# Unit - IV

## Introduction to Kinetics of Rigid Bodies:-

In the case of a particle moving in a plane, the relation between the forces acting on the particle and the resulting translatory motion is given by a Single Vector Equation.

$$\Sigma F = ma$$

The corresponding scalar equations are,

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

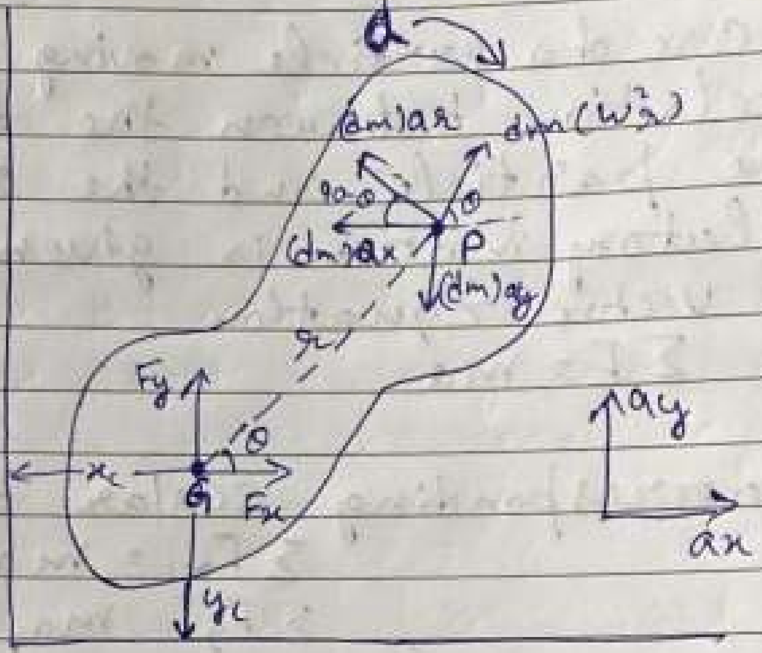
In case of a rigid body in plane motion, its rotatory motion is also to be considered. This results in an increase in one equation of motion to account for this rotary motion.

## Plane motion of a Rigid Body

### Equations of Motion:-

Consider a rigid body of mass m in plane motion under the action of the applied external forces.
Let the resultant of these external forces reduce to
a force $F_x$ in the x- direction
a force $F_y$ in the y- direction
a moment $M_G$ about the mass centre

Let the mass Centre G of the body move parallel to the $x$-$y$ plane.




Consider the motion of any particle P of mass dm situated at a distance $r$ from the axis passing through G and normal to the plane of the motion $x$-$y$

Choose the mass centre G as pole and let its coordinates be $x_c, y_c$. Let the angle which the line PG makes with the x axis be $\theta$.

Let the linear acceleration of the pole G be,

$a_x$ in the $x$-direction

$a_y$ in the $y$-direction

These are also the components of linear acceleration on the particle P.

The inertia force acting on the particle P :-

$(dm)a_x$ acting in direction opposite to $a_x$

$(dm)a_y$ acting in direction opposite to $a_y$

Let the angular velocity of the body be $\omega$ and the angular acceleration be $\alpha$.

As the particle P rotates about an axis through G and at a distance $r$ from it, inertia force acting on it are :-

$dm(\omega^2 r)$, acting along GP

$dm(\alpha r)$, acting normal to GP

The inertia forces acting on the entire Body can be calculated by adding the inertia forces acting on all the particles of the Body.

