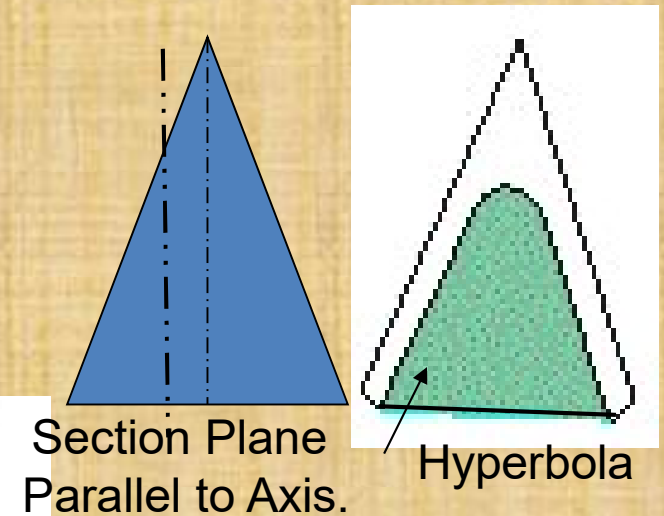
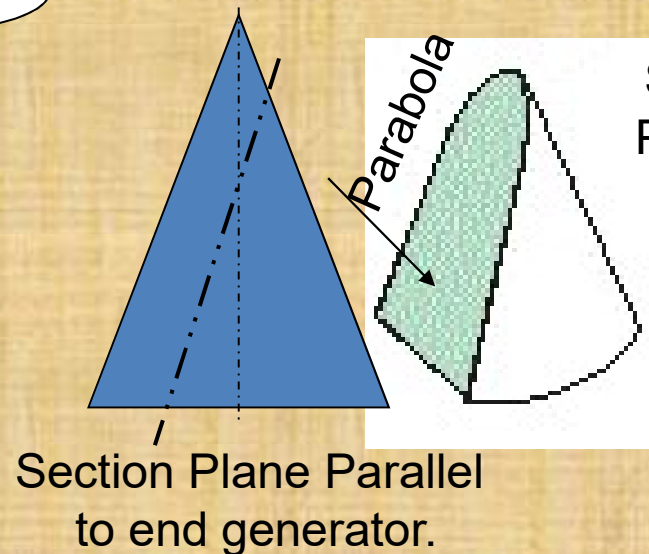
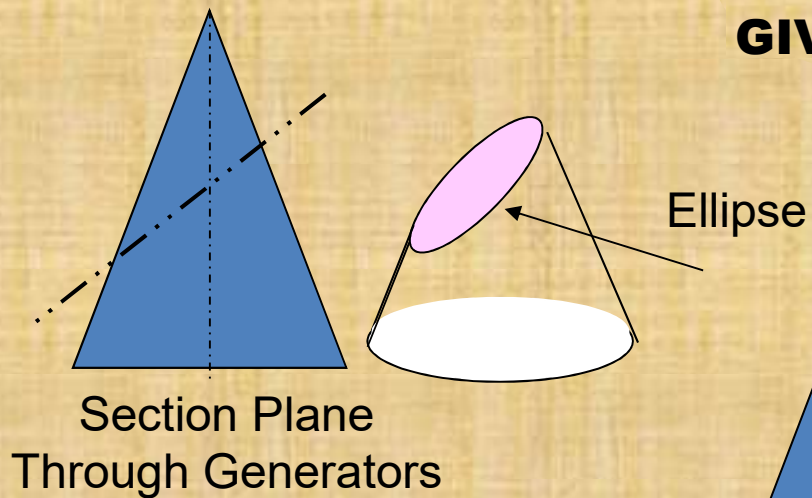


To construct  $115^\circ$  angle at O.

## CONIC SECTIONS

ELLIPSE, PARABOLA AND HYPERBOLA ARE CALLED CONIC SECTIONS  
BECAUSE  
THESE CURVES APPEAR ON THE SURFACE OF A CONE  
WHEN IT IS CUT BY SOME TYPICAL CUTTING PLANES.

**OBSERVE  
ILLUSTRATIONS  
GIVEN BELOW..**





## PROBLEM 12: Construct $25^\circ$ and $115^\circ$ angles with a horizontal line, by using scale of cords

### CONSTRUCTION:

First prepare Scale of Cords for the problem.

Then Draw a horizontal line. Mark point O on it.

To construct  $25^\circ$  angle at O.

Take O-A distance in compass from cords scale and mark it on the line drawn, from O. Name O & A as shown. Then O as center, O-A radius draw an arc upward..

Take cord length of  $25^\circ$  angle from scale of cords in compass and from A cut the arc at point B. Join B with O. The angle AOB is thus  $25^\circ$

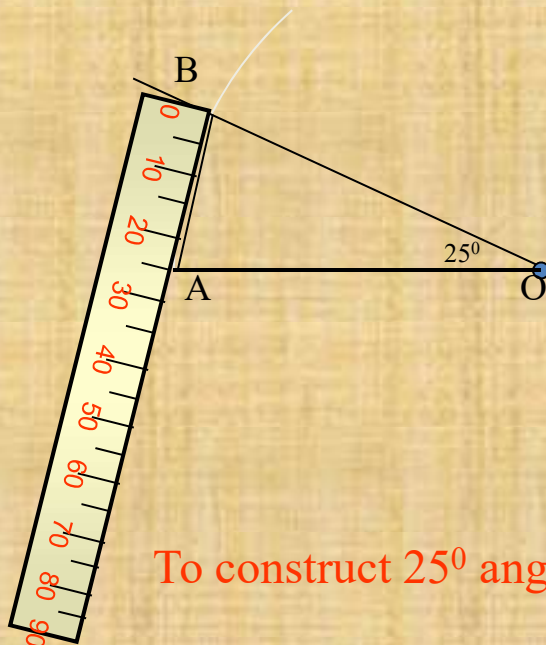
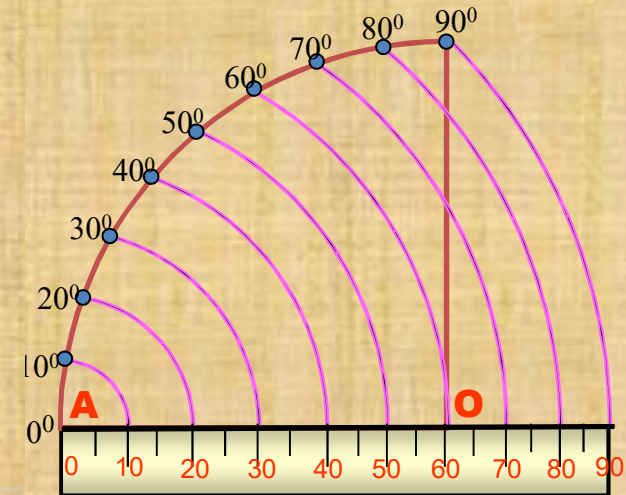
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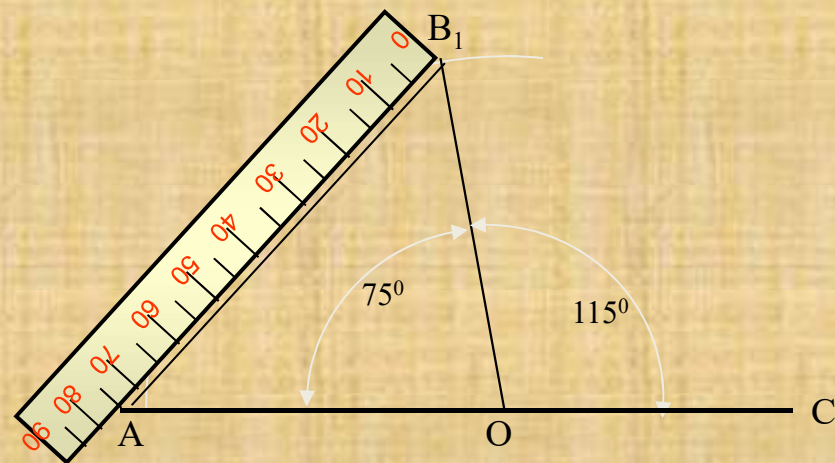
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Extend previous arc of OA radius and taking cord length of  $75^\circ$  in compass cut this arc at  $B_1$  with A as center. Join  $B_1$  with O. Now angle  $AOB_1$  is  $75^\circ$  and angle  $COB_1$  is  $115^\circ$ .

### SCALE OF CORDS



To construct  $25^\circ$  angle at O.



To construct  $115^\circ$  angle at O.

## COMMON DEFINATION OF ELLIPSE, PARABOLA & HYPERBOLA:

These are the loci of points moving in a plane such that the ratio of it's distances from a *fixed point* And a *fixed line* always remains constant.

The Ratio is called ECCENTRICITY. (E)

- A) For Ellipse  $E < 1$
- B) For Parabola  $E = 1$
- C) For Hyperbola  $E > 1$

Refer Problem nos. 6. 9 & 12

## SECOND DEFINATION OF AN ELLIPSE:-

It is a locus of a point moving in a plane such that the SUM of it's distances from TWO fixed points always remains constant.

{And this *sum equals* to the length of *major axis*.}

These TWO fixed points are FOCUS 1 & FOCUS 2

Refer Problem no.4

Ellipse by Arcs of Circles Method.



# **ELLIPSE**

## ***BY CONCENTRIC CIRCLE METHOD***

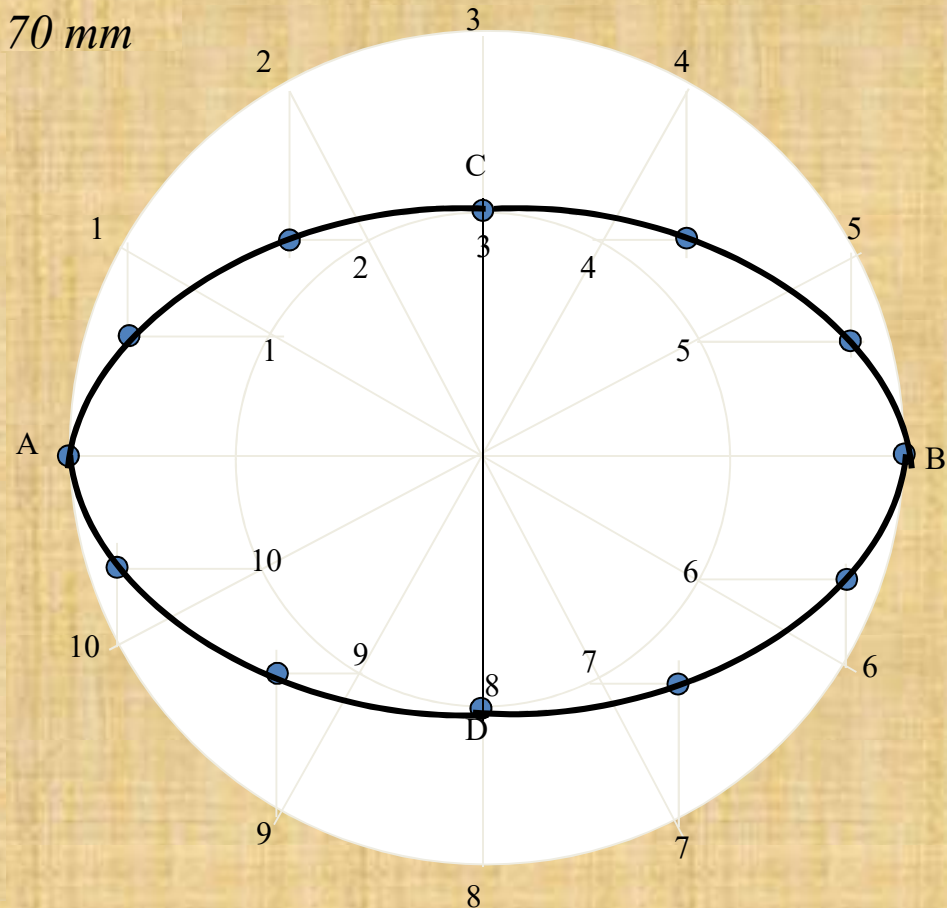
Problem 1 :-

*Draw ellipse by concentric circle method.*

*Take major axis 100 mm and minor axis 70 mm long.*

Steps:

1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
3. Divide both circles in 12 equal parts & name as shown.
4. From all points of outer circle draw vertical lines downwards and upwards respectively.
5. From all points of inner circle draw horizontal lines to intersect those vertical lines.
6. Mark all intersecting points properly as those are the points on ellipse.
7. Join all these points along with the ends of both axes in smooth possible curve. It is required ellipse.



## ELLIPSE

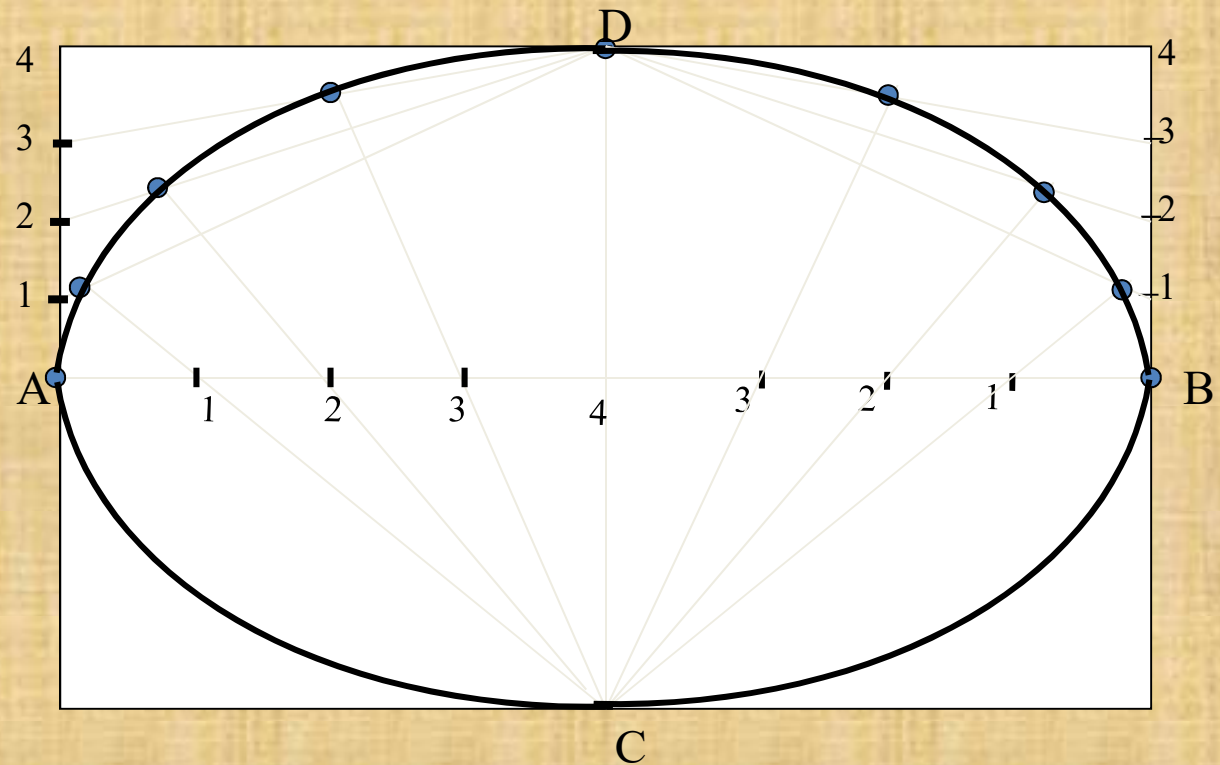
BY RECTANGLE METHOD

Steps:

- 1 Draw a rectangle taking major and minor axes as sides.
  2. In this rectangle draw both axes as perpendicular bisectors of each other..
  3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts.( here divided in four parts)
  4. Name those as shown..
  5. Now join all vertical points 1,2,3,4, to the upper end of minor axis. And all horizontal points i.e.1,2,3,4 to the lower end of minor axis.
  6. Then extend C-1 line upto D-1 and mark that point. Similarly extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
  7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side part.along with lower half of the rectangle.Join all points in smooth curve.
- It is required ellipse.

### Problem 2

*Draw ellipse by Rectangle method.  
Take major axis 100 mm and minor axis 70 mm long.*





## ELLIPSE

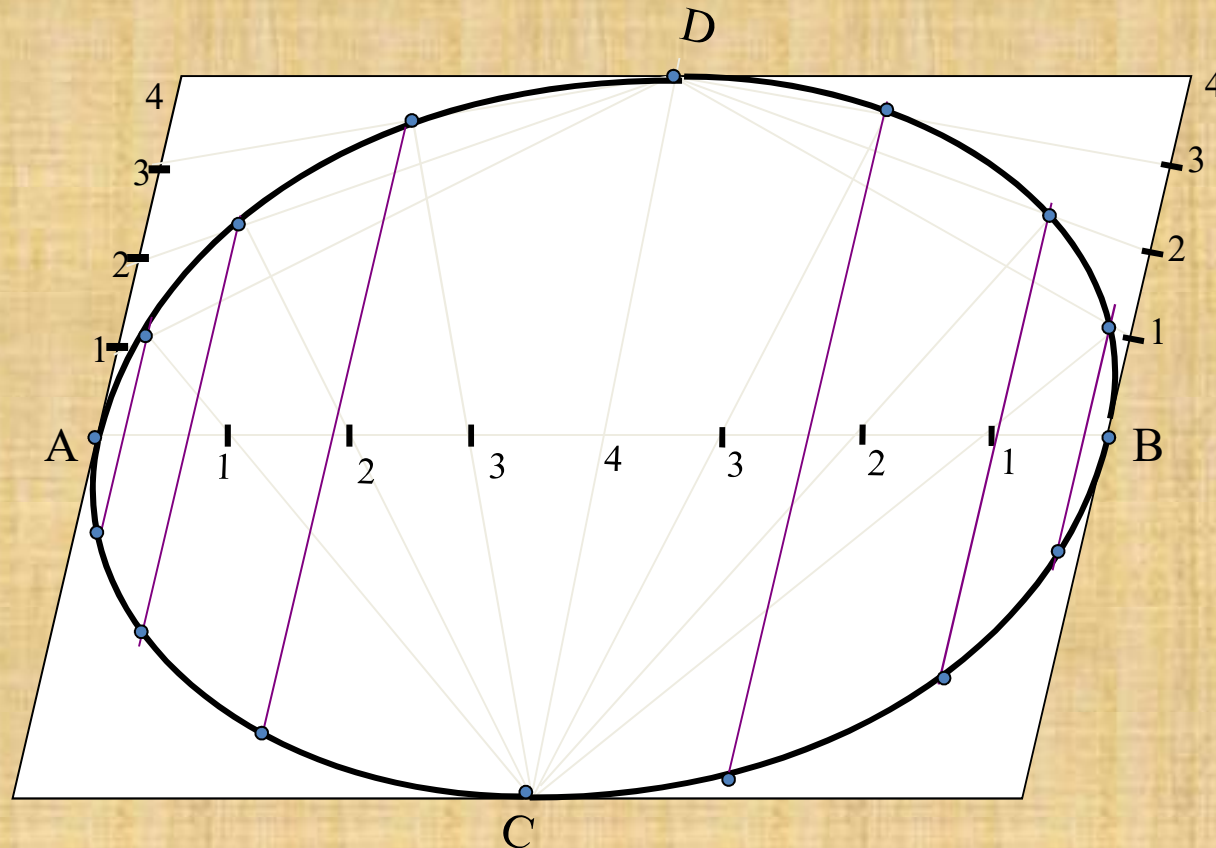
BY OBLONG METHOD

Problem 3:-

*Draw ellipse by Oblong method.*

*Draw a parallelogram of 100 mm and 70 mm long sides with included angle of  $75^\circ$ . Inscribe*

STEPS ARE SIMILAR TO  
THE PREVIOUS CASE  
(RECTANGLE METHOD)  
ONLY IN PLACE OF RECTANGLE,  
HERE IS A PARALLELOGRAM.



#### PROBLEM 4.

MAJOR AXIS AB & MINOR AXIS CD ARE  
100 AND 70MM LONG RESPECTIVELY  
.DRAW ELLIPSE BY ARCS OF CIRCLES  
METHOD.

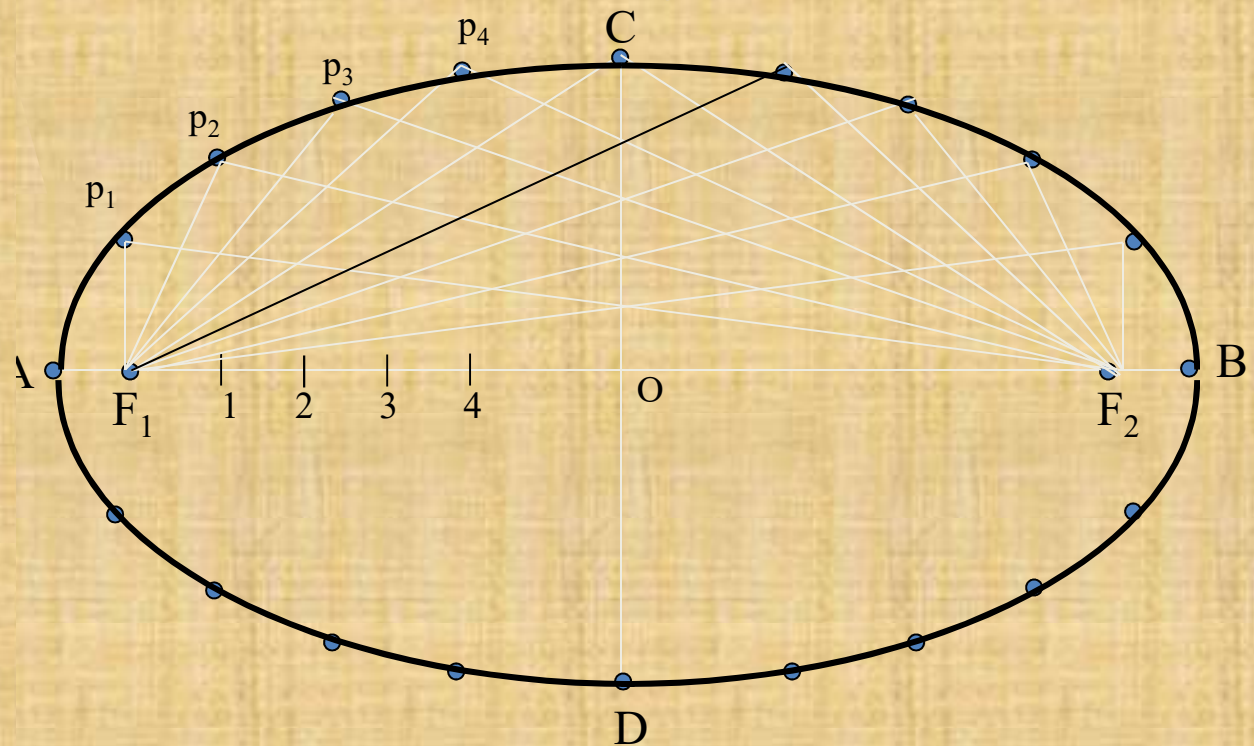
#### STEPS:

1. Draw both axes as usual. Name the ends & intersecting point
2. Taking AO distance I.e. half major axis, from C, mark  $F_1$  &  $F_2$  On AB .  
( focus 1 and 2.)
3. On line  $F_1$ - O taking any distance, mark points 1,2,3, & 4
4. Taking  $F_1$  center, with distance A-1 draw an arc above AB and taking  $F_2$  center, with B-1 distance cut this arc. Name the point  $p_1$
5. Repeat this step with same centers but taking now A-2 & B-2 distances for drawing arcs. Name the point  $p_2$
6. Similarly get all other P points.  
With same steps positions of P can be located below AB.
7. Join all points by smooth curve to get an ellipse/

## ELLIPSE

*BY ARCS OF CIRCLE METHOD*

As per the definition Ellipse is locus of point P moving in a plane such that the **SUM** of it's distances from two fixed points ( $F_1$  &  $F_2$ ) remains constant and equals to the length of major axis AB. (Note  $A . 1 + B . 1 = A . 2 + B . 2 = AB$ )





## **ELLIPSE**

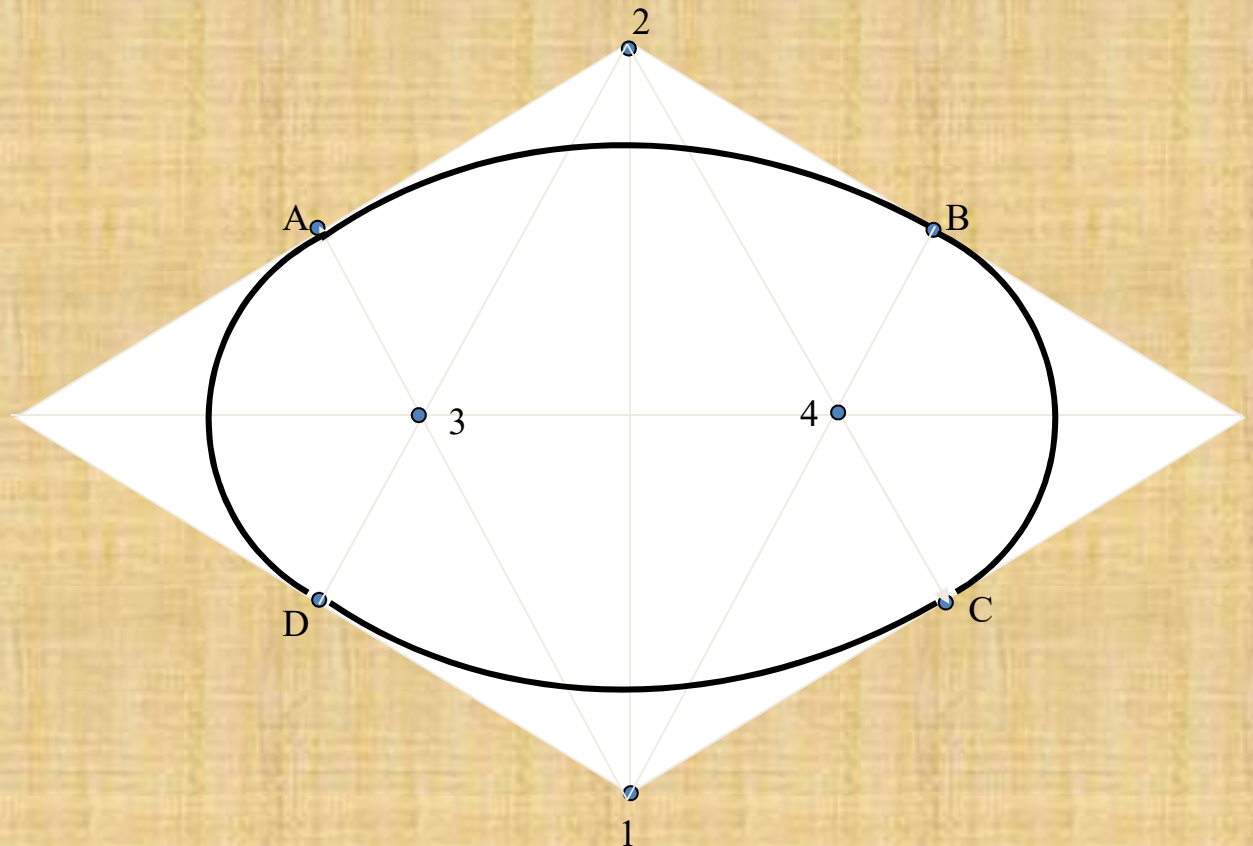
*BY RHOMBUS METHOD*

### PROBLEM 5.

DRAW RHOMBUS OF 100 MM & 70 MM LONG  
DIAGONALS AND INSCRIBE AN ELLIPSE IN IT.

