Page No.

Unit-2 Part-I

Equilibrium in three Dimension :-

The conditions for equilibrium of a rigid body subjected to a three-dimensional force system requires that both the resultant force and resultant couple moment acting on the body be equal to Zero.

The two conditions of equilibrium of a rigid body may be expressed as

$$\boxed{\begin{array}{c}\Sigma F = 0 \\ \Sigma M = 0\end{array}} \quad \text{--- (i)}$$

where, ΣF = Sum of all external forces
ΣM = Sum of all moments

If all the applied external forces and Couple moments are expressed in vector form and substituted in eqn. (i)., Then we have,

$$\Sigma F = \Sigma F_x + \Sigma F_y + \Sigma F_z = 0$$
$$\Sigma M = \Sigma M_x + \Sigma M_y + \Sigma M_z = 0$$

These six scalar equilibrium equations may be used to solve for at most six unknowns shown on the free body diagram.

Charlie

Date

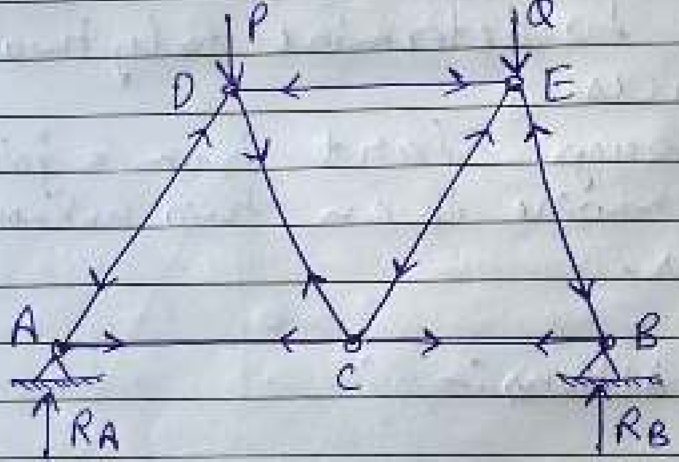
Page No.

Method of Joints :-

In this method the equilibrium of each joint is considered. The procedure is as follows:-

→ Consider the free body diagram of the entire truss and compute the support reactions using the equation of equilibrium.

→ Assume and mark the direction of the axial forces in the members on the diagram as shown in fig.



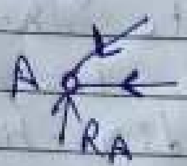
Representation of compression in the member EB, Arrows point towards the joint E and B (Push at the Joint)

Representation of tension member AC Arrows point away from joints A and C (Pull at the joint).

→ If in the solution the magnitude of a force comes out to be negative the assumed direction of the force in the member is simply reversed.

→ Consider a joint, let us consider joint A. Then draw its Free body diagram.

Joint A



F.B.D

Therefore, only two equations of equilibrium can be written for the joint and can be solved to determine only two unknown forces.

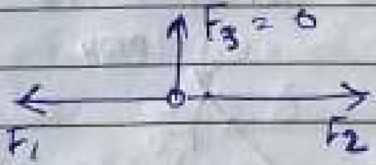
→ So we always start from a joint where not more than two unknown forces appear.

Special Conditions :-

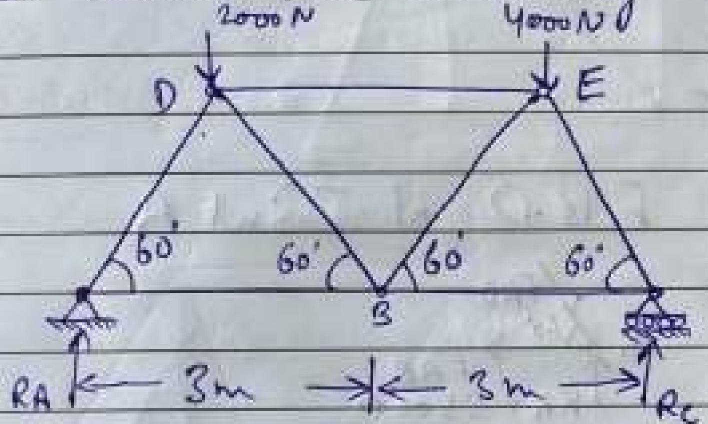
1. when two members meeting at a joint are not collinear and there is no external forces acting at the joint then the forces in both the members are zero.

2. When there are three members meeting at a joint of which two are collinear and third be

at an angle and if there is no load at the Joint the force in the third member is Zero



Q: Using the methods of Joints, find the axial forces in all the members of a truss with the loading shown in fig.