The Equation of dynamic Equilibrium of the body can be written as,

$\Sigma F_x = 0 : F_x - a_x \int dm - \alpha \int r\cos(90-\theta)\,dm + \omega^2 \int r\cos\theta\,dm = 0$

$\Sigma F_y = 0 : F_y - a_y \int dm + \alpha \int r\sin(90-\theta)\,dm + \omega^2 \int r\sin\theta\,dm = 0$

$\Sigma M_G = 0 : M_G + a_y \int r\cos\theta\,dm - a_x \int r\sin\theta\,dm - \alpha \int r^2\,dm = 0$

But, $\int dm = m$, $\int r\sin\theta\,dm = 0$, $\int r\cos\theta\,dm = 0$

and $\int r^2\,dm = I_G$

$I_G$ being the M.O.I of the Body about the axis through the mass centre G and normal to the plane of motion $x$-$y$

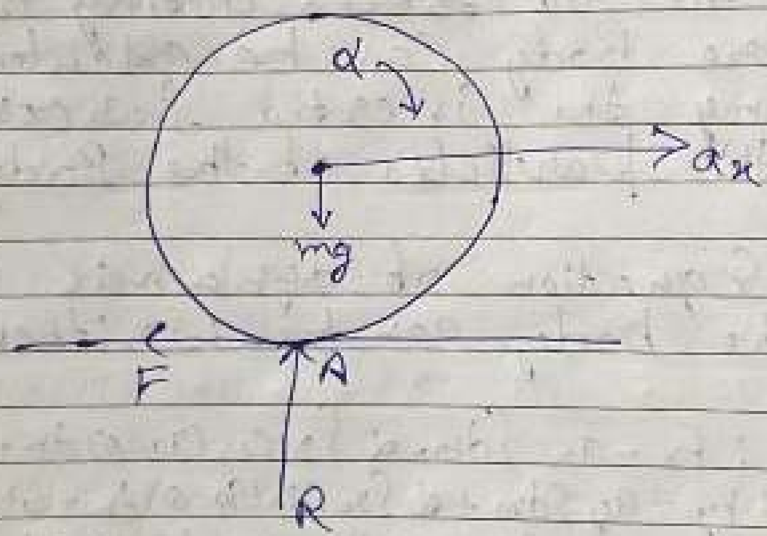
Therefore, the equations can be written as

$F_x - ma_x = 0 \Rightarrow F_x = ma_x$
$F_y - ma_y = 0 \Rightarrow F_y = ma_y$
$M_G - I_G\alpha = 0 \Rightarrow M_G = I_G\alpha$

Equation of motion of a rigid Body

* **Relation Between the translatory motion and Rotary motion of a Body in plane motion :-**

Consider the specific case of the plane motion of a right circular cylinder on a horizontal surface as shown in figure




let the mass of the cylinder be m, R be the normal reaction and F the force of friction acting on the cylinder. The acceleration of its mass centre G be $a_x$ and angular acceleration α.

The cylinder may:-

(i) roll without slipping
(ii) may slip without rolling
(iii) may roll as well as slip

(i) <u>Rolls without slipping</u>

In this case, the forward motion of the mass centre is related to its angular motion. The distance travelled and the angle turned by the body are related as

$$x = \theta r$$
$$\dot{x} = \dot{\theta} r = \omega r$$
$$\ddot{x} = \ddot{\theta} r = \alpha r \quad \text{or} \quad a_x = \alpha r$$

and the frictional force F is such that

$$F \leq \mu R$$

(ii) <u>Slips without Rolling</u>

There is no rotary motion involved and $F = \mu R$

(iii) <u>Rolls as well as slips</u>

In this case translatory and rotary motions are independent of each other and

$$F = \mu R$$

Q:- A grinding wheel has a rated speed of 1500 r.p.m and can be assumed to be a disc of 0.5m radius and of uniform thickness. It weights 300N. It is made to turn at 1500 r.p.m and then allowed to decelerate uniformly due to bearing friction. It was observed to come to rest in 120 seconds. Determine

(i) the number of revolutions that it shall execute before coming to rest

(ii) Frictional torque.