#### STEPS:

1. Draw rhombus of given dimensions.
2. Mark mid points of all sides & name Those A,B,C,& D
3. Join these points to the ends of smaller diagonals.
4. Mark points 1,2,3,4 as four centers.
5. Taking 1 as center and 1-A radius draw an arc AB.
6. Take 2 as center draw an arc CD.
7. Similarly taking 3 & 4 as centers and 3-D radius draw arcs DA & BC.



**PROBLEM 6:-** POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE RATIO OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO  $\frac{2}{3}$  DRAW LOCUS OF POINT P. { ECCENTRICITY =  $\frac{2}{3}$  }

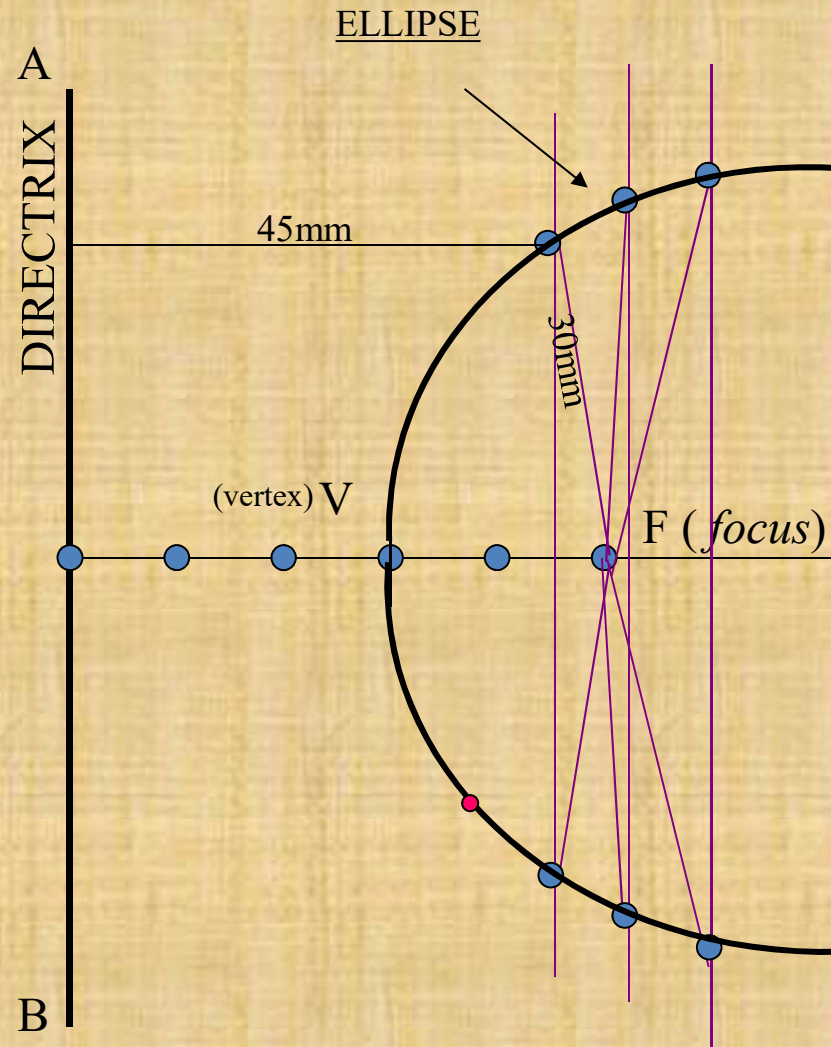
## **ELLIPSE**

*DIRECTRIX-FOCUS METHOD*

### STEPS:

1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2<sup>nd</sup> part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB  $\frac{2}{3}$  i.e  $\frac{20}{30}$
4. Form more points giving same ratio such as  $\frac{30}{45}$ ,  $\frac{40}{60}$ ,  $\frac{50}{75}$  etc.
5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.

This is required locus of P. It is an ELLIPSE.



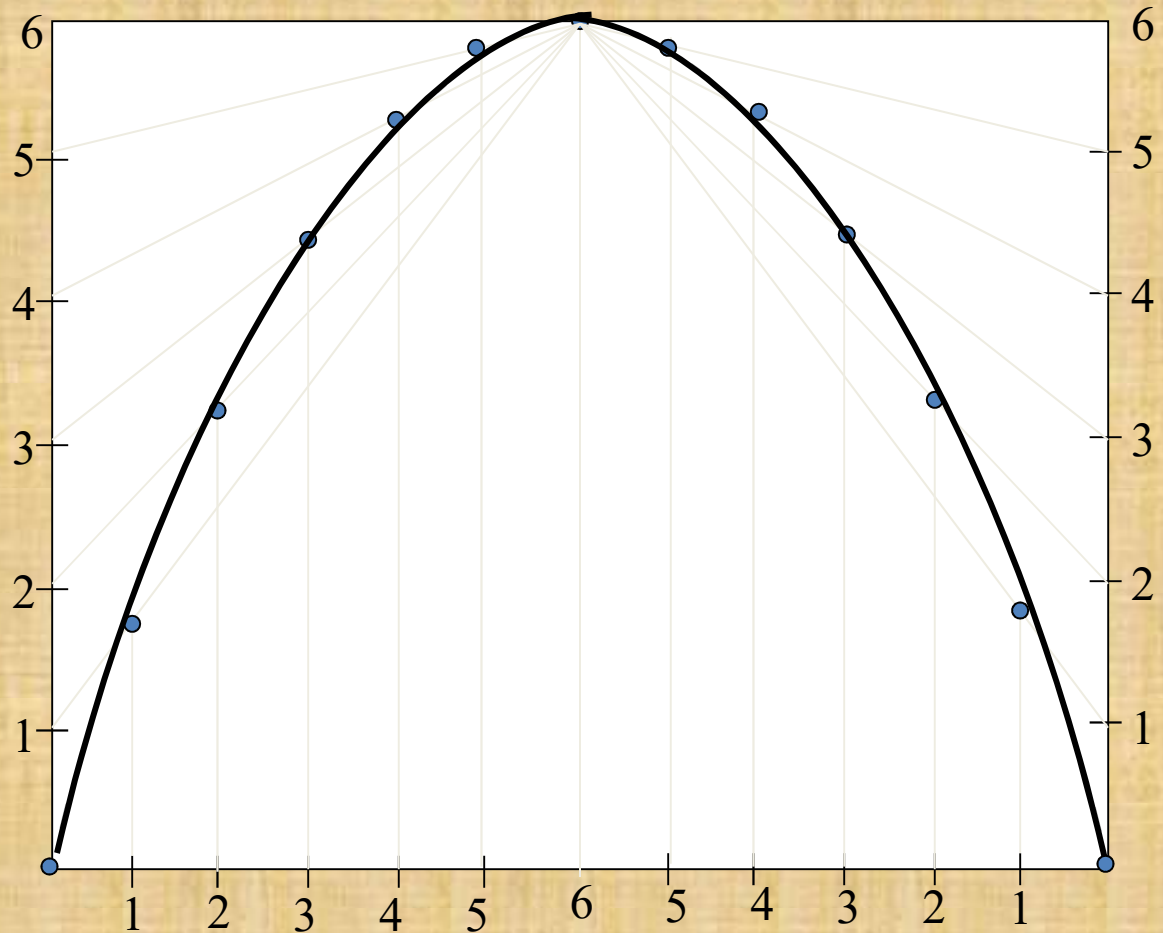


PROBLEM 7: A BALL THROWN IN AIR ATTAINS 100 M HIEGHT  
AND COVERS HORIZONTAL DISTANCE 150 M ON GROUND.  
Draw the path of the ball (projectile)-

## PARABOLA RECTANGLE METHOD

### STEPS:

1. Draw rectangle of above size and divide it in two equal vertical parts
2. Consider left part for construction.  
Divide height and length in equal number of parts and name those 1,2,3,4,5 & 6
3. Join vertical 1,2,3,4,5 & 6 to the top center of rectangle
4. Similarly draw upward vertical lines from horizontal 1,2,3,4,5  
And wherever these lines intersect previously drawn inclined lines in sequence Mark those points and further join in smooth possible curve.
5. Repeat the construction on right side rectangle also. Join all in sequence.  
This locus is Parabola.

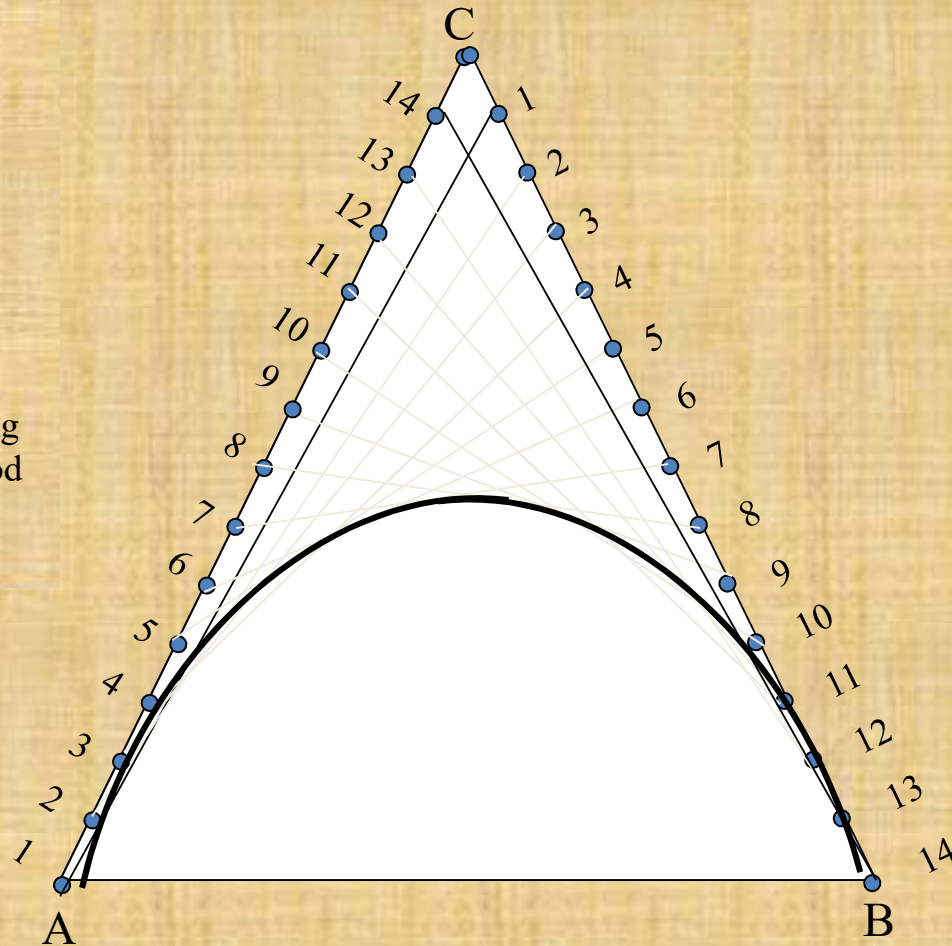


Problem no.8: Draw an isosceles triangle of 100 mm long base and 110 mm long altitude. Inscribe a parabola in it by method of tangents.

## PARABOLA METHOD OF TANGENTS

Solution Steps:

1. Construct triangle as per the given dimensions.
2. Divide its both sides into same no. of equal parts.
3. Name the parts in ascending and descending manner, as shown.
4. Join 1-1, 2-2, 3-3 and so on.
5. Draw the curve as shown i.e. tangent to all these lines. The above all lines being tangents to the curve, it is called method of tangents.





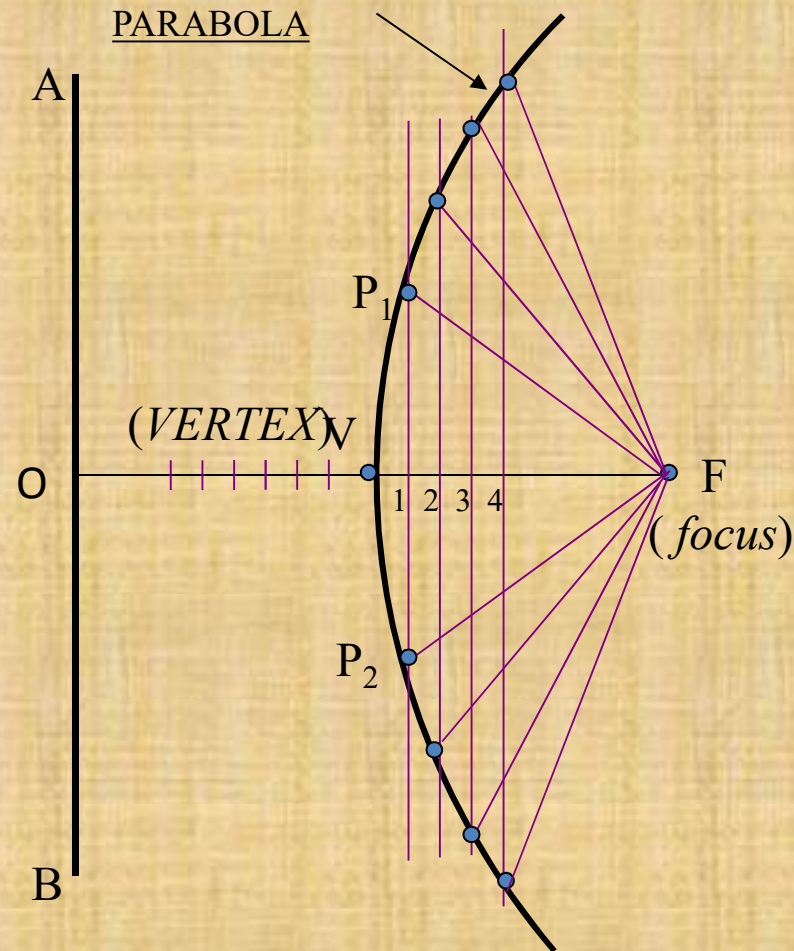
**PROBLEM 9:** Point F is 50 mm from a vertical straight line AB.  
 Draw locus of point P, moving in a plane such that  
 it always remains equidistant from point F and line AB.

## PARABOLA DIRECTRIX-FOCUS METHOD

### SOLUTION STEPS:

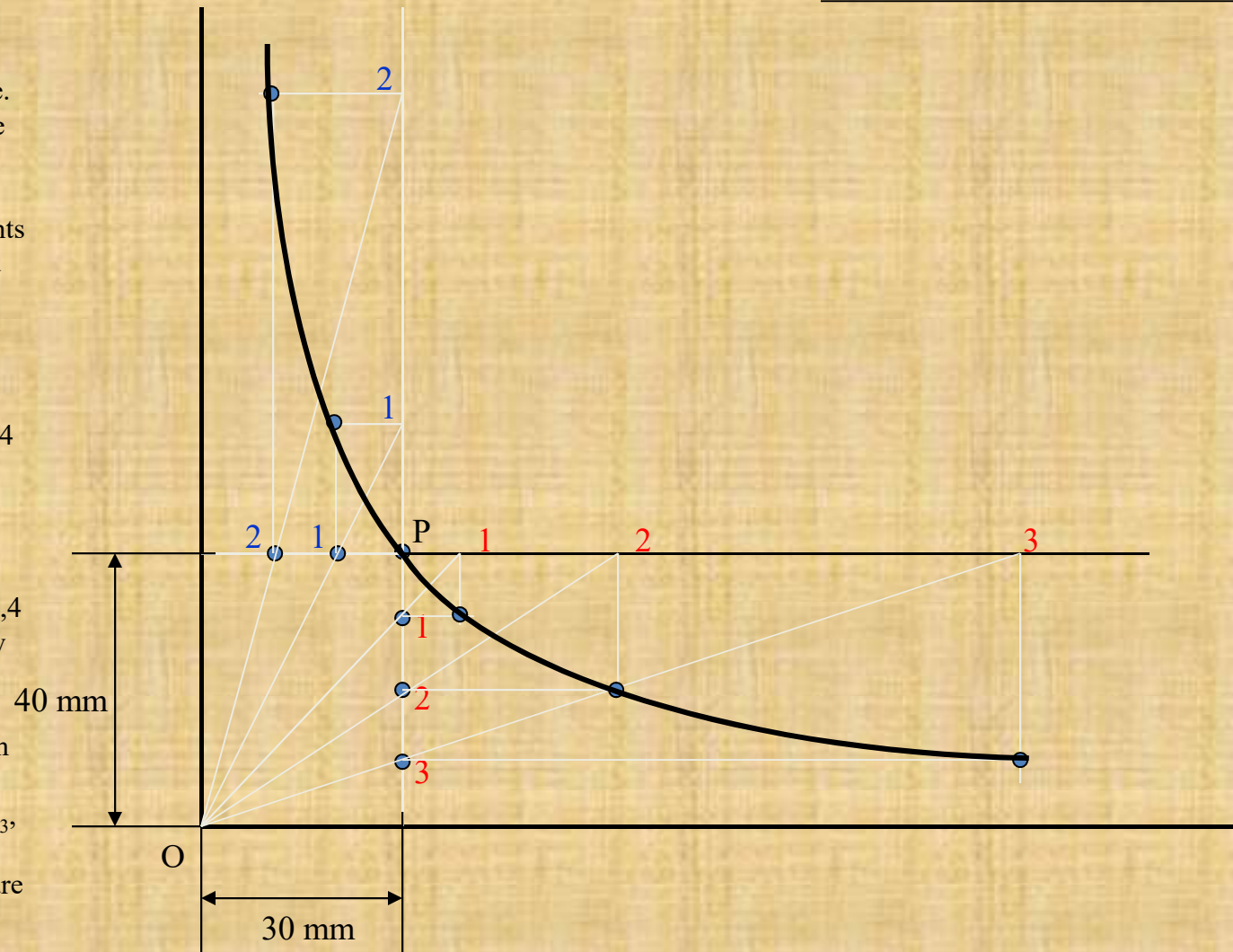
1. Locate center of line, perpendicular to AB from point F. This will be initial point P and also the vertex.
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.
3. Mark 5 mm distance to its left of P and name it 1.
4. Take O-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point  $P_1$  and lower point  $P_2$ .  
 $(FP_1 = O1)$
5. Similarly repeat this process by taking again 5mm to right and left and locate  $P_3P_4$ .
6. Join all these points in smooth curve.

It will be the locus of P equidistance from line AB and fixed point F.



## HYPERBOLA THROUGH A POINT OF KNOWN CO-ORDINATES

- 1) Extend horizontal line from P to right side.
- 2) Extend vertical line from P upward.
- 3) On horizontal line from P, mark some points taking any distance and name them after P-1, 2,3,4 etc.
- 4) Join 1-2-3-4 points to pole O. Let them cut part [P-B] also at 1,2,3,4 points.
- 5) From horizontal 1,2,3,4 draw vertical lines downwards and
- 6) From vertical 1,2,3,4 points [from P-B] draw horizontal lines.
- 7) Line from 1 horizontal and line from 1 vertical will meet at  $P_1$ . Similarly mark  $P_2, P_3, P_4$  points.
- 8) Repeat the procedure by marking four points on upward vertical line from P and joining all those to pole O. Name this points  $P_6, P_7, P_8$  etc. and join them by smooth curve.





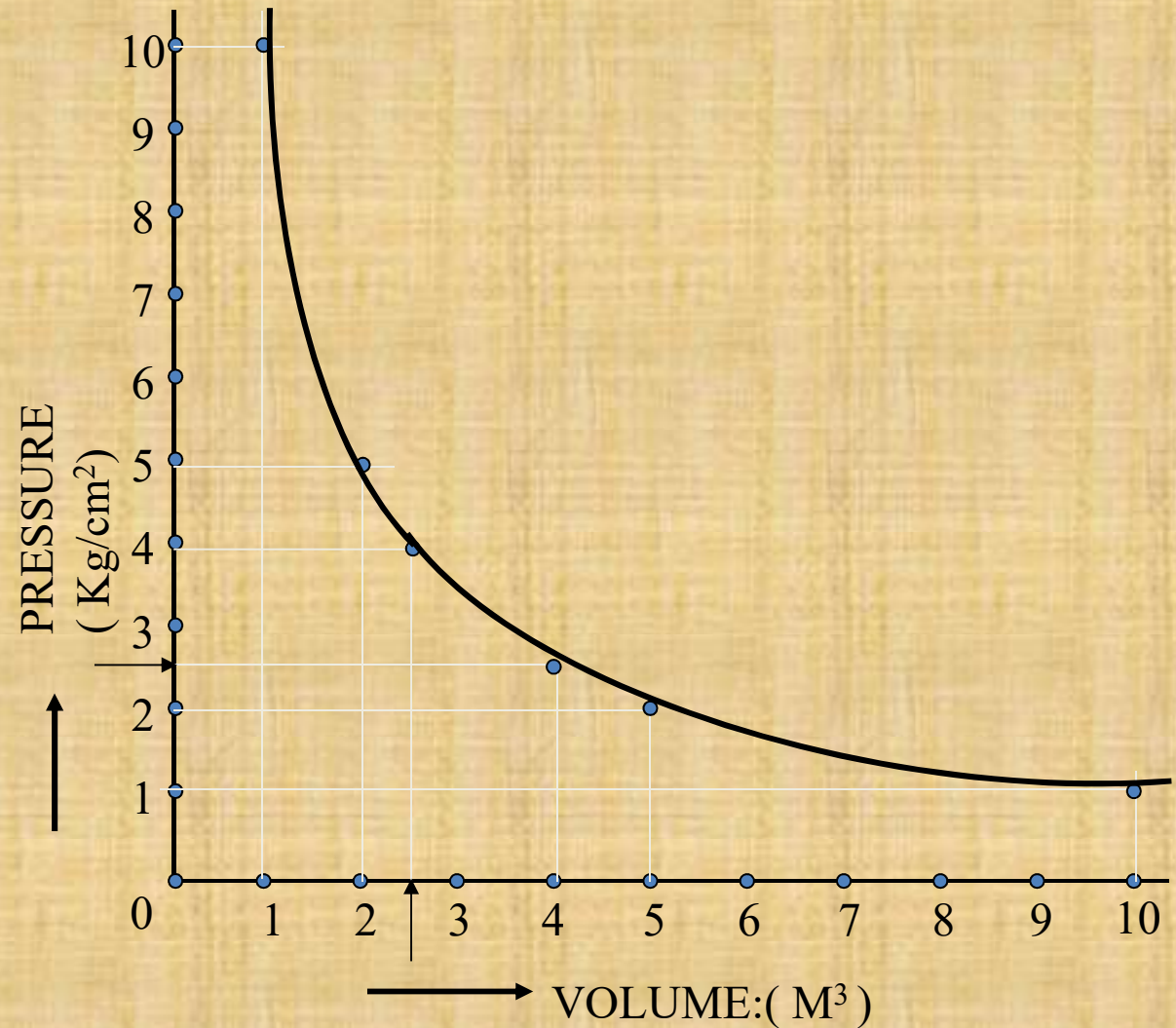
**Problem no.11:** A sample of gas is expanded in a cylinder from 10 unit pressure to 1 unit pressure. Expansion follows law  $PV=\text{Constant}$ . If initial volume being 1 unit, draw the curve of expansion. Also Name the curve.