Soln: To determine the support reactions consider the equilibrium of the entire truss.
In general, the reaction at a hinge can have two components acting in the horizontal and the vertical direction. As there is no horizontal ~~and~~ external force acting on the truss, so the horizontal component of reaction at A is Zero.
∴ Taking moment about A

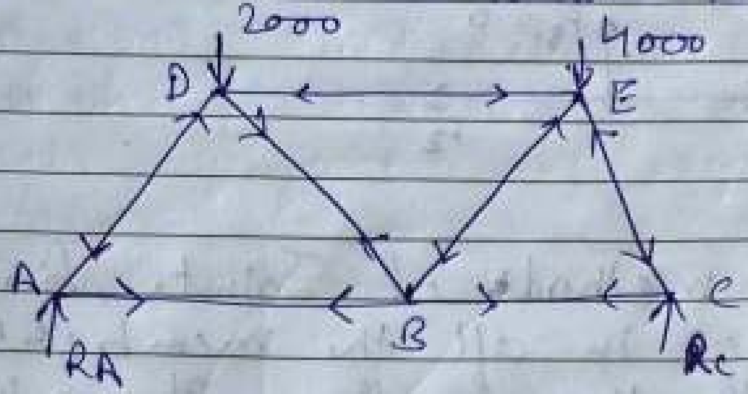
$\Sigma M_A = 0, \quad -2000(1.5) - 4000(4.5) + R_C(6) = 0$

$R_C = 3500 \text{ N}$

$\Sigma F_y = 0, \quad R_A + R_C - 2000 - 4000 = 0$

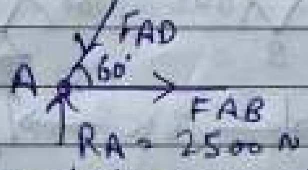
$R_A = 2500 \text{ N}$

Before considering equilibrium of the Joints The direction of axial forces in all the members as shown in fig.



At Joint A :-

Consider F.B.D of Joint A



Equation of Equilibrium can be written as

$\Sigma F_x = 0$:- $\quad F_{AB} - F_{AD} \cos 60° = 0 \quad$ -(i)

$\Sigma F_y = 0$:- $\quad R_A - F_{AD} \sin 60° = 0 \quad$ -(ii)

$$F_{AD} = \frac{R_A}{\sin 60°} = \frac{2500}{0.866} = 2887\,N$$

$F_{AD} = 2887\,N$ (Compression)

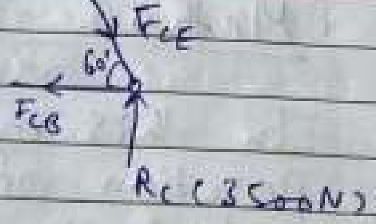
using eqn. (i)

$$F_{AB} = F_{AD} \cos 60°$$
$$= 2887 \times 0.5$$
$$= 1443\,N \text{ (Tension)}$$

The magnitudes of the forces F_{AB} and F_{AD} are both coming out be positive therefore the assumed direction of the forces are correct.

Date
Page No.

At Joint C :-



$\Sigma F_x = 0$:- $R_{CE} \cos 60° - F_{CB} = 0$ — (iii)

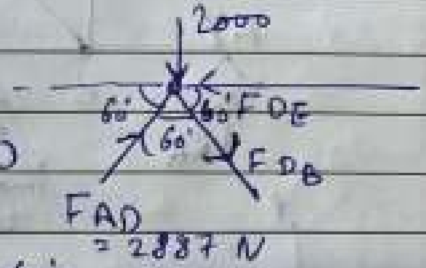
$\Sigma F_y = 0$:- $R_C - R_{CE} \sin 60° = 0$ — (iv)

$$F_{CE} = \frac{R_C}{\sin 60°} = \frac{3500}{0.866} = 4041\,N\ (C)$$

From (iii)

$$F_{CB} = \frac{F_{CE}}{\cos 60°} = \frac{4041}{0.5} = 2020.5\,N\ (T)$$

At Joint D :-



$\Sigma F_x = 0$:- $F_{DB} \cos 60° + F_{AD} \cos 60° - F_{DE} = 0$ — (v)

$\Sigma F_y = 0$

$F_{AD} \sin 60° - F_{DB} \sin 60° - 2000 = 0$ — (vi)

$$F_{DB} = \frac{2887 \times 0.866 - 2000}{0.866}$$

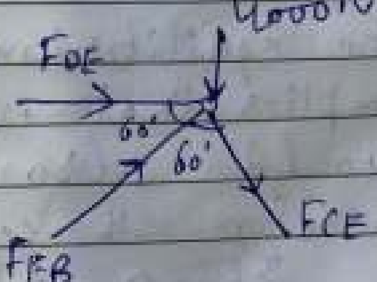
$$F_{DB} = 577\,N\ (T)$$

from (v)