Solution:- $N = 1500$ r.p.m

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60} \text{ rad/s}$$

$\omega = 0$ (final)

$t = 120$ seconds

as $\omega = \omega_0 + \alpha t$

$$\alpha = -\frac{\omega_0}{t} = -\frac{2\pi \times 1500}{60 \times 120}$$

Angular deceleration

$$\alpha = -1.308 \text{ rad/s}^2$$

Using, $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$0 = \left(\frac{2\pi \times 1500}{60}\right)^2 + 2(-1.308)\theta$$

$$\theta = 9417 \text{ radians}$$

Number of revolution $= \dfrac{9417}{2\pi}$

No. of revolution before coming to rest $= 1499$

Frictional Torque

$$M = I_G \alpha$$

$$I_G = \frac{1}{2} m r^2 = \frac{1}{2}\left(\frac{300}{9.81}\right)(0.5)^2$$

$$M = -\frac{1}{2}\left(\frac{300}{9.81}\right)(0.5)^2 \times (1.308)$$

$$\underline{M = -5.0\,Nm}\ \text{Ans}$$

D'alembert's principle in plane Motion :-

D'alembert principle can also be applied to study the plane motion of rigid bodies. In this method, the body is brought to a dynamic Equilibrium by applying inertia forces and inertia couples. The equation of dynamic Equilibrium then can be written for the Body.

Inertia forces are to be applied to the mass centre of the Body in a direction opposite to the direction of the acceleration.

Inertia couple can be applied anywhere in the plane of motion of the body but in a direction opposite to the direction of the angular acceleration of the Body.

Date
Page

Equations of dynamic equilibrium can be written as,

$\Sigma F_x = 0$ :- $X + (-m a_x) = 0$

$\Sigma F_y = 0$ :- $Y + (-m a_y) = 0$

$\Sigma M = 0$ :- $M + (-I\alpha) = 0$

X and Y are the External forces acting on the Body and M is the moment due to the External forces.

$a_x, a_y$ are the components of the acceleration in the $x$ and $y$ directions

$\alpha$ is the angular acceleration and I the M.O.I of the body about the axis of rotation.

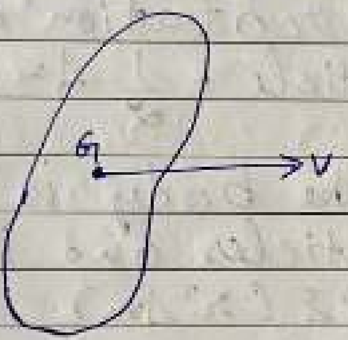
Kinetic Energy of a Rigid Body

Consider a rigid body in plane motion. Let the body be made up of a large number of particles each of mass $\Delta m$. The mass $m$ of the Body is the Sum of the masses of its particles therefore, $m = \Sigma \Delta m$.
The Kinetic Energy of the rigid body would be equal to the Kinetic Energy of all its particles.

But the plane motion of a rigid body consists of a translatory and a rotary motion.
Therefore, the kinetic Energy of a rigid body is the sum of the Kinetic Energy in translation and the kinetic energy in rotation of all its particles.

## Kinetic Energy in Translation :-

Let all the particles of the rigid body be moving with the same velocity as the velocity v of its mass centre G.



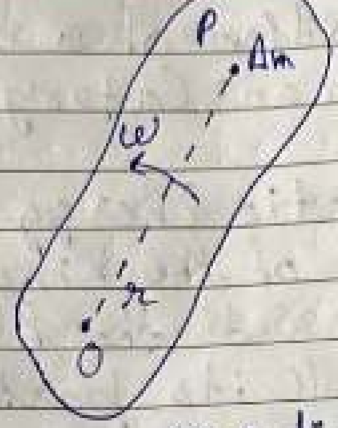

Kinetic Energy of particle of mass $\Delta m = \frac{1}{2} \Delta m v^2$

Kinetic Energy of the body $= \frac{1}{2} (\Sigma \Delta m) v^2$

Kinetic Energy of the body in translation $= \frac{1}{2} m v^2$

## Kinetic Energy in Rotation :-

Consider a rigid body rotating at an angular velocity $\omega$ about a fixed axis intersecting the plane of motion at O as shown in figure.




Consider a particle P of mass $\Delta m$ situated at a distance $r$ from the axis of rotation.