## HYPERBOLA P-V DIAGRAM

Form a table giving few more values of P & V

$$\begin{aligned} P \times V &= C \\ 10 \times 1 &= 10 \\ 5 \times 2 &= 10 \\ 4 \times 2.5 &= 10 \\ 2.5 \times 4 &= 10 \\ 2 \times 5 &= 10 \\ 1 \times 10 &= 10 \end{aligned}$$

Now draw a Graph of Pressure against Volume.  
It is a PV Diagram and it is Hyperbola.  
Take pressure on vertical axis and Volume on horizontal axis.



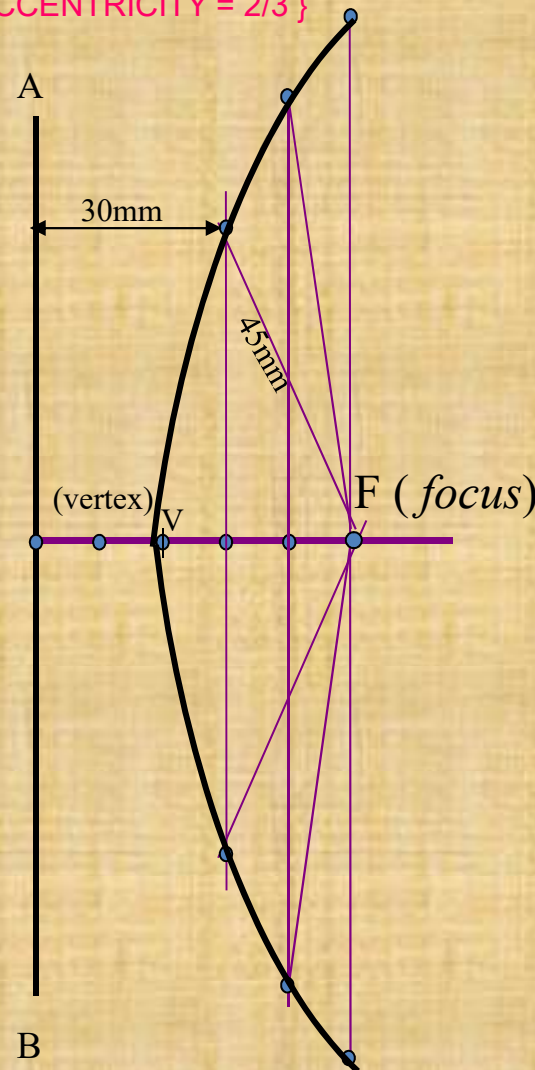
HYPERBOLA  
DIRECTRIX  
FOCUS METHOD

**PROBLEM 12:-** POINT F IS 50 MM FROM A LINE AB. A POINT P IS MOVING IN A PLANE SUCH THAT THE RATIO OF IT'S DISTANCES FROM F AND LINE AB REMAINS CONSTANT AND EQUALS TO  $\frac{2}{3}$  DRAW LOCUS OF POINT P. { **ECCENTRICITY =  $\frac{2}{3}$**  }

**STEPS:**

1. Draw a vertical line AB and point F 50 mm from it.
2. Divide 50 mm distance in 5 parts.
3. Name 2<sup>nd</sup> part from F as V. It is 20mm and 30mm from F and AB line resp. It is first point giving ratio of it's distances from F and AB  $\frac{2}{3}$  i.e  $\frac{20}{30}$
4. Form more points giving same ratio such as  $\frac{30}{45}$ ,  $\frac{40}{60}$ ,  $\frac{50}{75}$  etc.
5. Taking 45, 60 and 75mm distances from line AB, draw three vertical lines to the right side of it.
6. Now with 30, 40 and 50mm distances in compass cut these lines above and below, with F as center.
7. Join these points through V in smooth curve.

This is required locus of P. It is an ELLIPSE.



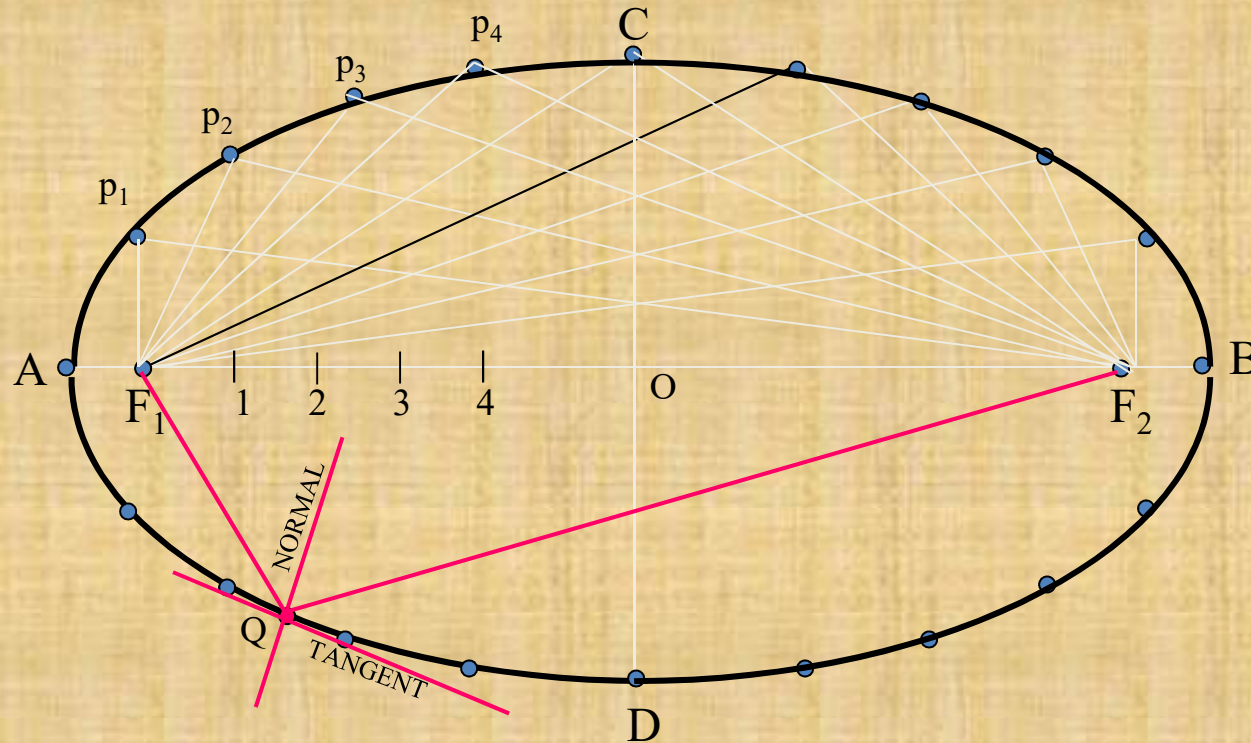


Problem 13:

**ELLIPSE**  
**TANGENT & NORMAL**

*TO DRAW TANGENT & NORMAL  
TO THE CURVE FROM A GIVEN POINT ( Q )*

1. JOIN POINT Q TO  $F_1$  &  $F_2$
2. BISECT ANGLE  $F_1QF_2$  THE ANGLE BISECTOR IS NORMAL
3. A PERPENDICULAR LINE DRAWN TO IT IS TANGENT TO THE CURVE.

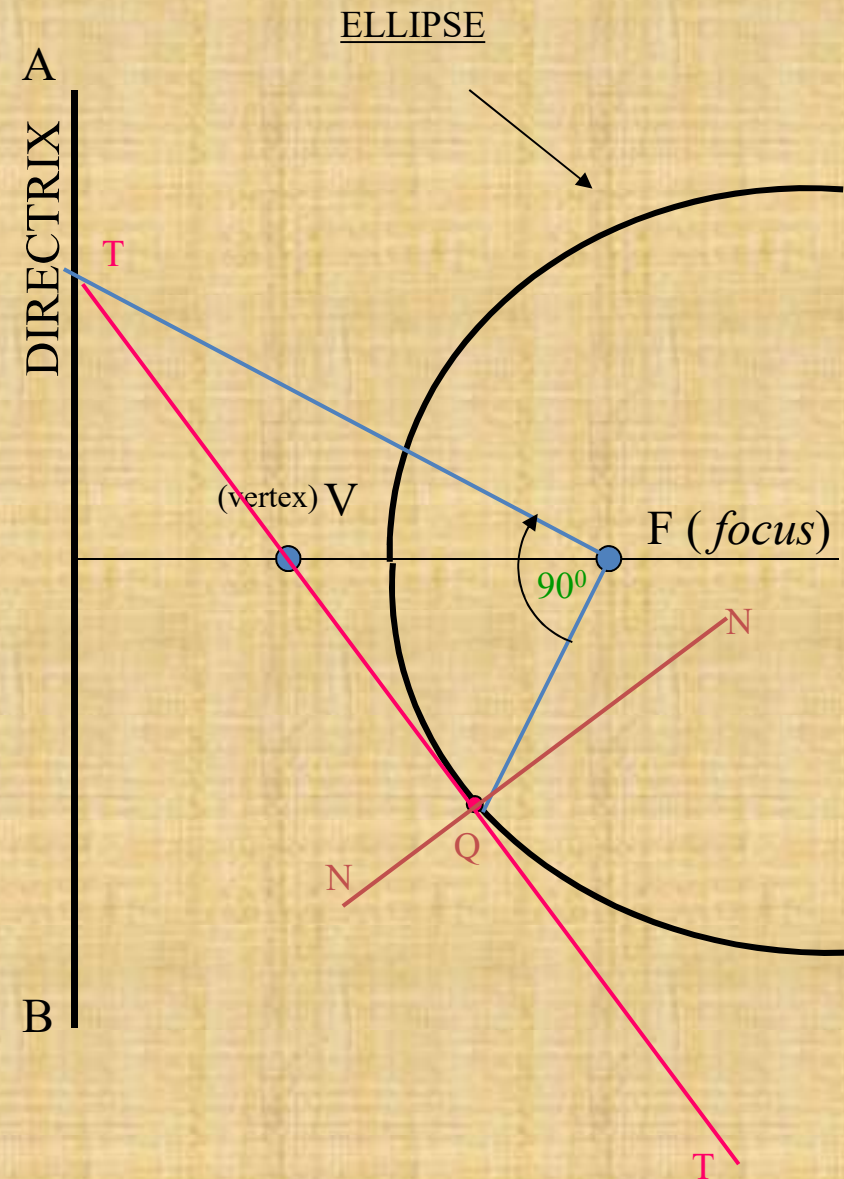


### Problem 14:

TO DRAW TANGENT & NORMAL  
TO THE CURVE  
FROM A GIVEN POINT ( Q )

1. JOIN POINT Q TO F.
2. CONSTRUCT 90° ANGLE WITH THIS LINE AT POINT F
3. EXTEND THE LINE TO MEET DIRECTRIX AT T
4. JOIN THIS POINT TO Q AND EXTEND. THIS IS TANGENT TO ELLIPSE FROM Q
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM Q. IT IS NORMAL TO CURVE.

### ELLIPSE TANGENT & NORMAL



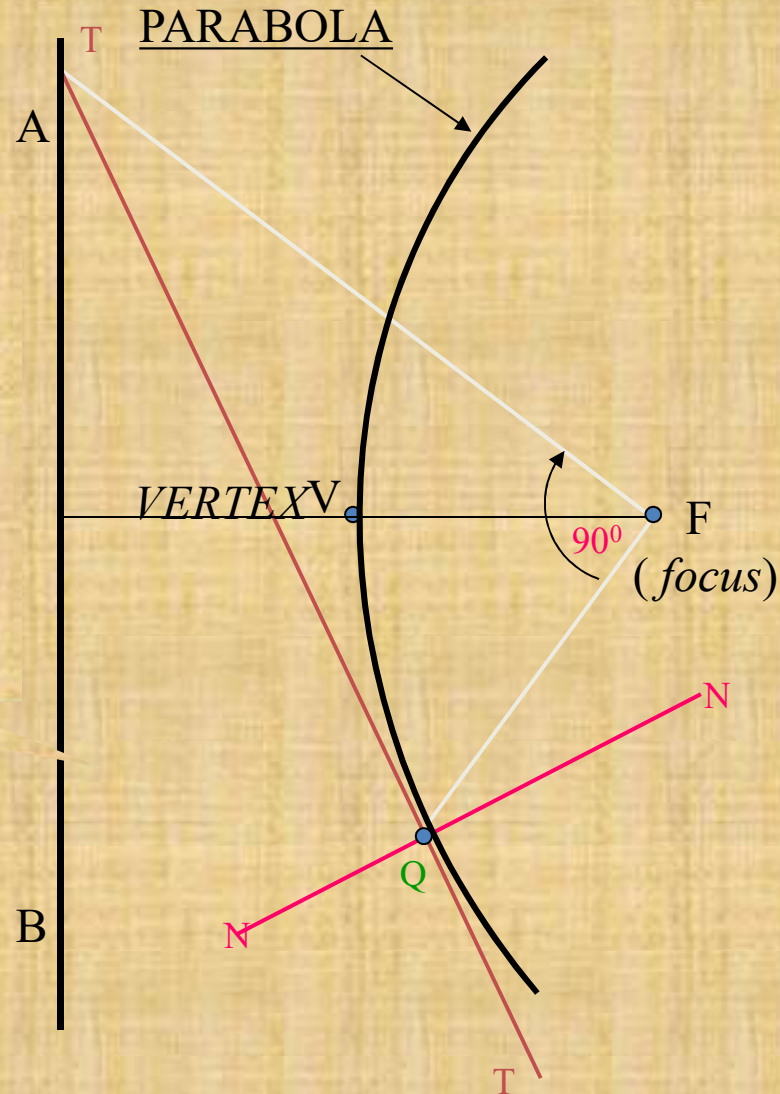


## PARABOLA TANGENT & NORMAL

### Problem 15:

TO DRAW TANGENT & NORMAL  
TO THE CURVE  
FROM A GIVEN POINT ( Q )

1. JOIN POINT Q TO F.
2. CONSTRUCT  $90^\circ$  ANGLE WITH THIS LINE AT POINT F
3. EXTEND THE LINE TO MEET DIRECTRIX AT T
4. JOIN THIS POINT TO Q AND EXTEND. THIS IS TANGENT TO THE CURVE FROM Q
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM Q. IT IS NORMAL TO CURVE.

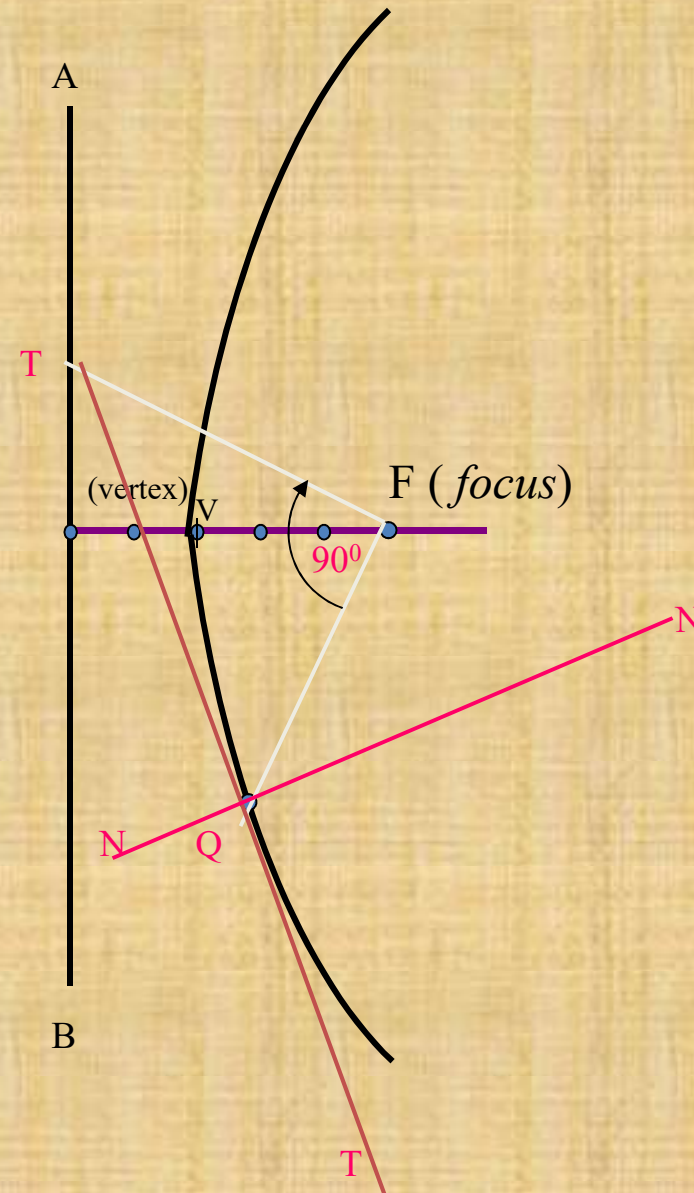


## Problem 16

TO DRAW TANGENT & NORMAL  
TO THE CURVE  
FROM A GIVEN POINT ( Q )

1. JOIN POINT Q TO F.
2. CONSTRUCT  $90^\circ$  ANGLE WITH THIS LINE AT POINT F
3. EXTEND THE LINE TO MEET DIRECTRIX AT T
4. JOIN THIS POINT TO Q AND EXTEND. THIS IS TANGENT TO CURVE FROM Q
5. TO THIS TANGENT DRAW PERPENDICULAR LINE FROM Q. IT IS NORMAL TO CURVE.

## HYPERBOLA TANGENT & NORMAL





# ENGINEERING CURVES

## Part-II

(Point undergoing two types of displacements)

### INVOLUTE

1. Involute of a circle
  - a) String Length =  $\pi D$
  - b) String Length >  $\pi D$
  - c) String Length <  $\pi D$
2. Pole having Composite shape.
3. Rod Rolling over a Semicircular Pole.

### CYCLOID

1. General Cycloid
2. Trochoid  
( superior)
3. Trochoid  
( Inferior)
4. Epi-Cycloid
5. Hypo-Cycloid

### SPIRAL

1. Spiral of One Convolution.
2. Spiral of Two Convolution.

### HELIX

1. On Cylinder
2. On a Cone

**AND**

Methods of Drawing  
Tangents & Normal  
To These Curves.

# DEFINITIONS

## **CYCLOID:**

IT IS A LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A STRAIGHT LINE PATH.

## **INVOLUTE:**

IT IS A LOCUS OF A FREE END OF A STRING WHEN IT IS WOUND ROUND A CIRCULAR POLE

## **SPIRAL:**

IT IS A CURVE GENERATED BY A POINT WHICH REVOLVES AROUND A FIXED POINT AND AT THE SAME MOVES TOWARDS IT.

## **HELIX:**

IT IS A CURVE GENERATED BY A POINT WHICH MOVES AROUND THE SURFACE OF A RIGHT CIRCULAR CYLINDER / CONE AND AT THE SAME TIME ADVANCES IN AXIAL DIRECTION AT A SPEED BEARING A CONSTANT RATIO TO THE SPEED OF ROTATION.

### SUPERIOR TROCHOID:

IF THE POINT IN THE DEFINITION OF CYCLOID IS OUTSIDE THE CIRCLE

### INFERIOR TROCHOID:

IF IT IS INSIDE THE CIRCLE

### EPI-CYCLOID

IF THE CIRCLE IS ROLLING ON ANOTHER CIRCLE FROM OUTSIDE

### HYPO-CYCLOID.

IF THE CIRCLE IS ROLLING FROM INSIDE THE OTHER CIRCLE,



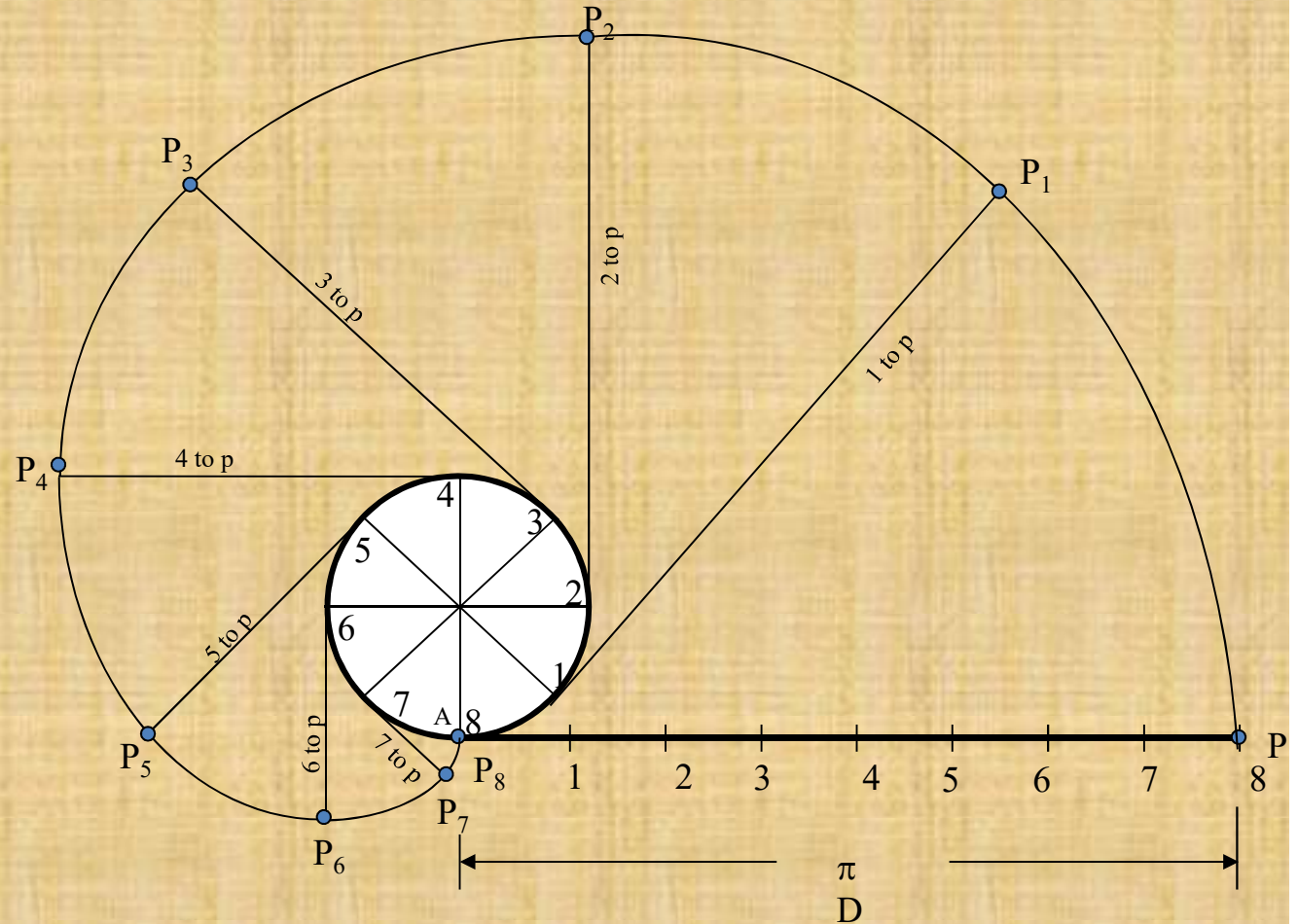
## INVOLUTE OF A CIRCLE

### Problem no 17: Draw Involutees of a circle.

String length is equal to the circumference of circle.

#### *Solution Steps:*

- 1) Point or end P of string AP is exactly  $\pi D$  distance away from A. Means if this string is wound round the circle, it will completely cover given circle. B will meet A after winding.
- 2) Divide  $\pi D$  (AP) distance into 8 number of equal parts.
- 3) Divide circle also into 8 number of equal parts.
- 4) Name after A, 1, 2, 3, 4, etc. up to 8 on  $\pi D$  line AP as well as on circle (in anticlockwise direction).
- 5) To radius C-1, C-2, C-3 up to C-8 draw tangents (from 1,2,3,4,etc to circle).
- 6) Take distance 1 to P in compass and mark it on tangent from point 1 on circle (means one division less than distance AP).
- 7) Name this point P1
- 8) Take 2-B distance in compass and mark it on the tangent from point 2. Name it point P2.
- 9) Similarly take 3 to P, 4 to P, 5 to P up to 7 to P distance in compass and mark on respective tangents and locate P3, P4, P5 up to P8 (i.e. A) points and join them in smooth curve it is an INVOLUTE of a given circle.



Problem 18: Draw Involute of a circle.

String length is MORE than the circumference of circle.

### INVOLUTE OF A CIRCLE

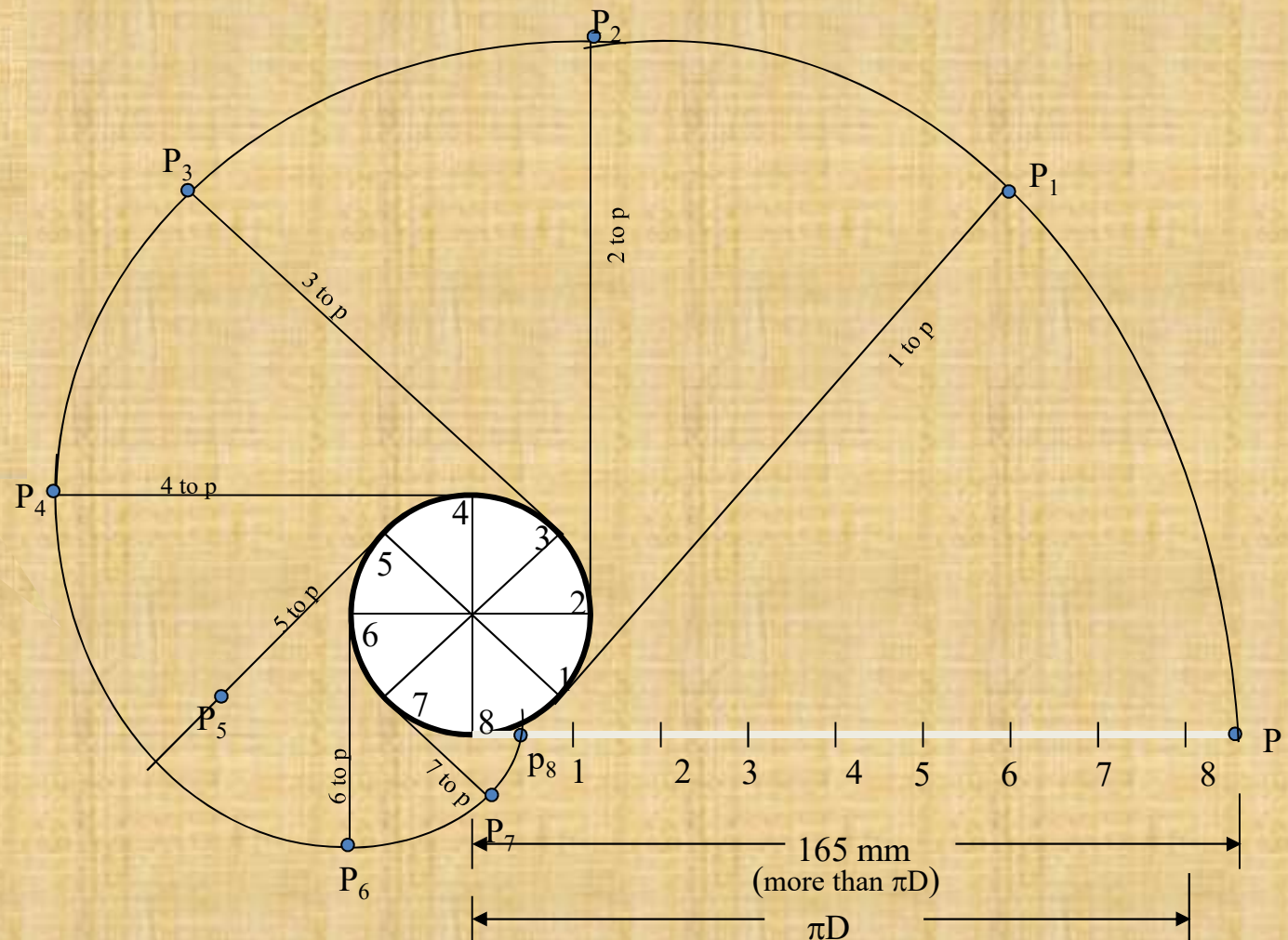
String length MORE than  $\pi D$

#### *Solution Steps:*

In this case string length is more than  $\pi D$ .

But remember!

Whatever may be the length of string, mark  $\pi D$  distance horizontal i.e. along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.





## INVOLUTE OF A CIRCLE

Problem 19: Draw Involute of a circle.

String length is LESS than the circumference of circle.

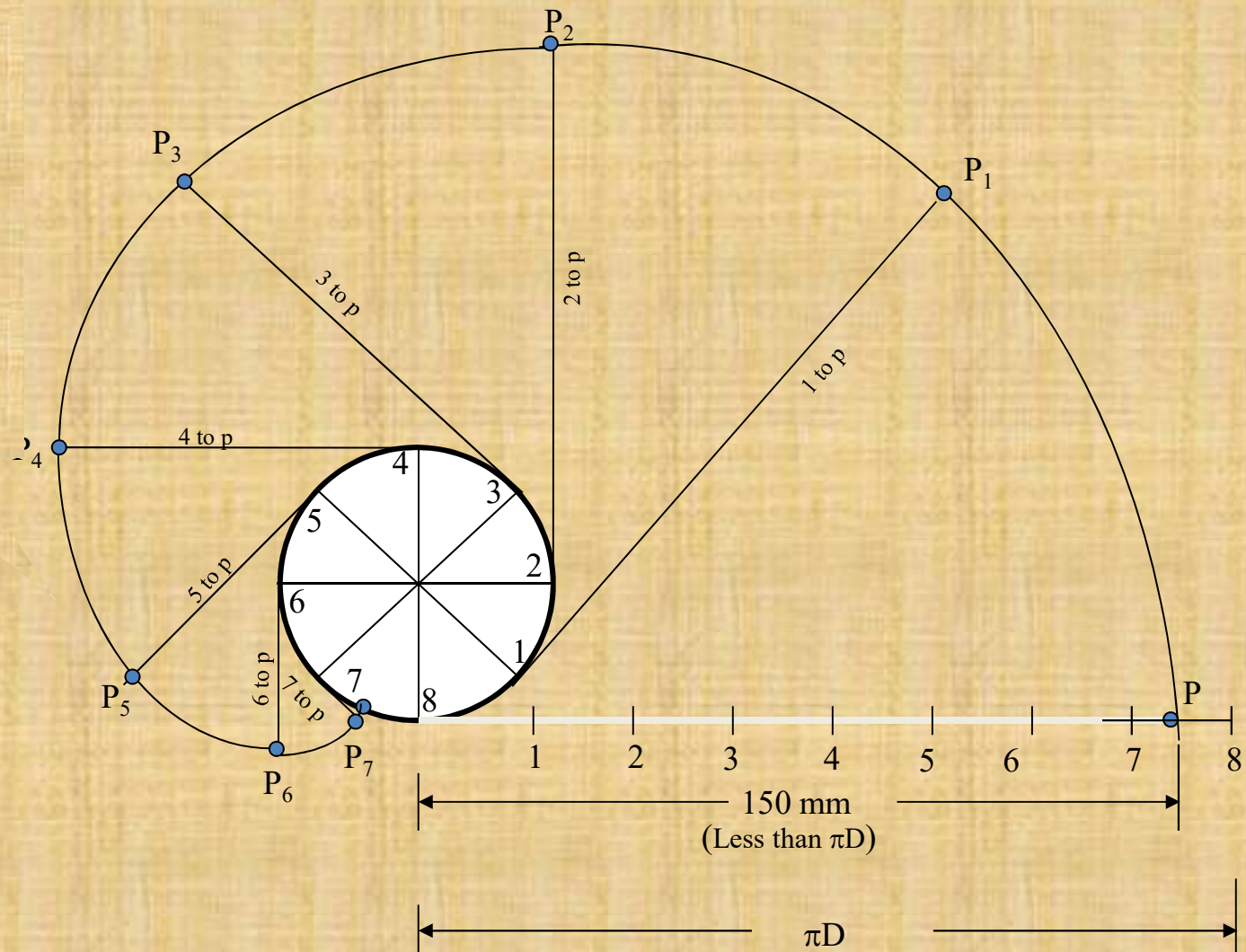
String length LESS than  $\pi D$

### Solution Steps:

In this case string length is Less than  $\pi D$ .

But remember!

Whatever may be the length of string, mark  $\pi D$  distance horizontal i.e. along the string and divide it in 8 number of equal parts, and not any other distance. Rest all steps are same as previous INVOLUTE. Draw the curve completely.



PROBLEM 20 : A POLE IS OF A SHAPE OF HALF HEXAGON AND SEMICIRCLE.  
 A STRING IS TO BE WOUND HAVING LENGTH EQUAL TO THE POLE PERIMETER  
 DRAW PATH OF FREE END  $P$  OF STRING WHEN WOUND COMPLETELY.

(Take hex 30 mm sides and semicircle of 60 mm diameter.)

# INVOLUTE OF COMPOSIT SHAPED POLE

## SOLUTION STEPS:

Draw pole shape as per dimensions.

Divide semicircle in 4 parts and name those along with corners of hexagon.

Calculate perimeter length.

Show it as string  $AP$ .

On this line mark 30mm from  $A$

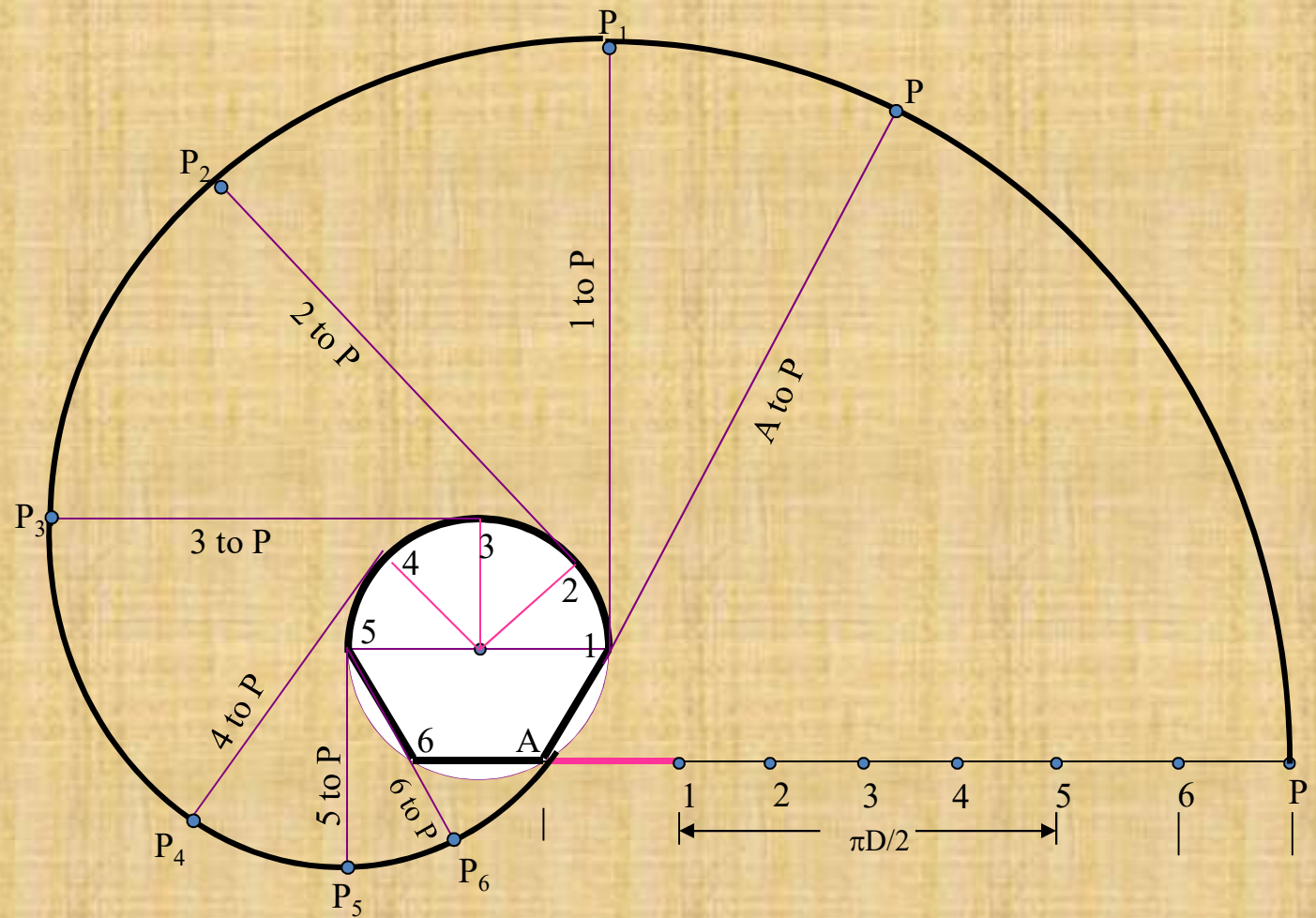
Mark and name it 1

Mark  $\pi D/2$  distance on it from 1

And dividing it in 4 parts name 2,3,4,5.

Mark point 6 on line 30 mm from 5

Now draw tangents from all points of pole and proper lengths as done in all previous involute's problems and complete the curve.

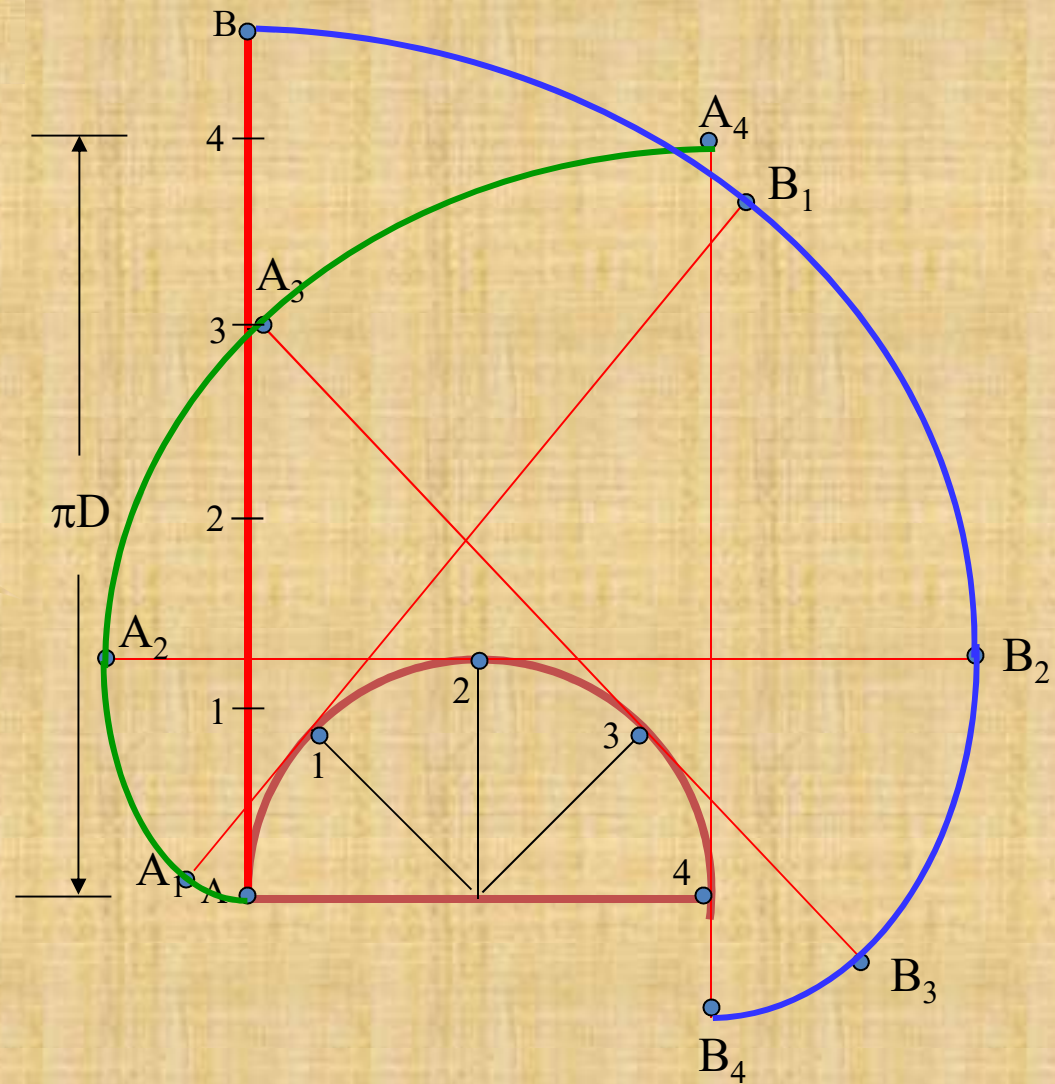




PROBLEM 21 : Rod AB 85 mm long rolls over a semicircular pole without slipping from its initially vertical position till it becomes up-side-down vertical.  
Draw locus of both ends A & B.

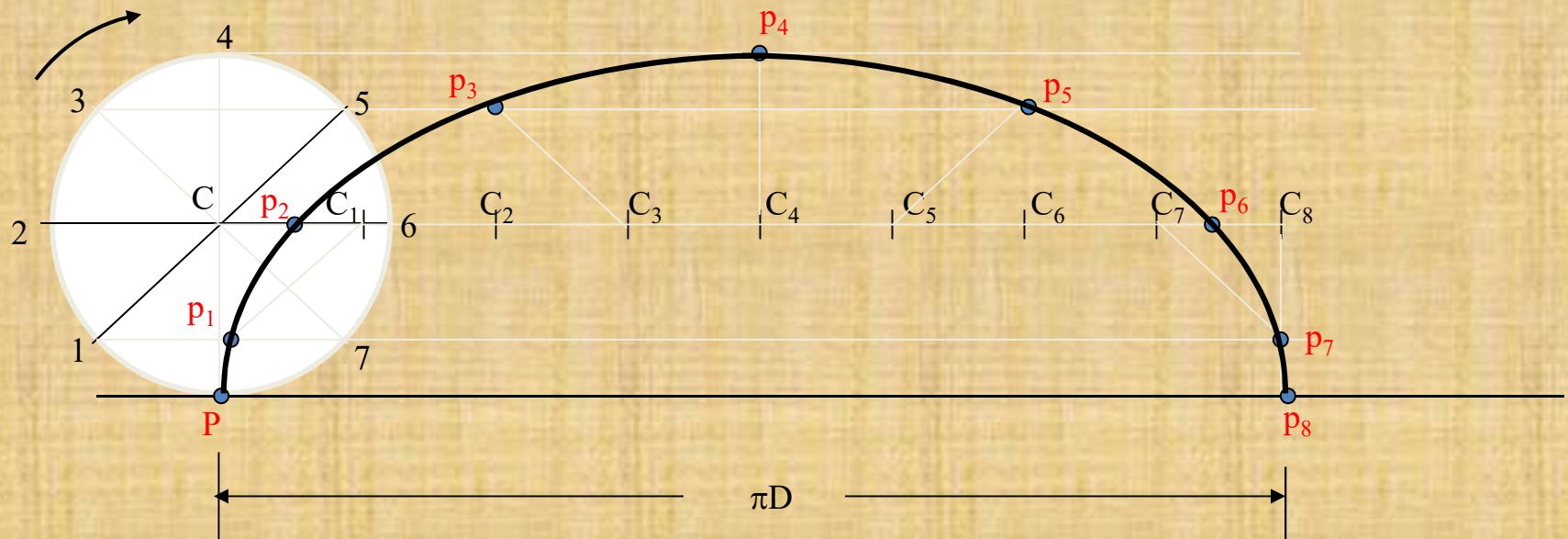
#### Solution Steps?

If you have studied previous problems properly, you can surely solve this also. Simply remember that this being a rod, it will roll over the surface of pole. Means when one end is approaching, other end will move away from pole. OBSERVE ILLUSTRATION CAREFULLY!



PROBLEM 22: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON STRAIGHT LINE PATH. Take Circle diameter as 50 mm

## CYCLOID

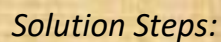


### *Solution Steps:*

- 1) From center C draw a horizontal line equal to  $\pi D$  distance.
- 2) Divide  $\pi D$  distance into 8 number of equal parts and name them C1, C2, C3\_\_ etc.
- 3) Divide the circle also into 8 number of equal parts and in clock wise direction, after P name 1, 2, 3 up to 8.
- 4) From all these points on circle draw horizontal lines. (parallel to locus of C)
- 5) With a fixed distance C-P in compass, C1 as center, mark a point on horizontal line from 1. Name it P.
- 6) Repeat this procedure from C2, C3, C4 upto C8 as centers. Mark points P2, P3, P4, P5 up to P8 on the horizontal lines drawn from 2, 3, 4, 5, 6, 7 respectively.
- 7) Join all these points by curve. It is Cycloid.

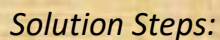


## SUPERIOR TROCHOID



- 1) Draw circle of given diameter and draw a horizontal line from its center C of length  $\Pi D$  and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
- 2) Draw circle by CP radius, as in this case CP is larger than radius of circle.
- 3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join
- 4) This curve is called Superior Trochoid.

## INFERIOR TROCHOID



- 1) Draw circle of given diameter and draw a horizontal line from its center C of length II D and divide it in 8 number of equal parts and name them C1, C2, C3, up to C8.
- 2) Draw circle by CP radius, as in this case CP is SHORTER than radius of circle.
- 3) Now repeat steps as per the previous problem of cycloid, by dividing this new circle into 8 number of equal parts and drawing lines from all these points parallel to locus of C and taking CP radius with different positions of C as centers, cut these lines and get different positions of P and join those in curvature.
- 4) This curve is called Inferior Trochoid.