Velocity of the particle $= \omega r$

Kinetic Energy of rotation of the particle $= \frac{1}{2} \Delta m (\omega r)^2$

Kinetic Energy of the body is made up of similar particles and is

$$= \frac{1}{2} \omega^2 \Sigma (r^2 \Delta m)$$

Kinetic Energy of rotation of the body $= \frac{1}{2} I \omega^2$

Total Kinetic Energy of the body

= K.E in translation + K.E in rotation

Total Kinetic Energy of the body

$$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Scanned with CamScanner

## Principle of work and Energy for a Rigid Body :-

Mathematically this principle for a rigid body can be written in the same form as that for a particle.

$$U_{1-2} = (T_2 - T_1)$$

Work done = change in Kinetic Energy

In the case of rigid body,

Work done = {work resulting due to the linear displacement from initial position $S_1$ to final position $S_2$} + {work resulting due to the angular displacement from position $\theta_1$ to $\theta_2$}

$$U_{1-2} = \int_{S_1}^{S_2} (F \cos \alpha) ds + \int_{\theta_1}^{\theta_2} M \, d\theta$$

$$U_{1-2} = Fs + M\theta$$

(when force F and moment M are constant.)

$(T_2 - T_1)$ is the change in kinetic Energy of the body resulting from translation and rotation between the initial and final position.

Change in K.E = $(T_2 - T_1)$

$$= \left(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2\right) + \left(\frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2\right)$$

## Principle of Conservation of Energy :-

The Sum of the Kinetic and Potential Energy of a rigid body under the action of conservative forces remains constant.

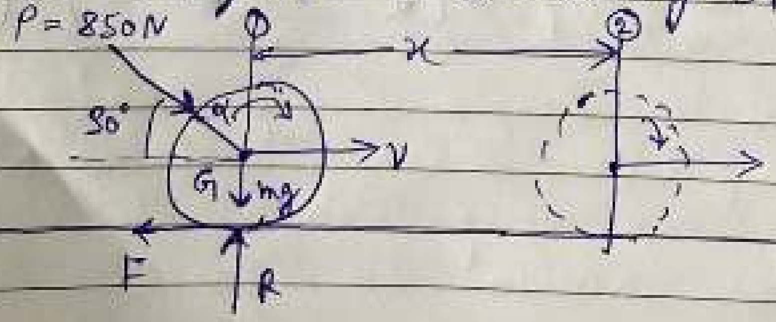
$$(P.E + K.E)_{Initial} = (P.E + K.E)_{Final}$$

$$U_1 + T_1 = U_2 + T_2$$

$$T_1 + V_1 = T_2 + V_2$$

In the case of a rigid body, K.E means the sum of the K.E in translation and in rotation.
The various conservative forces are :- gravitation forces, spring force, Elasticity force.

Q:- A roller of mass m = 600 kg and radius r = 0.25 m is pushed with a Constant Force P = 850N on a Rough horizontal plane as shown in Figure. If the roller Starts from rest and rolls without slipping, Find the distance required to be rolled if it is to acquire a velocity of 3 m/s.



Solution :- $m = 600 kg$, $r = 0.25 m$
$P = 850 N$ $v = 3 m/s$

Force involved and work done :-

(i) gravity force mg (It does no work as it acts normal to the direction of motion)

(ii) Normal Reaction R (It does no work)

(iii) friction force F (In the case of a body that rolls without sliding, it does no work).

(iv) External force 'P'.

Change in K.E between position 1 and 2.

position 1, $K.E = 0$, $T_1 = 0$

position 2, $v = 3 m/s$, $\omega = \frac{v}{r}$

K.E,

$$T_2 = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$T_2 = \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$T_2 = \frac{3}{4} mv^2$$

Change in K.E

$$(T_2 - T_1) = \frac{3}{4} mv^2$$

classmate
Date
Page

Work done between positions 1 and 2

$$U_{1-2} = (P\cos\theta)x$$
$$= (850\cos 30^\circ)x$$

principle of work and Energy gives

$$U_{1-2} = (T_2 - T_1)$$
$$(850\cos 30^\circ)x = \frac{3}{4} mv^2$$
$$(850 \times 0.866)x = \frac{3}{4} 600 \times (3)^2$$
$$x = 5.5\,m$$