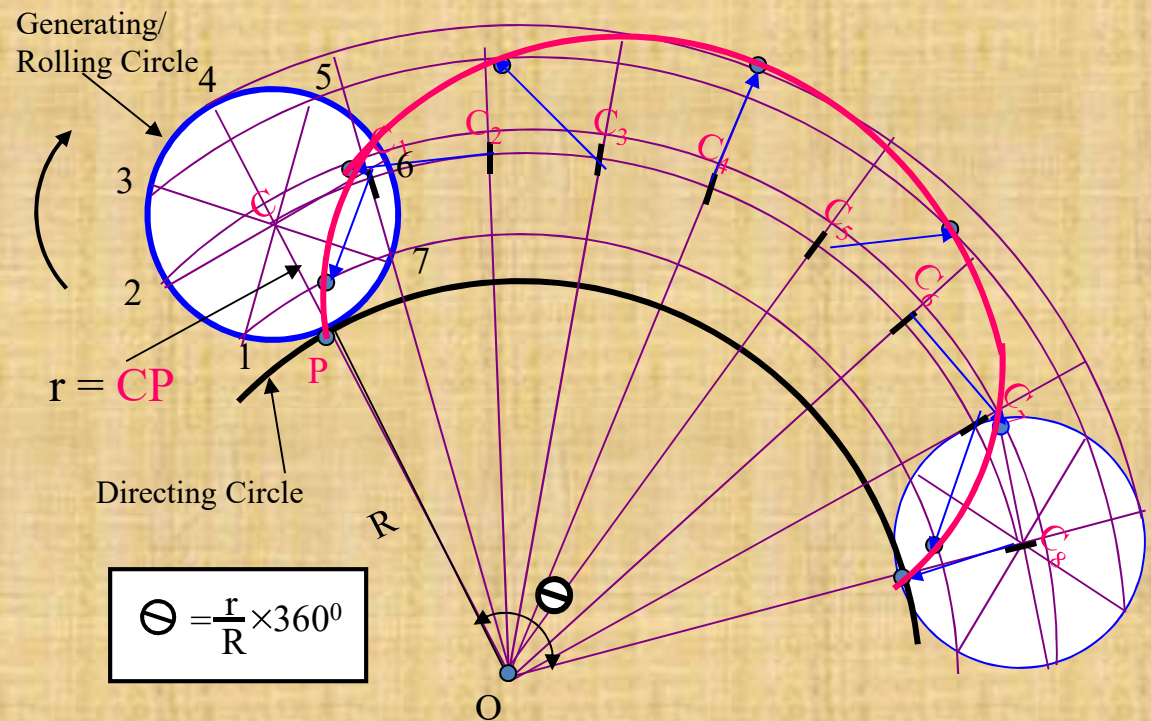


**PROBLEM 25: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS ON A CURVED PATH. Take diameter of rolling Circle 50 mm And radius of directing circle i.e. curved path, 75 mm.**

## EPI CYCLOID

### Solution Steps:

- 1) When smaller circle will roll on larger circle for one revolution it will cover  $\Pi D$  distance on arc and it will be decided by included arc angle  $\theta$ .
  - 2) Calculate  $\theta$  by formula  $\theta = (r/R) \times 3600$ .
  - 3) Construct angle  $\theta$  with radius OC and draw an arc by taking O as center OC as radius and form sector of angle  $\theta$ .
  - 4) Divide this sector into 8 number of equal angular parts. And from C onward name them C1, C2, C3 up to C8.
  - 5) Divide smaller circle (Generating circle) also in 8 number of equal parts. And next to P in clockwise direction name those 1, 2, 3, up to 8.
  - 6) With O as center, O-1 as radius draw an arc in the sector. Take O-2, O-3, O-4, O-5 up to O-8 distances with center O, draw all concentric arcs in sector. Take fixed distance C-P in compass, C1 center, cut arc of 1 at P1.
- Repeat procedure and locate P2, P3, P4, P5 unto P8 (as in cycloid) and join them by smooth curve. This is EPI – CYCLOID.

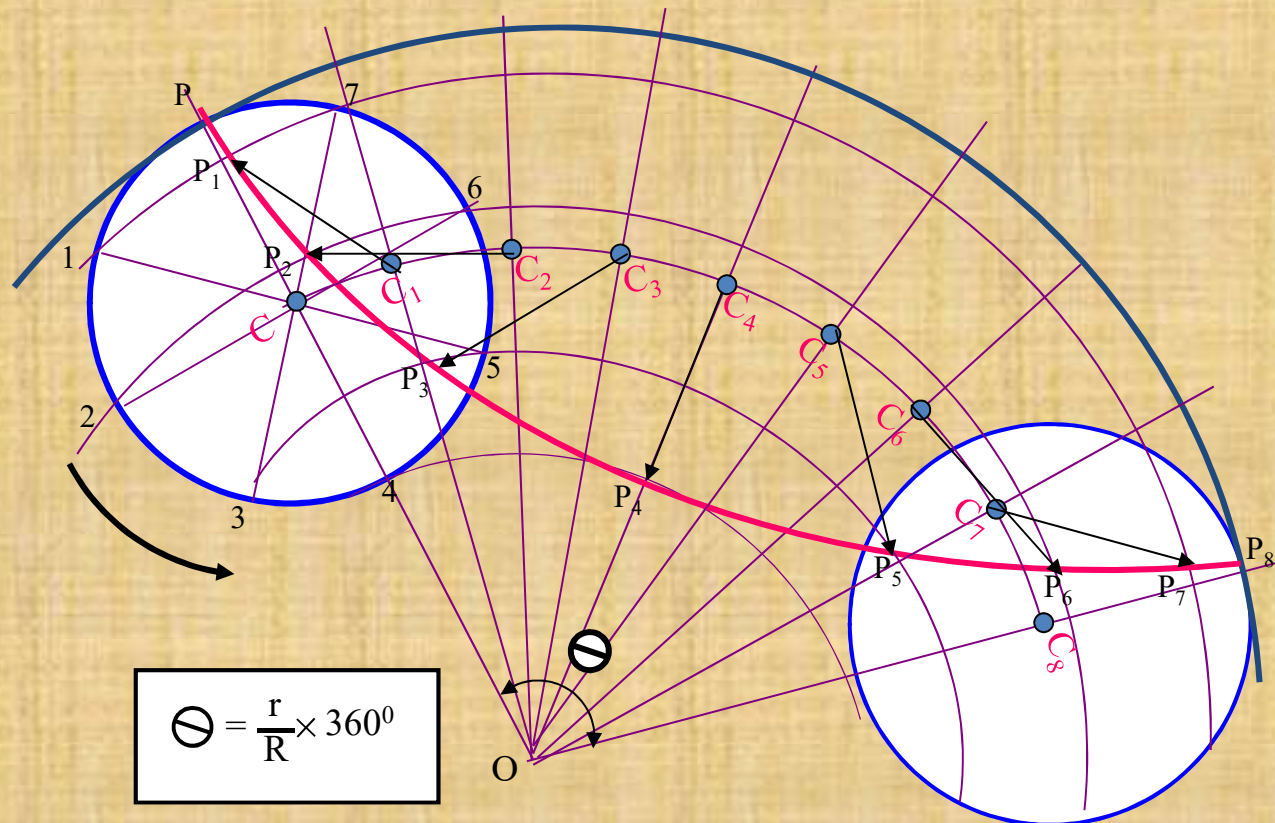


PROBLEM 26: DRAW LOCUS OF A POINT ON THE PERIPHERY OF A CIRCLE WHICH ROLLS FROM THE INSIDE OF A CURVED PATH. Take diameter of rolling circle 50 mm and radius of directing circle (curved path) 75 mm.

## HYPO CYCLOID

### *Solution Steps:*

- 1) Smaller circle is rolling here, inside the larger circle. It has to rotate anticlockwise to move ahead.
- 2) Same steps should be taken as in case of EPI – CYCLOID. Only change is in numbering direction of 8 number of equal parts on the smaller circle.
- 3) From next to P in anticlockwise direction, name 1,2,3,4,5,6,7,8.
- 4) Further all steps are that of epi – cycloid. This is called HYPO – CYCLOID.



OC = R (Radius of Directing Circle)  
CP = r (Radius of Generating Circle)



Problem 27: Draw a spiral of one convolution. Take distance PO 40 mm.

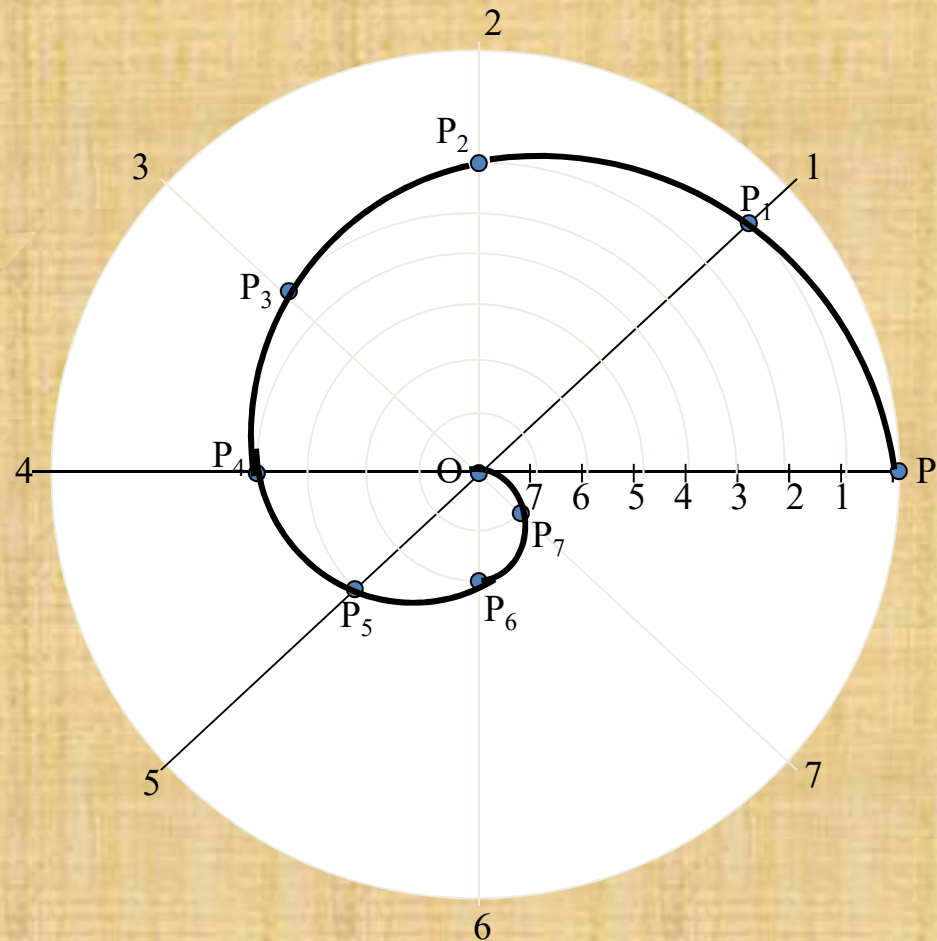
## SPIRAL

### IMPORTANT APPROACH FOR CONSTRUCTION!

FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT  
AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

#### Solution Steps

1. With PO radius draw a circle and divide it in EIGHT parts. Name those 1,2,3,4, etc. up to 8
2. Similarly divided line PO also in EIGHT parts and name those 1,2,3,-- as shown.
3. Take o-1 distance from op line and draw an arc up to O1 radius vector. Name the point  $P_1$
4. Similarly mark points  $P_2, P_3, P_4$  up to  $P_8$   
And join those in a smooth curve.  
It is a SPIRAL of one convolution.



## Problem 28

Point P is 80 mm from point O. It starts moving towards O and reaches it in two revolutions around it. Draw locus of point P (To draw a Spiral of TWO convolutions).

## SPIRAL of two convolutions

IMPORTANT APPROACH FOR CONSTRUCTION!  
FIND TOTAL ANGULAR AND TOTAL LINEAR DISPLACEMENT  
AND DIVIDE BOTH IN TO SAME NUMBER OF EQUAL PARTS.

### SOLUTION STEPS:

Total angular displacement here is two revolutions And  
Total Linear displacement here is distance PO.

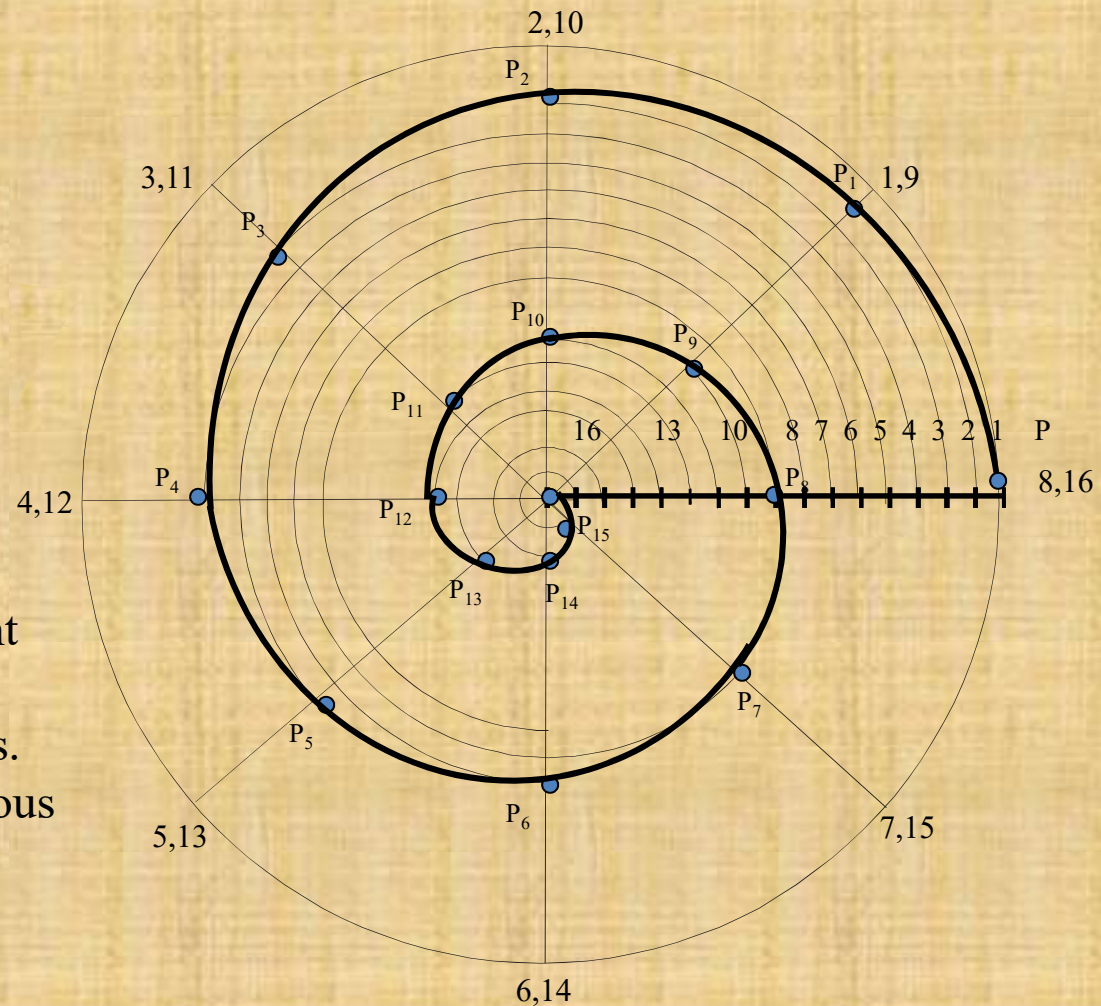
Just divide both in same parts i.e.

Circle in EIGHT parts.

( means total angular displacement in SIXTEEN parts)

Divide PO also in SIXTEEN parts.

Rest steps are similar to the previous problem.





## HELIX (UPON A CYLINDER)

**PROBLEM:** Draw a helix of one convolution, upon a cylinder.  
Given 80 mm pitch and 50 mm diameter of a cylinder.  
(The axial advance during one complete revolution is called  
The *pitch* of the helix)

### SOLUTION:

Draw projections of a cylinder.

Divide circle and axis in to same no. of equal parts. ( 8 )

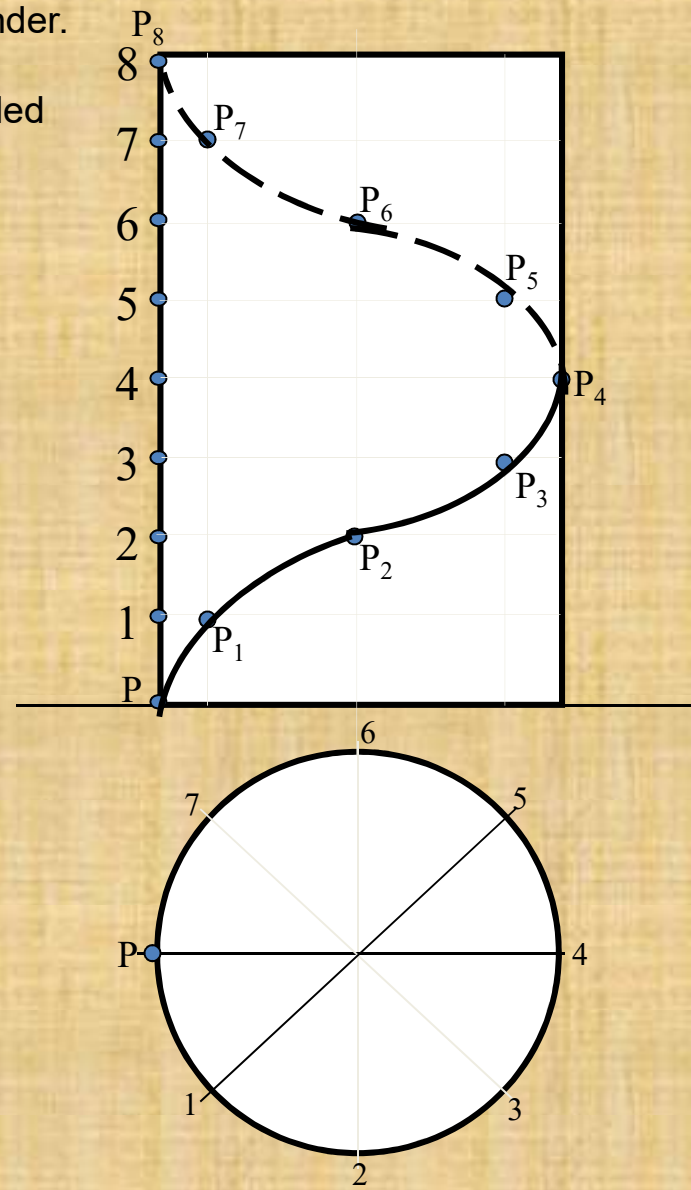
Name those as shown.

Mark initial position of point 'P'

Mark various positions of *P* as shown in animation.

Join all points by smooth possible curve.

Make upper half dotted, as it is going behind the solid  
and hence will not be seen from front side.



## HELIX (UPON A CONE)

**PROBLEM:** Draw a helix of one convolution, upon a cone, diameter of base 70 mm, axis 90 mm and 90 mm pitch.  
(The axial advance during one complete revolution is called The *pitch* of the helix)

### SOLUTION:

Draw projections of a cone

Divide circle and axis in to same no. of equal parts. ( 8 )

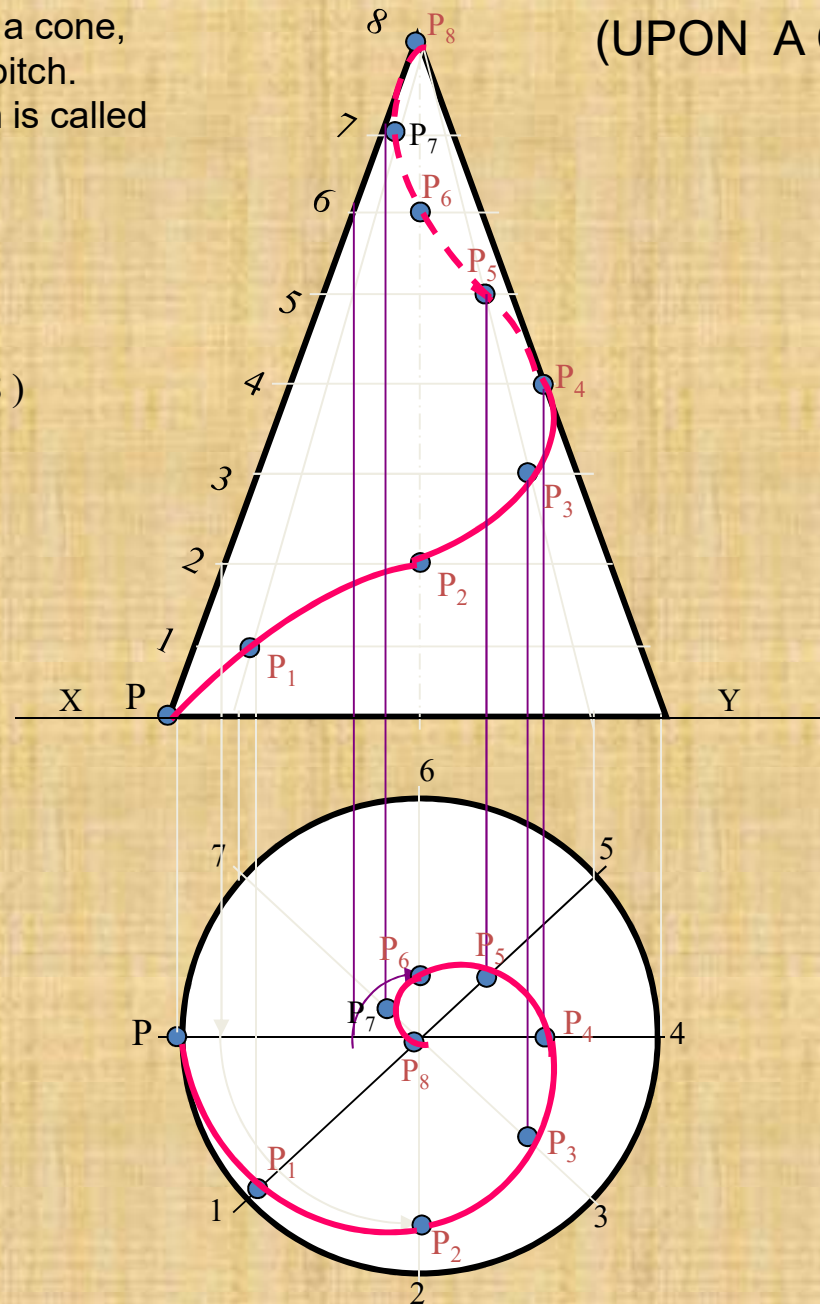
Name those as shown.

Mark initial position of point 'P'

Mark various positions of *P* as shown in animation.

Join all points by smooth possible curve.

Make upper half dotted, as it is going behind the solid and hence will not be seen from front side.





STEPS:

DRAW INVOLUTE AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

JOIN **Q** TO THE CENTER OF CIRCLE **C**.  
CONSIDERING **CQ** DIAMETER, DRAW  
A SEMICIRCLE AS SHOWN.

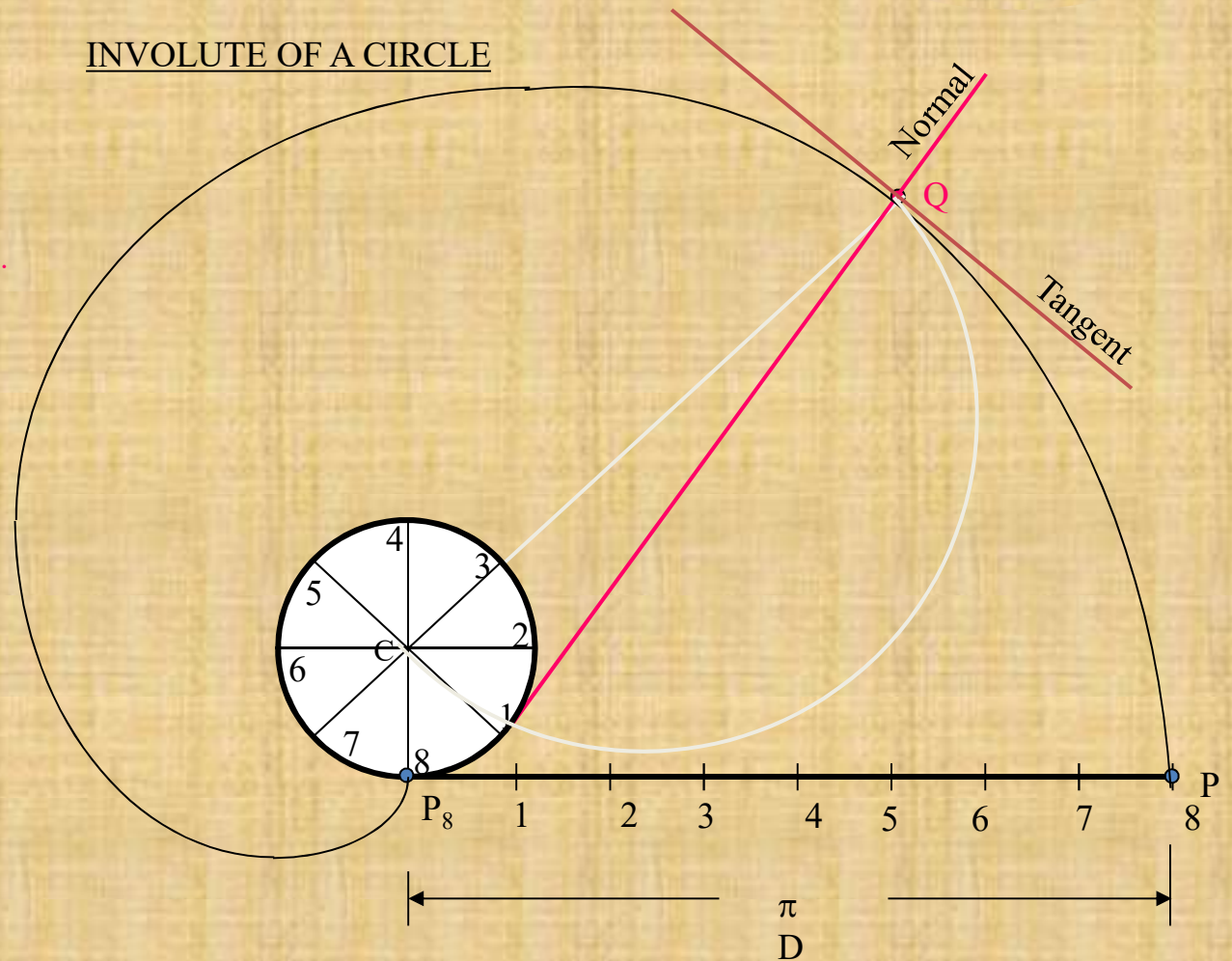
MARK POINT OF INTERSECTION OF  
THIS SEMICIRCLE AND POLE CIRCLE  
AND JOIN IT TO **Q**.

THIS WILL BE **NORMAL TO INVOLUTE**.

DRAW A LINE AT RIGHT ANGLE TO  
THIS LINE FROM **Q**.

IT WILL BE **TANGENT TO INVOLUTE**.

### INVOLUTE OF A CIRCLE



STEPS:

DRAW CYCLOID AS USUAL.

MARK POINT **Q** ON IT AS DIRECTED.

WITH CP DISTANCE, FROM **Q**. CUT THE  
POINT ON LOCUS OF **C** AND JOIN IT TO **Q**.

FROM THIS POINT DROP A PERPENDICULAR  
ON GROUND LINE AND NAME IT N

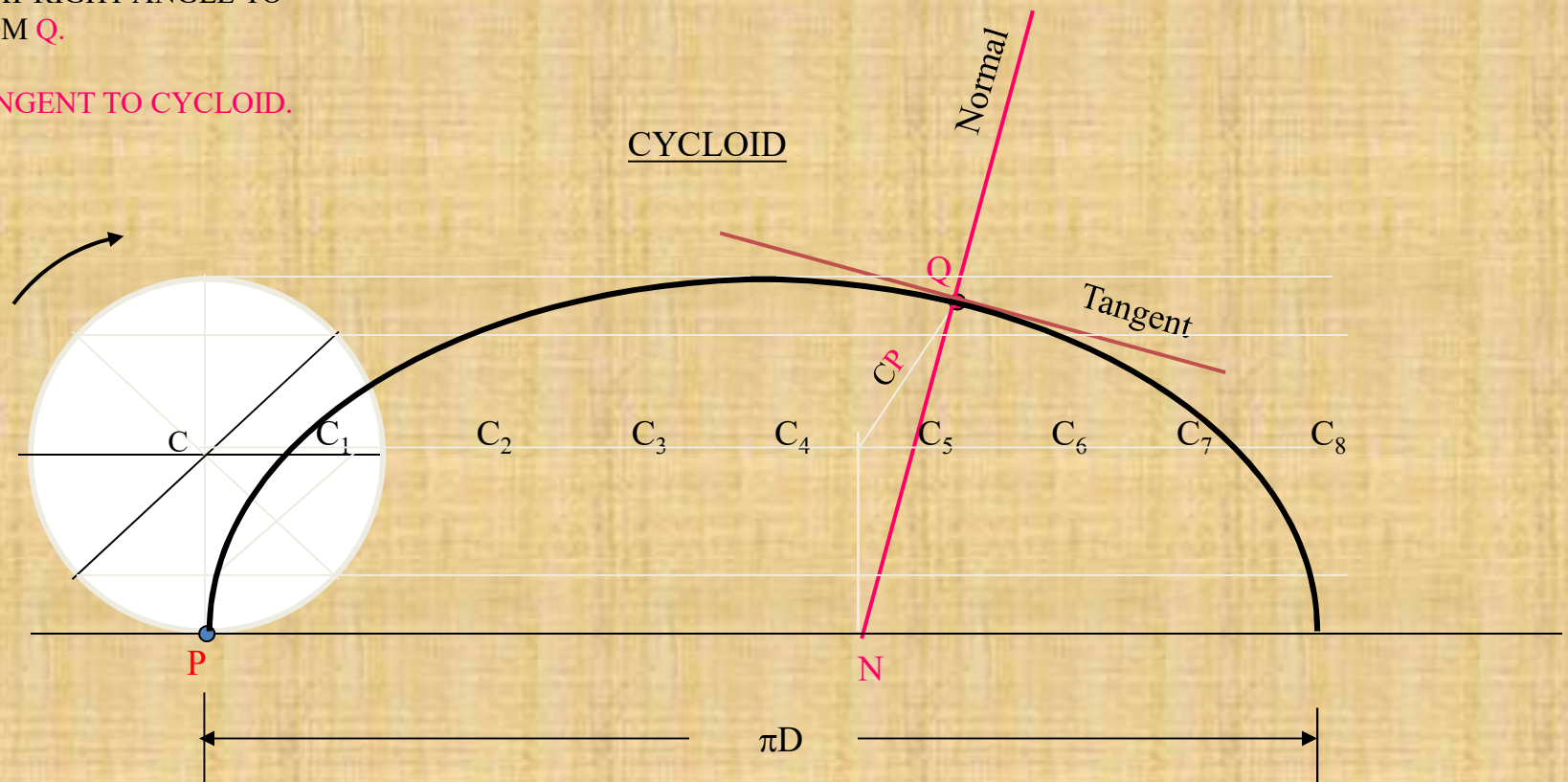
JOIN N WITH Q. THIS WILL BE **NORMAL TO  
CYCLOID**.

DRAW A LINE AT RIGHT ANGLE TO  
THIS LINE FROM **Q**.

**IT WILL BE TANGENT TO CYCLOID.**

## CYCLOID

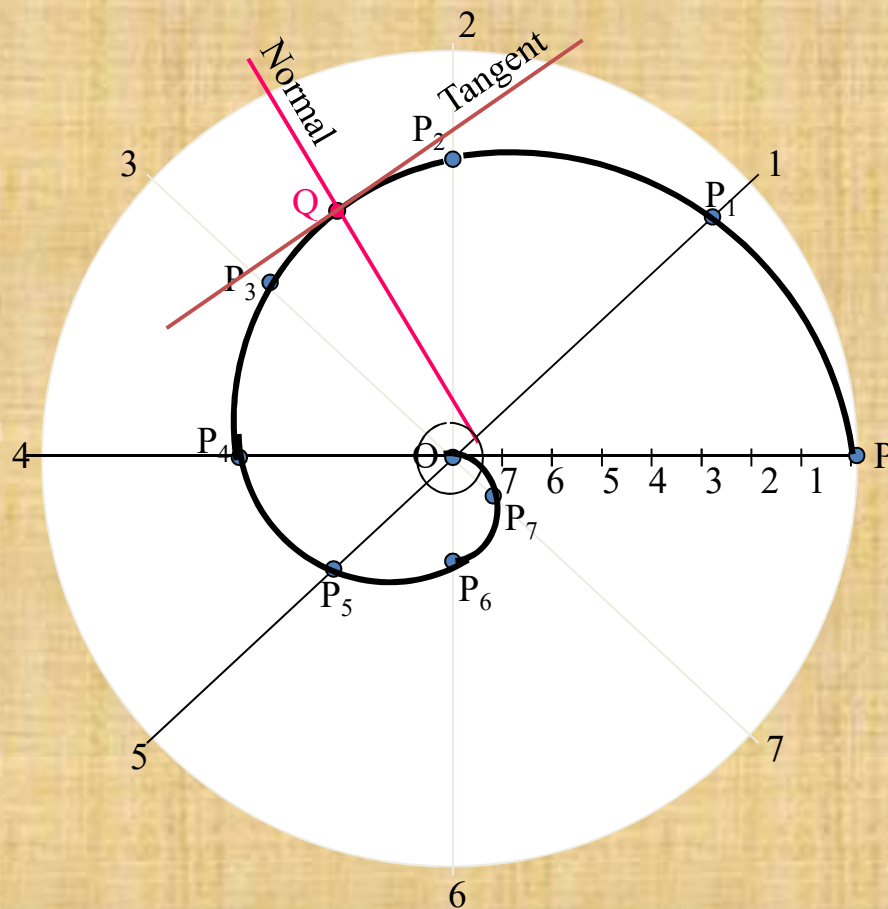
### Method of Drawing Tangent & Normal





# Spiral. Method of Drawing Tangent & Normal

SPIRAL (ONE CONVOLUTION.)



Constant of the Curve =  $\frac{\text{Difference in length of any radius vectors}}{\text{Angle between the corresponding radius vector in radian.}}$

$$= \frac{OP - OP_2}{\pi/2} = \frac{OP - OP_2}{1.57}$$

$$= 3.185 \text{ m.m.}$$

## STEPS:

- \*DRAW SPIRAL AS USUAL.  
DRAW A SMALL CIRCLE OF RADIUS EQUAL TO THE CONSTANT OF CURVE CALCULATED ABOVE.
- \* LOCATE POINT Q AS DISCRIBED IN PROBLEM AND THROUGH IT DRAW A TANGENT TO THIS SMALLER CIRCLE. THIS IS A **NORMAL** TO THE SPIRAL.
- \*DRAW A LINE AT RIGHT ANGLE
- \*TO THIS LINE FROM Q.  
**IT WILL BE TANGENT TO CYCLOID.**

# LOCUS

It is a path traced out by a point moving in a plane, in a particular manner, for one cycle of operation.

The cases are classified in THREE categories for easy understanding.

- A} Basic Locus Cases.
- B} Oscillating Link.....
- C} Rotating Link.....

## Basic Locus Cases:

Here some geometrical objects like point, line, circle will be described with their relative positions. Then one point will be allowed to move in a plane maintaining specific relation with above objects. And studying situation carefully you will be asked to draw its locus.

## Oscillating & Rotating Link:

Here a link oscillating from one end or rotating around its center will be described. Then a point will be allowed to slide along the link in specific manner. And now studying the situation carefully you will be asked to draw its locus.

**STUDY TEN CASES GIVEN ON NEXT PAGES**



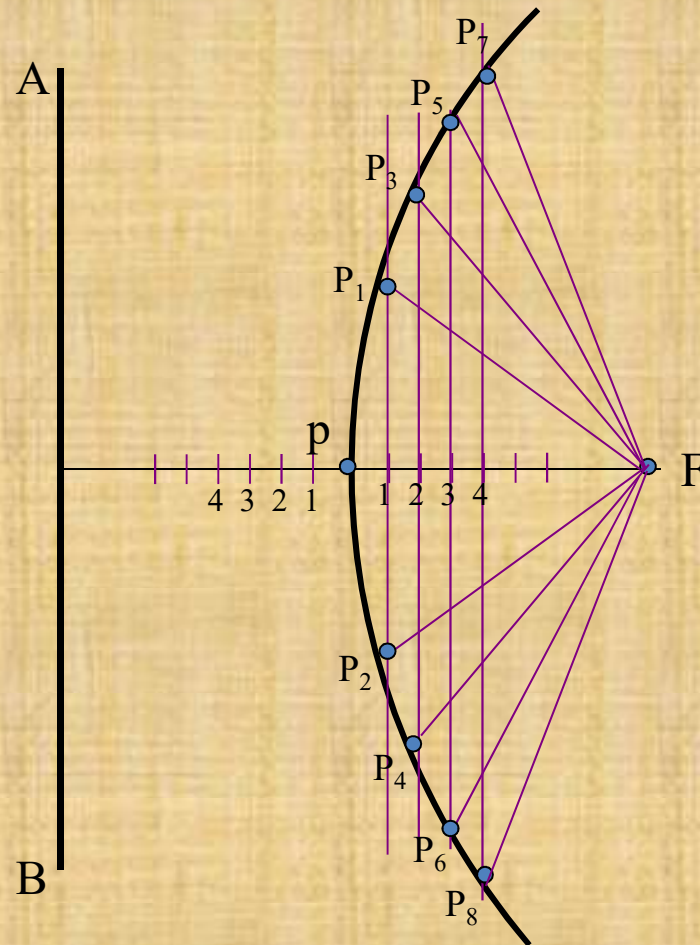
## Basic Locus Cases:

PROBLEM 1.: Point F is 50 mm from a vertical straight line AB.  
Draw locus of point P, moving in a plane such that  
it always remains equidistant from point F and line AB.

### SOLUTION STEPS:

1. Locate center of line, perpendicular to AB from point F. This will be initial point P.
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.
3. Mark 5 mm distance to its left of P and name it 1.
4. Take F-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point  $P_1$  and lower point  $P_2$ .
5. Similarly repeat this process by taking again 5mm to right and left and locate  $P_3P_4$ .
6. Join all these points in smooth curve.

It will be the locus of P equidistance  
from line AB and fixed point F.



## Basic Locus Cases:

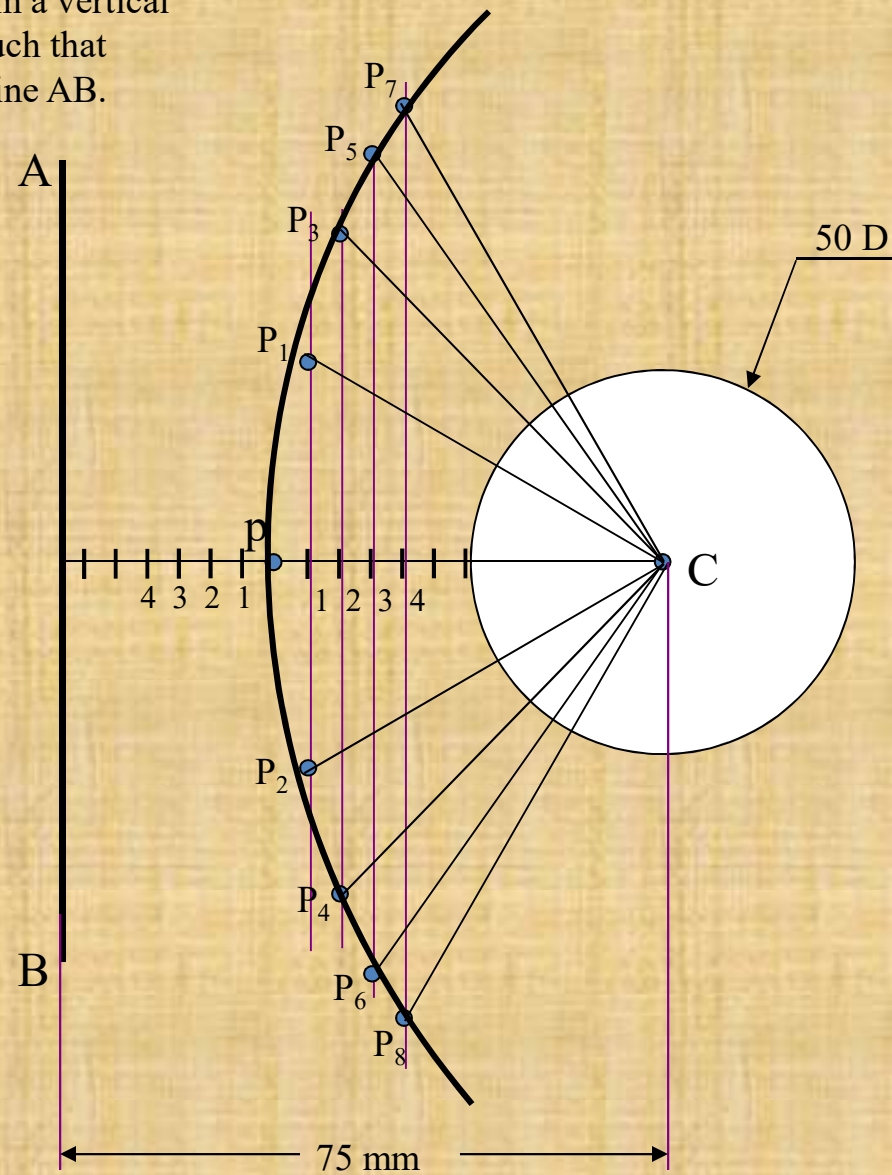
### PROBLEM 2 :

A circle of 50 mm diameter has its center 75 mm from a vertical line AB. Draw locus of point P, moving in a plane such that it always remains equidistant from given circle and line AB.

### SOLUTION STEPS:

1. Locate center of line, perpendicular to AB from the periphery of circle. This will be initial point P.
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.
3. Mark 5 mm distance to its left of P and name it 1,2,3,4.
4. Take C-1 distance as radius and C as center draw an arc cutting first parallel line to AB. Name upper point  $P_1$  and lower point  $P_2$ .
5. Similarly repeat this process by taking again 5 mm to right and left and locate  $P_3, P_4$ .
6. Join all these points in smooth curve.

It will be the locus of P equidistance from line AB and given circle.





## Basic Locus Cases:

### PROBLEM 3 :

Center of a circle of 30 mm diameter is 90 mm away from center of another circle of 60 mm diameter.

Draw locus of point P, moving in a plane such that it always remains equidistant from given two circles.

### SOLUTION STEPS:

1. Locate center of line, joining two centers but part in between periphery of two circles. Name it P. This will be initial point P.

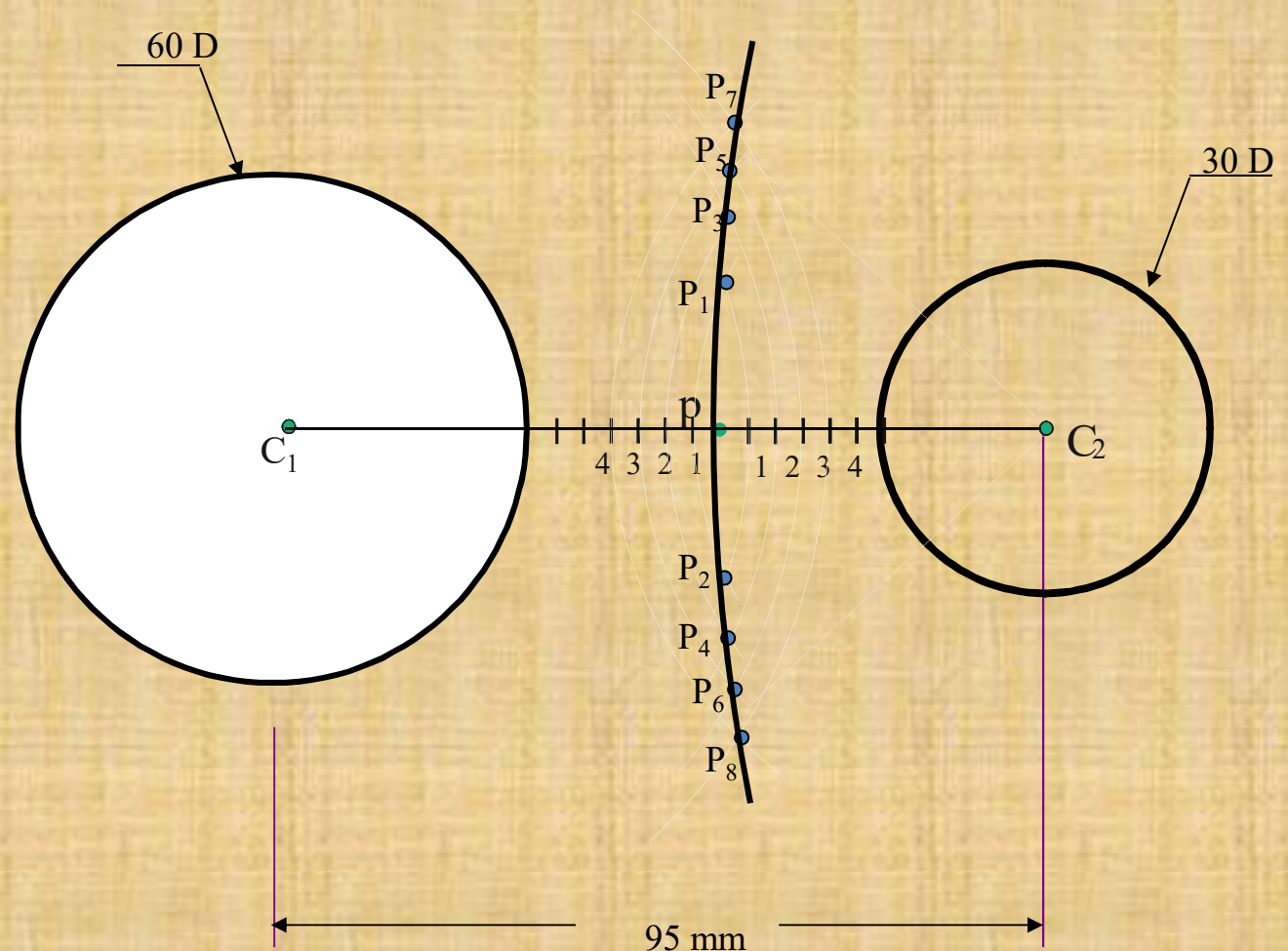
2. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw arcs from  $C_1$  As center.

3. Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw arcs from  $C_2$  As center.

4. Mark various positions of P as per previous problems and name those similarly.

5. Join all these points in smooth curve.

It will be the locus of P equidistance from given two circles.

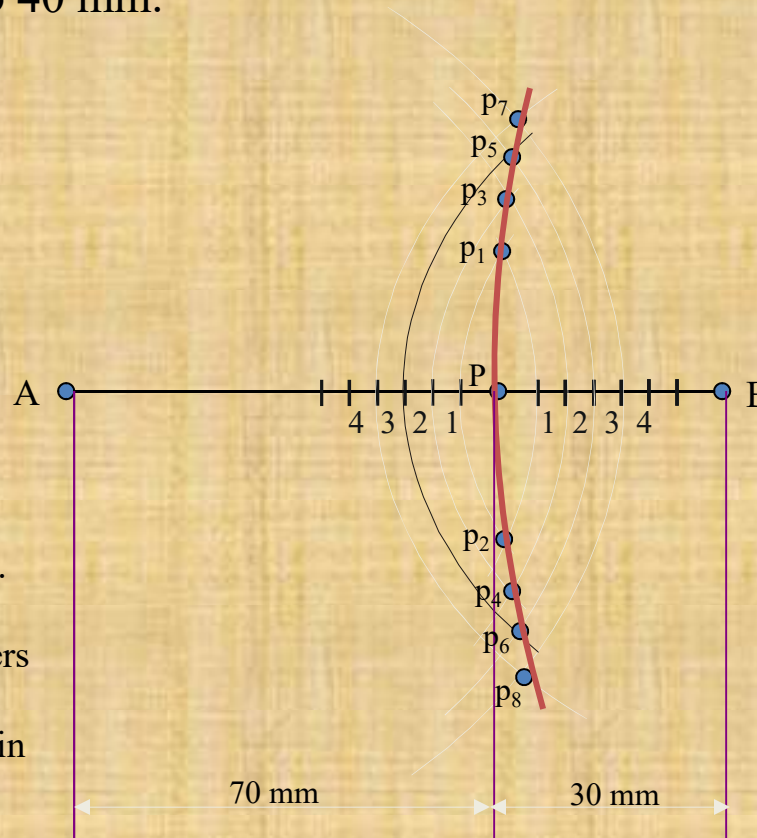


## Basic Locus Cases:

**Problem 5:-**Two points A and B are 100 mm apart.  
There is a point P, moving in a plane such that the difference of its distances from A and B always remains constant and equals to 40 mm.  
Draw locus of point P.

**Solution Steps:**

1. Locate A & B points 100 mm apart.
2. Locate point P on AB line,  
70 mm from A and 30 mm from B  
As  $PA - PB = 40$  ( $AB = 100$  mm)
3. On both sides of P mark points 5 mm apart. Name those 1,2,3,4 as usual.
4. Now similar to steps of Problem 2,  
Draw different arcs taking A & B centers and A-1, B-1, A-2, B-2 etc as radius.
5. Mark various positions of p i.e. and join them in smooth possible curve.  
It will be locus of P



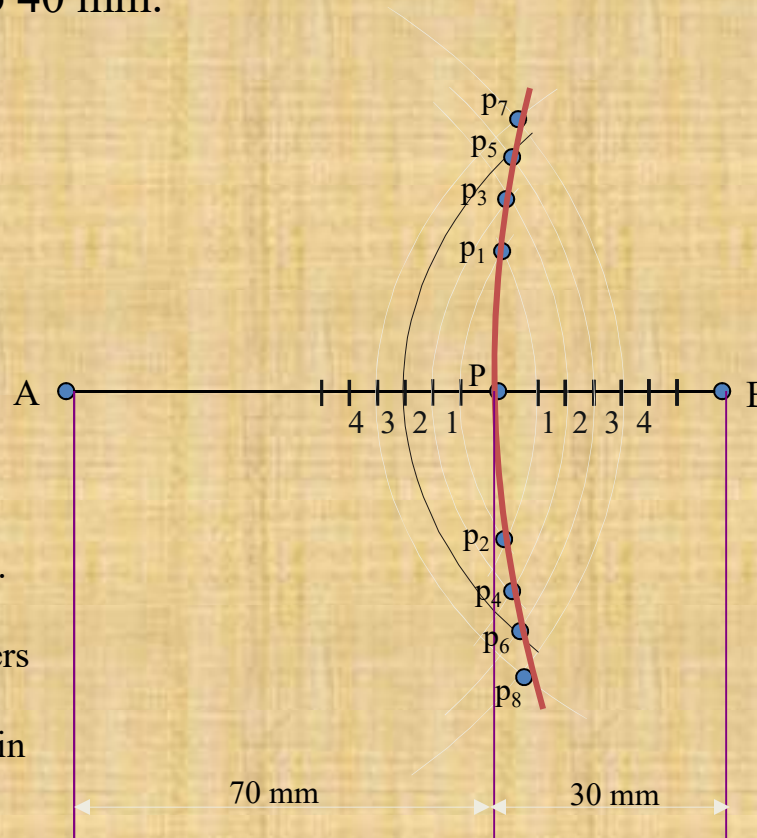


## Basic Locus Cases:

**Problem 5:-**Two points A and B are 100 mm apart.  
There is a point P, moving in a plane such that the difference of its distances from A and B always remains constant and equals to 40 mm.  
Draw locus of point P.

**Solution Steps:**

1. Locate A & B points 100 mm apart.
2. Locate point P on AB line,  
70 mm from A and 30 mm from B  
As  $PA - PB = 40$  ( $AB = 100$  mm)
3. On both sides of P mark points 5 mm apart. Name those 1,2,3,4 as usual.
4. Now similar to steps of Problem 2,  
Draw different arcs taking A & B centers and A-1, B-1, A-2, B-2 etc as radius.
5. Mark various positions of p i.e. and join them in smooth possible curve.  
It will be locus of P

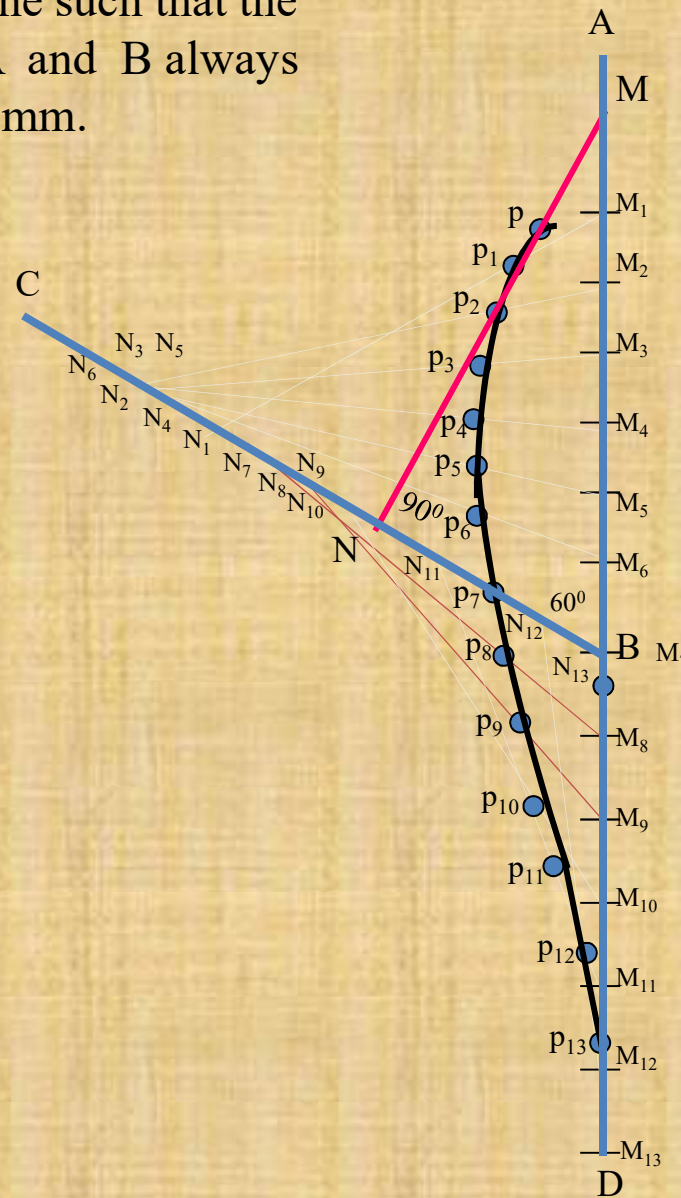


## FORK & SLIDER

Problem 6:-Two points A and B are 100 mm apart.  
There is a point P, moving in a plane such that the difference of it's distances from A and B always remains constant and equals to 40 mm.  
Draw locus of point P.

Solution Steps:

- 1) Mark lower most position of M on extension of AB (downward) by taking distance MN (40 mm) from point B (because N can not go beyond B ).
- 2) Divide line (M initial and M lower most ) into eight to ten parts and mark them  $M_1, M_2, M_3$  up to the last position of M .
- 3) Now take MN (40 mm) as fixed distance in compass,  $M_1$  center cut line CB in  $N_1$ .
- 4) Mark point  $P_1$  on  $M_1N_1$  with same distance of MP from  $M_1$ .
- 5) Similarly locate  $M_2P_2, M_3P_3, M_4P_4$  and join all P points.  
It will be locus of P.





## OSCILLATING LINK

Problem No.7:

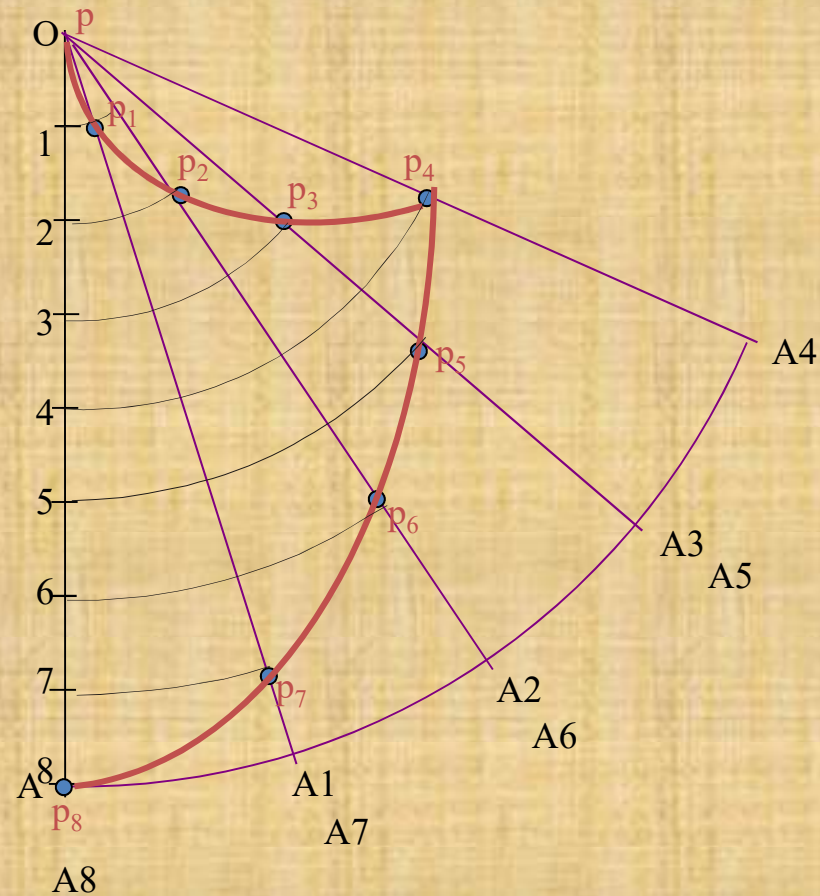
A Link OA, 80 mm long oscillates around O,  $60^\circ$  to right side and returns to its initial vertical Position with uniform velocity. Mean while point P initially on O starts sliding downwards and reaches end A with uniform velocity.

Draw locus of point P

### Solution Steps:

*Point P- Reaches End A (Downwards)*

- 1) Divide OA in EIGHT equal parts and from O to A after O name 1, 2, 3, 4 up to 8. (i.e. up to point A).
- 2) Divide  $60^\circ$  angle into four parts ( $15^\circ$  each) and mark each point by  $A_1, A_2, A_3, A_4$  and for return  $A_5, A_6, A_7$  and  $A_8$ . (Initial A point).
- 3) Take center O, distance in compass O-1 draw an arc upto  $OA_1$ . Name this point as  $P_1$ .
- 1) Similarly O center O-2 distance mark  $P_2$  on line O- $A_2$ .
- 2) This way locate  $P_3, P_4, P_5, P_6, P_7$  and  $P_8$  and join them. (It will be the desired locus of P)



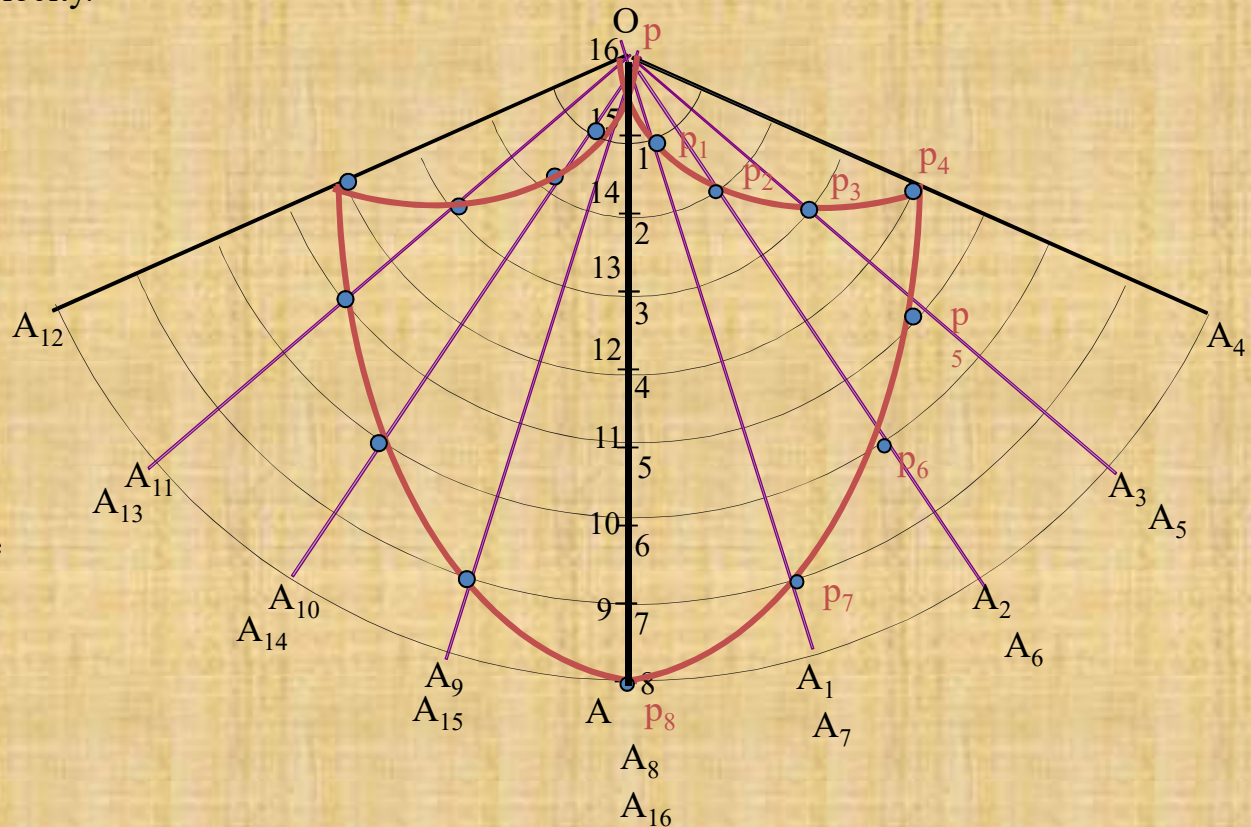
# OSCILLATING LINK

## Problem No 8:

A Link OA, 80 mm long oscillates around O,  $60^\circ$  to right side,  $120^\circ$  to left and returns to it's initial vertical Position with uniform velocity. Mean while point P initially on O starts sliding downwards, reaches end A and returns to O again with uniform velocity. Draw locus of point P

## Solution Steps:

- ( P reaches A i.e. moving downwards. & returns to O again i.e.moves upwards )
1. Here distance traveled by point P is PA. plus AP. Hence divide it into eight equal parts. ( so total linear displacement gets divided in 16 parts) Name those as shown.
2. Link OA goes  $60^\circ$  to right, comes back to original (Vertical) position, goes  $60^\circ$  to left and returns to original vertical position. Hence total angular displacement is  $240^\circ$ . Divide this also in 16 parts. ( $15^\circ$  each.) Name as per previous problem. (A,  $A_1$   $A_2$  etc)
3. Mark different positions of P as per the procedure adopted in previous case. and complete the problem.





## ROTATING LINK

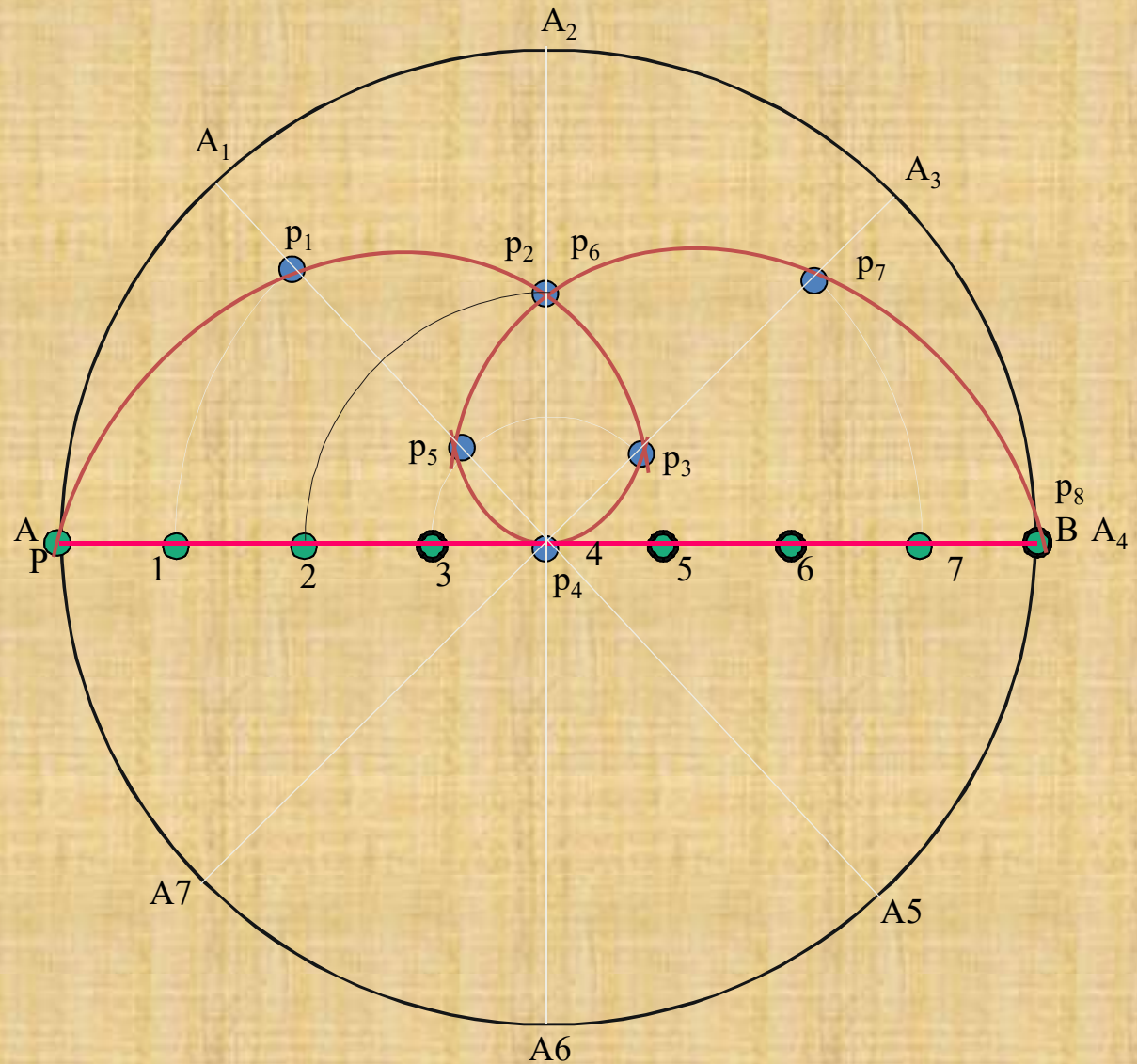
### Problem 9:

Rod AB, 100 mm long, revolves in clockwise direction for one revolution.

Meanwhile point P, initially on A starts moving towards B and reaches B.

Draw locus of point P.

- 1) AB Rod revolves around center O for one revolution and point P slides along AB rod and reaches end B in one revolution.
- 2) Divide circle in 8 number of equal parts and name in arrow direction after A-A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, up to A<sub>8</sub>.
- 3) Distance traveled by point P is AB mm. Divide this also into 8 number of equal parts.
- 4) Initially P is on end A. When A moves to A<sub>1</sub>, point P goes one linear division (part) away from A<sub>1</sub>. Mark it from A<sub>1</sub> and name the point P<sub>1</sub>.
- 5) When A moves to A<sub>2</sub>, P will be two parts away from A<sub>2</sub> (Name it P<sub>2</sub>). Mark it as above from A<sub>2</sub>.
- 6) From A<sub>3</sub> mark P<sub>3</sub> three parts away from P<sub>3</sub>.
- 7) Similarly locate P<sub>4</sub>, P<sub>5</sub>, P<sub>6</sub>, P<sub>7</sub> and P<sub>8</sub> which will be eight parts away from A<sub>8</sub>. [Means P has reached B].
- 8) Join all P points by smooth curve. It will be locus of P.



## ROTATING LINK

### Problem 10 :

Rod AB, 100 mm long, revolves in clockwise direction for one revolution.

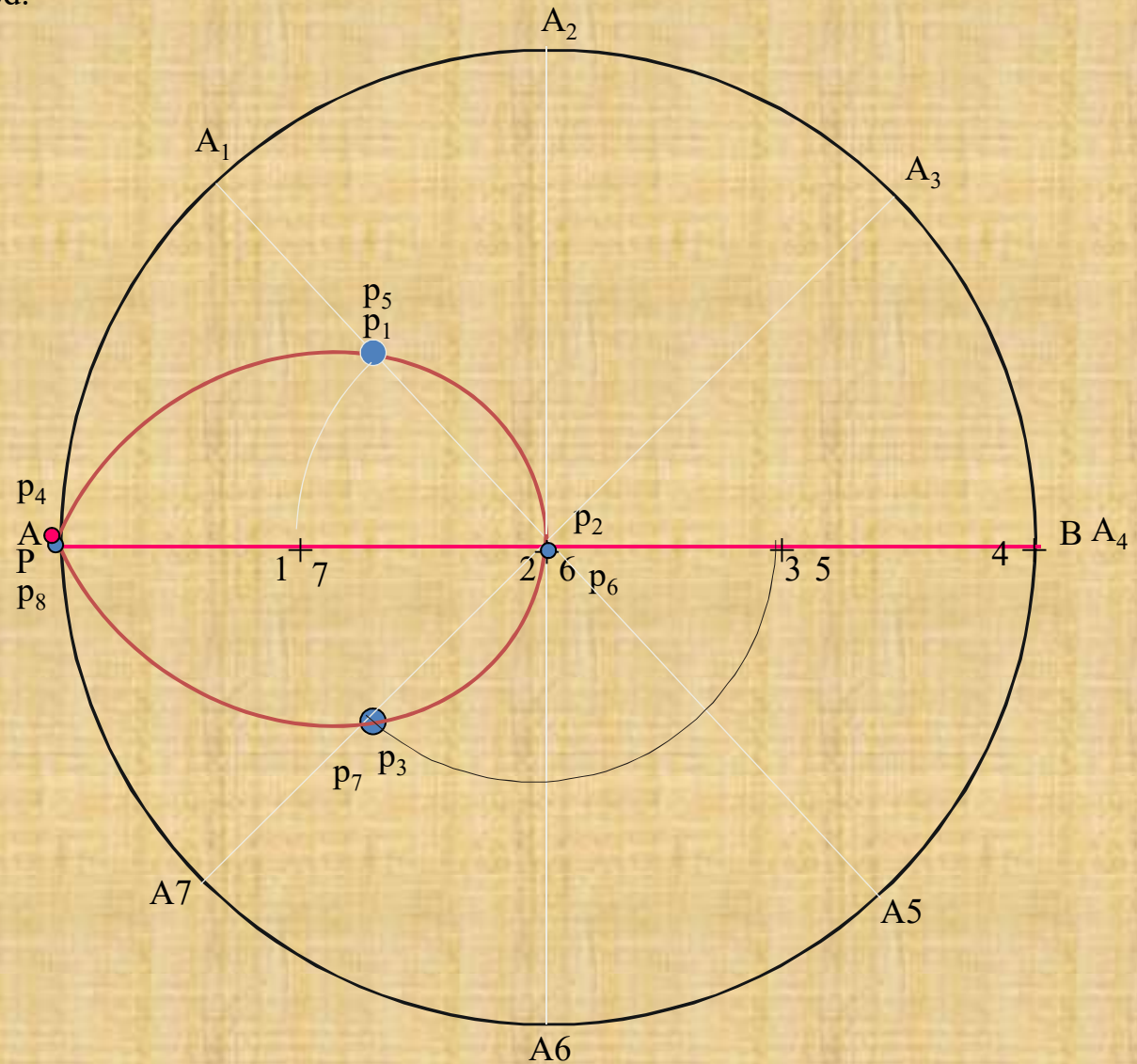
Meanwhile point P, initially on A starts moving towards B, reaches B

And returns to A in one revolution of rod.

Draw locus of point P.

### Solution Steps

- 1) AB Rod revolves around center O for one revolution and point P slides along rod AB reaches end B and returns to A.
  - 2) Divide circle in 8 number of equal parts and name in arrow direction after A-A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, up to A<sub>8</sub>.
  - 3) Distance traveled by point P is AB plus AB mm. Divide AB in 4 parts so those will be 8 equal parts on return.
  - 4) Initially P is on end A. When A moves to A<sub>1</sub>, point P goes one linear division (part) away from A<sub>1</sub>. Mark it from A<sub>1</sub> and name the point P<sub>1</sub>.
  - 5) When A moves to A<sub>2</sub>, P will be two parts away from A<sub>2</sub> (Name it P<sub>2</sub>). Mark it as above from A<sub>2</sub>.
  - 6) From A<sub>3</sub> mark P<sub>3</sub> three parts away from P<sub>3</sub>.
  - 7) Similarly locate P<sub>4</sub>, P<sub>5</sub>, P<sub>6</sub>, P<sub>7</sub> and P<sub>8</sub> which will be eight parts away from A<sub>8</sub>. [Means P has reached B].
  - 8) Join all P points by smooth curve. It will be locus of P
- The Locus will follow the loop path two times in one revolution.





# **DRAWINGS:**

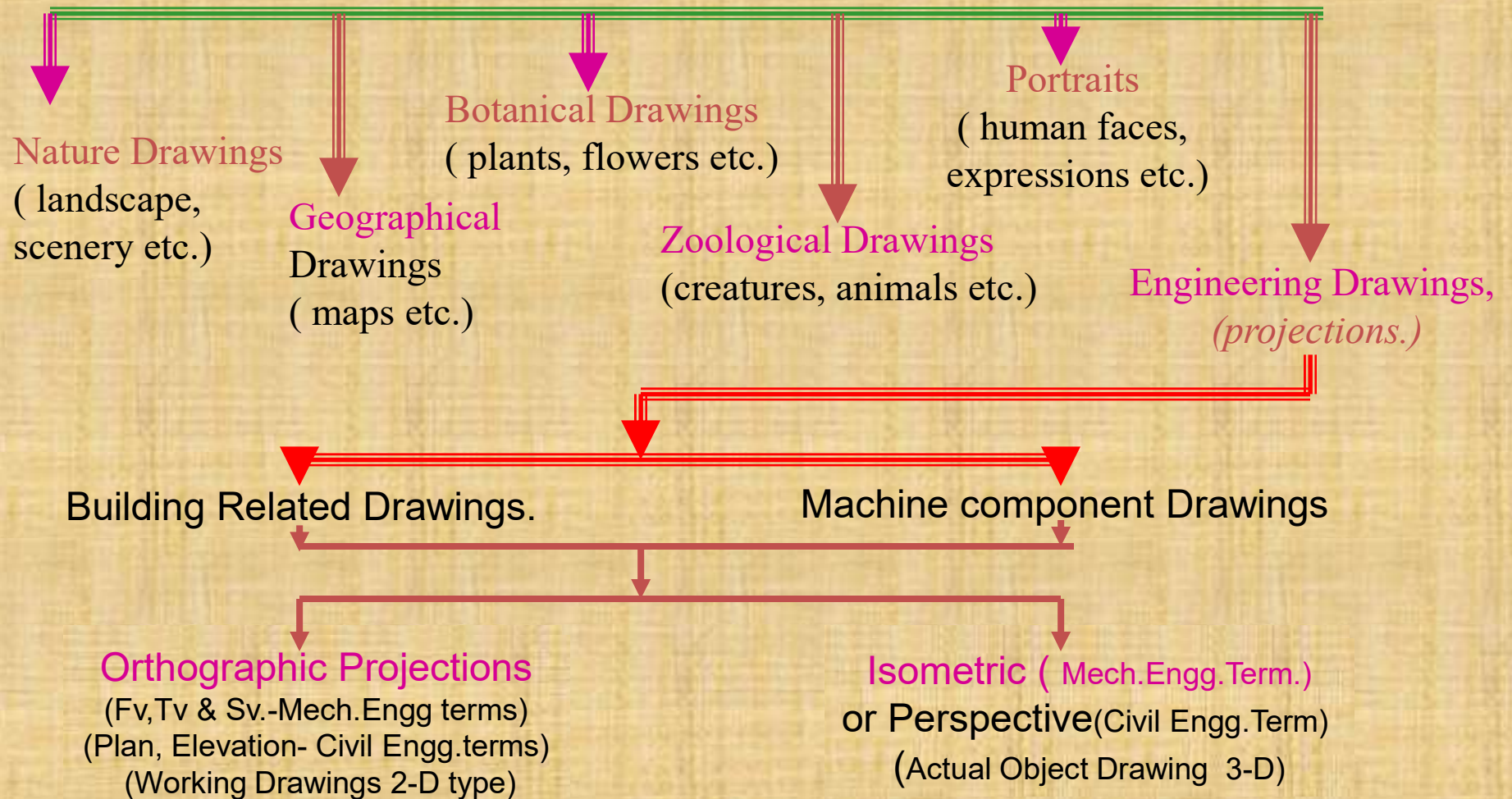
*( A Graphical Representation)*

**The Fact about:**

If compared with Verbal or Written Description,  
Drawings offer far better idea about the Shape, Size & Appearance of  
any object or situation or location, that too in quite a less time.

*Hence it has become the Best Media of Communication  
not only in Engineering but in almost all Fields.*

## Drawings (Some Types)





## ORTHOGRAPHIC PROJECTIONS:

IT IS A TECHNICAL DRAWING IN WHICH DIFFERENT VIEWS OF AN OBJECT  
ARE PROJECTED ON DIFFERENT REFERENCE PLANES  
OBSERVING PERPENDICULAR TO RESPECTIVE REFERENCE PLANE

Different Reference planes are

**Horizontal Plane (HP),  
Vertical Frontal Plane ( VP )  
Side Or Profile Plane ( PP)**

And

Different Views are Front View (FV), Top View (TV) and Side View (SV)

**FV is a view projected on VP.**

**TV is a view projected on HP.**

**SV is a view projected on PP.**

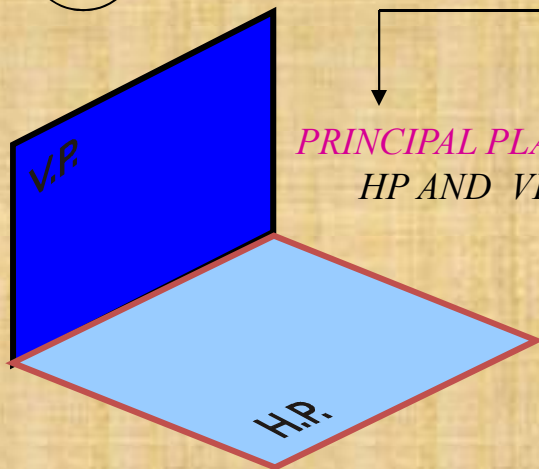
### *IMPORTANT TERMS OF ORTHOGRAPHIC PROJECTIONS:*

- ① Planes.
- ② Pattern of planes & Pattern of views
- ③ Methods of drawing Orthographic Projections

1

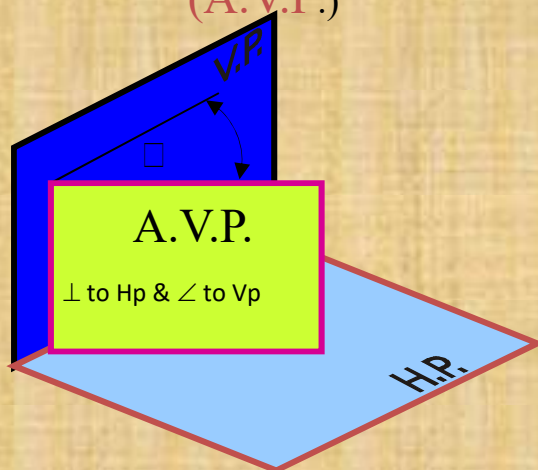
## PLANES

*PRINCIPAL PLANES*  
HP AND VP

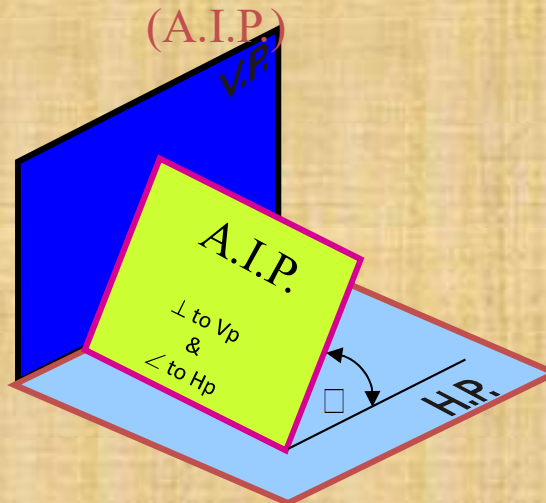


### *AUXILIARY PLANES*

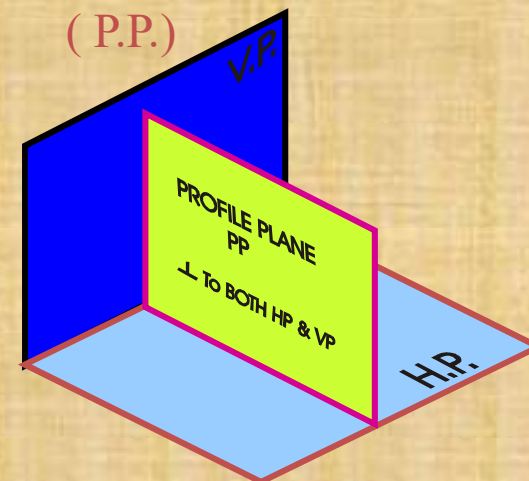
Auxiliary Vertical Plane  
(A.V.P.)



Auxiliary Inclined Plane  
(A.I.P.)



Profile Plane  
(P.P.)





2

## PATTERN OF PLANES & VIEWS (First Angle Method)

THIS IS A PICTORIAL SET-UP OF ALL THREE PLANES.  
ARROW DIRECTION IS A NORMAL WAY OF OBSERVING THE OBJECT.  
BUT IN THIS DIRECTION ONLY VP AND A VIEW ON IT (FV) CAN BE SEEN.  
THE OTHER PLANES AND VIEWS ON THOSE CAN NOT BE SEEN.

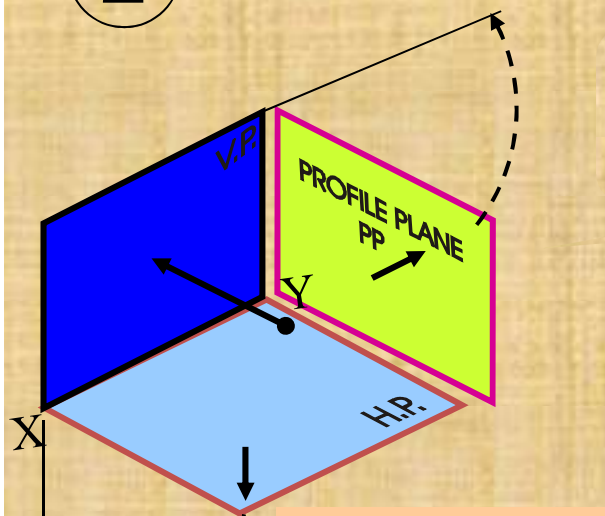
### PROCEDURE TO SOLVE ABOVE PROBLEM:-

TO MAKE THOSE PLANES ALSO VISIBLE FROM THE ARROW DIRECTION,

A) HP IS ROTATED  $90^\circ$  DOWNWARD

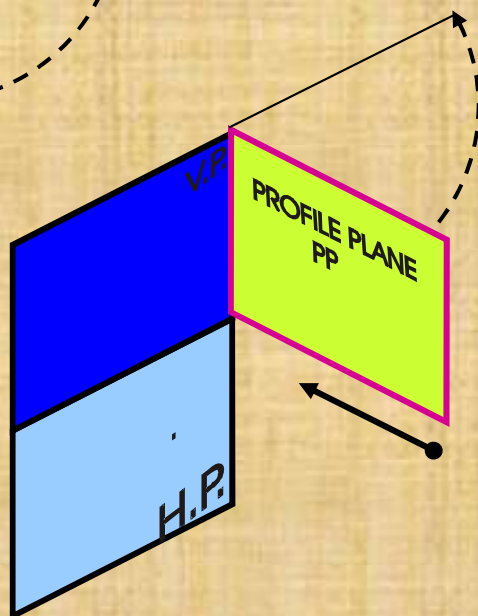
B) PP,  $90^\circ$  IN RIGHT SIDE DIRECTION.

THIS WAY BOTH PLANES ARE BROUGHT IN THE SAME PLANE CONTAINING VP.

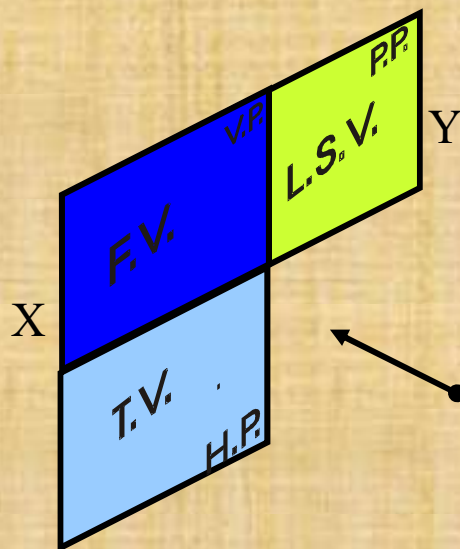


Click to view Animation

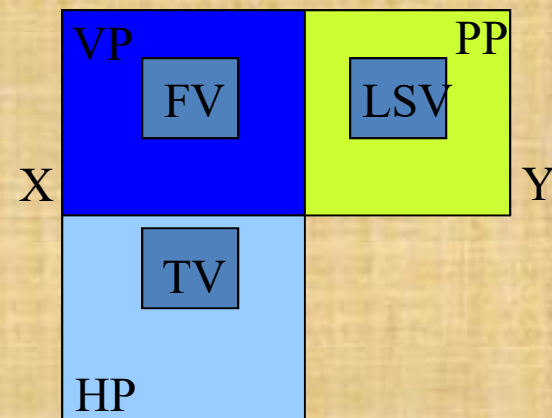
On clicking the button if a warning comes please click YES to continue, this program is safe for your pc.



HP IS ROTATED DOWNWARD  $90^\circ$   
AND  
BROUGHT IN THE PLANE OF VP.



PP IS ROTATED IN RIGHT SIDE  $90^\circ$   
AND  
BROUGHT IN THE PLANE OF VP.



ACTUAL PATTERN OF PLANES & VIEWS  
OF ORTHOGRAPHIC PROJECTIONS  
DRAWN IN  
FIRST ANGLE METHOD OF PROJECTIONS

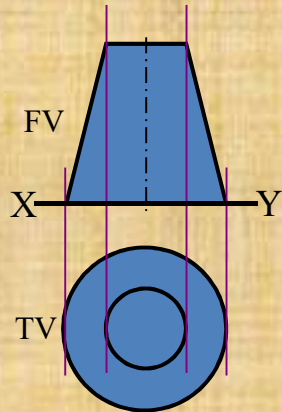
3

## Methods of Drawing Orthographic Projections

### First Angle Projections Method

Here views are drawn  
by placing object  
in 1<sup>st</sup> Quadrant

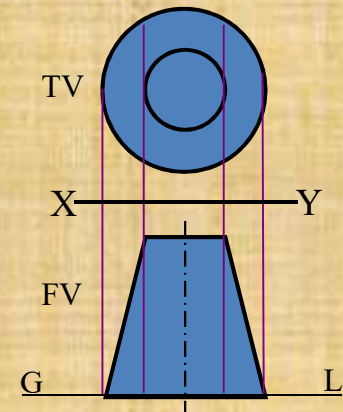
( Fv above X-y, Tv below X-y )



### Third Angle Projections Method

Here views are drawn  
by placing object  
in 3<sup>rd</sup> Quadrant.

( Tv above X-y, Fv below X-y )



SYMBOLIC  
PRESENTATION  
OF BOTH METHODS  
WITH AN OBJECT  
STANDING ON HP ( GROUND)  
ON IT'S BASE.

NOTE:-

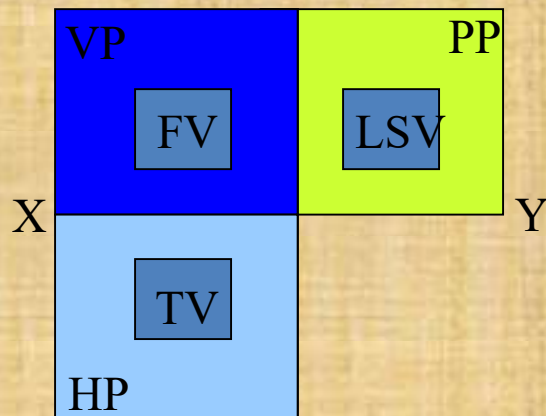
HP term is used in 1<sup>st</sup> Angle method  
&  
For the same  
Ground term is used  
in 3<sup>rd</sup> Angle method of projections



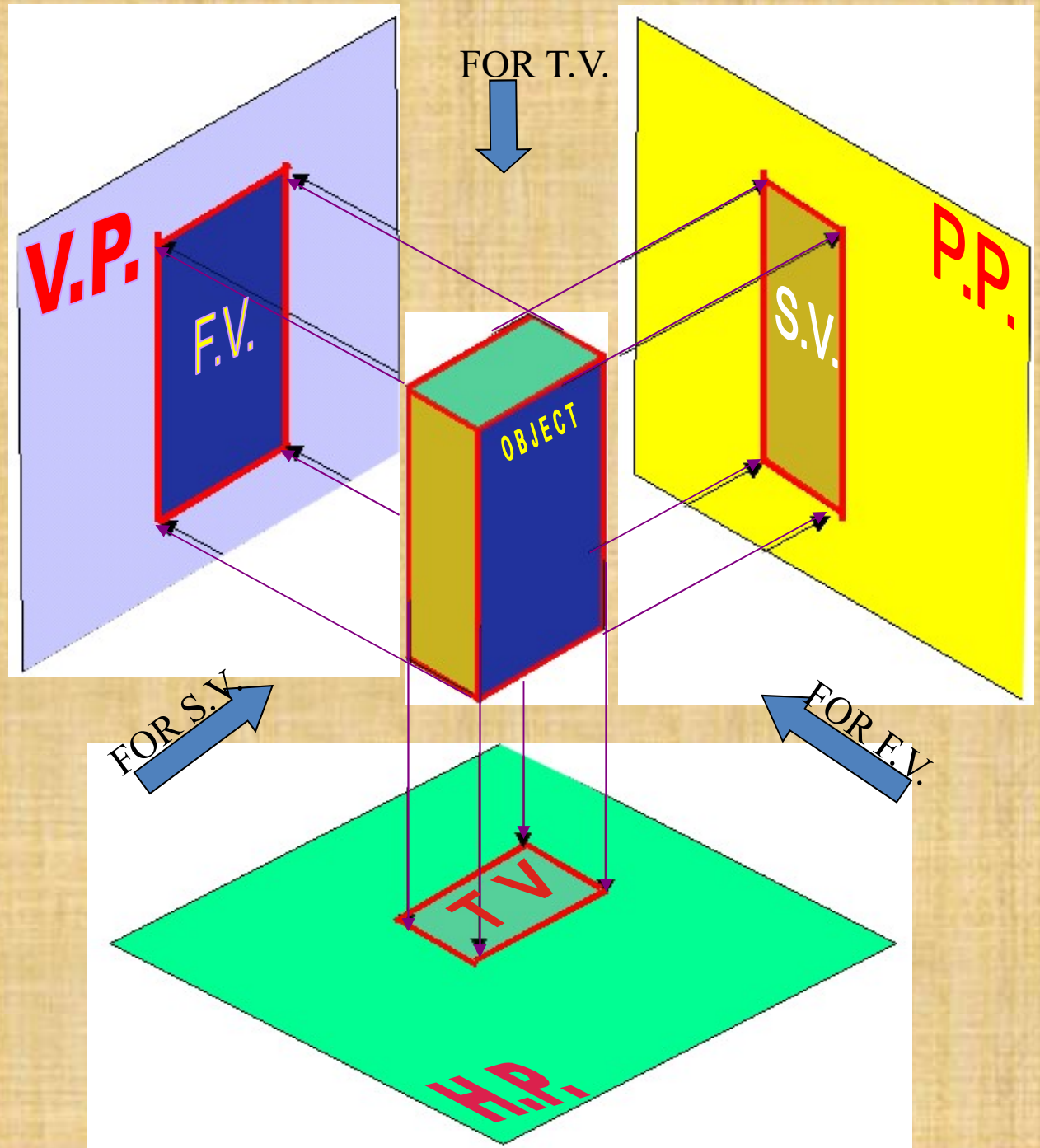
# FIRST ANGLE PROJECTION

IN THIS METHOD,  
THE OBJECT IS ASSUMED TO BE  
SITUATED IN FIRST QUADRANT  
MEANS  
ABOVE HP & INFRONT OF VP.

OBJECT IS IN BETWEEN  
OBSERVER & PLANE.



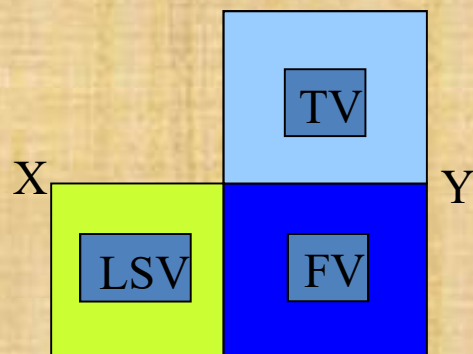
ACTUAL PATTERN OF  
PLANES & VIEWS  
IN  
FIRST ANGLE METHOD  
OF PROJECTIONS



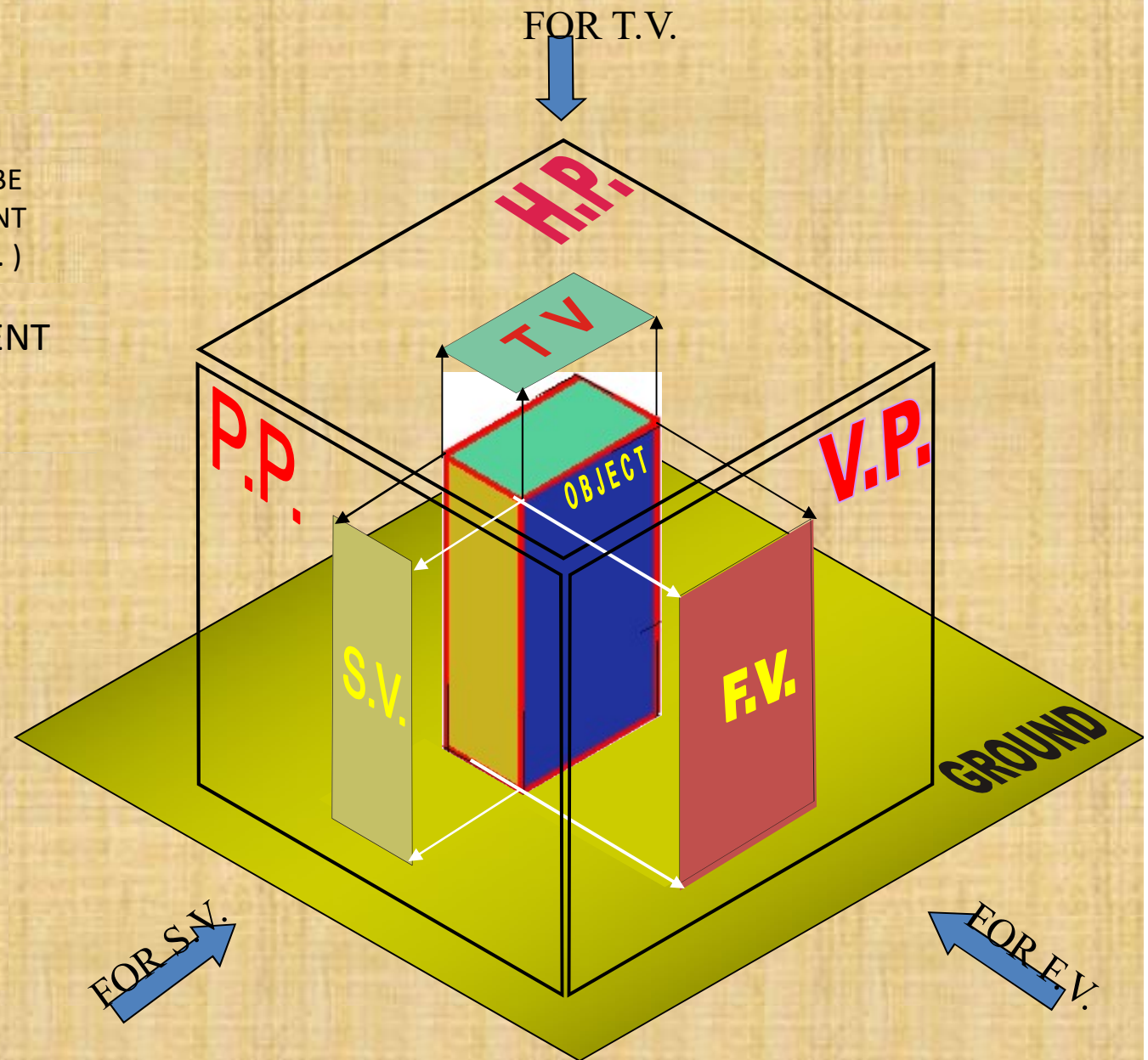
# THIRD ANGLE PROJECTION

IN THIS METHOD,  
THE OBJECT IS ASSUMED TO BE  
SITUATED IN THIRD QUADRANT  
( BELOW HP & BEHIND OF VP. )

PLANES BEING TRANSPERENT  
AND INBETWEEN  
OBSERVER & OBJECT.



ACTUAL PATTERN OF  
PLANES & VIEWS  
OF  
THIRD ANGLE PROJECTIONS





**Continued in next part**