$$F_{DE} = 577 \times 0.5 + 2887 \times 0.5$$

$$F_{DE} = 1732\,N\ (C)$$

At Joint E :-



$F_{DE} = 1732\,N$ } known forces
$F_{CE} = 4041\,N$

$\Sigma F_x = 0$:- $F_{DE} + F_{EB} \cos 60° - F_{CE} \cos 60° = 0$

$$F_{EB} \cos 60° = F_{CE} \cos 60° - F_{DE}$$

$$F_{EB} = \frac{4041 \times 0.5 - 1732}{0.5}$$

$$F_{EB} = 577\,N\ (C)$$

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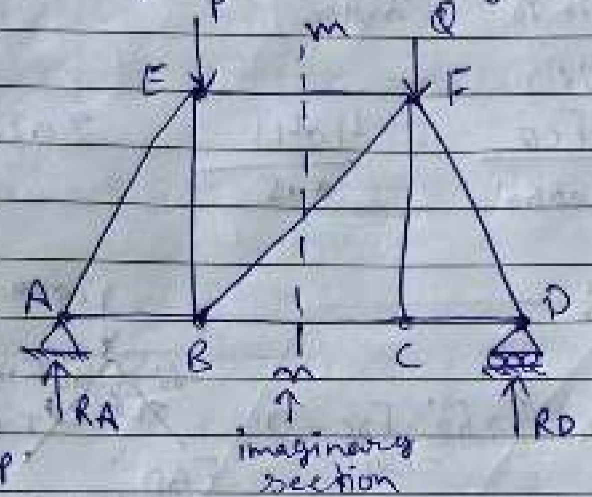
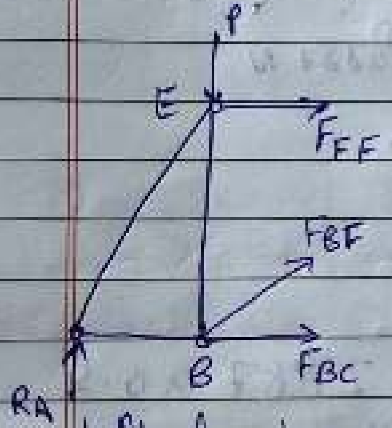
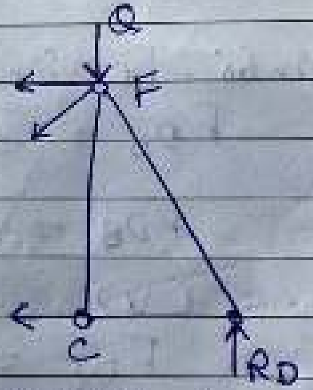
Method of Section :-

In this method, the equilibrium of a portion of the truss is considered which is obtained by cutting the truss by some imaginary section.





Left hand portion



Right Hand portion

The Equilibrium of the entire truss implies that every part of the truss would also be in equilibrium. Therefore three equations of equilibrium

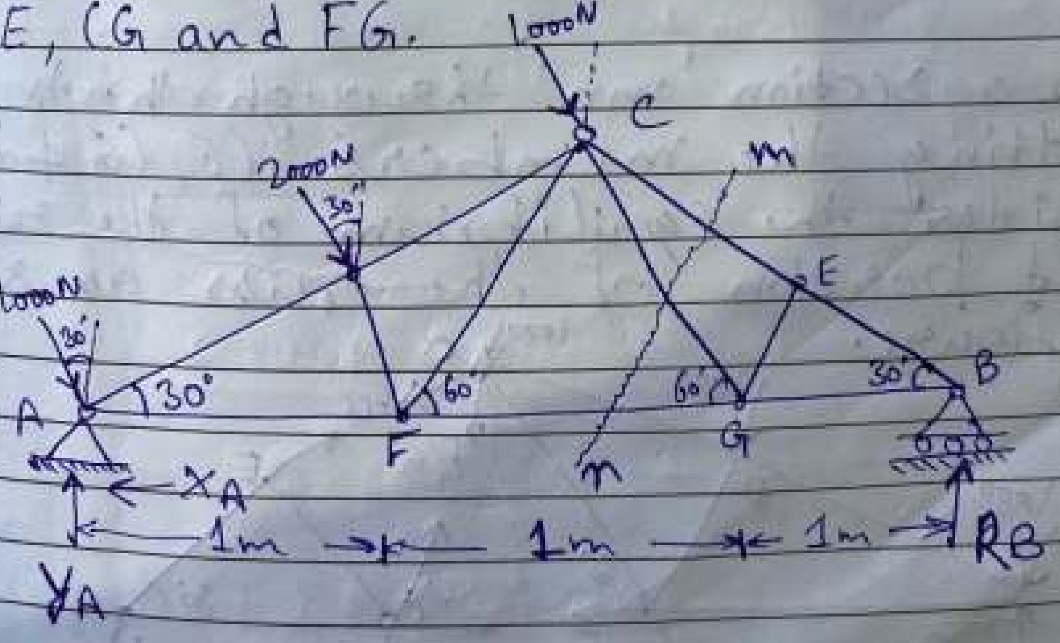
$$\Sigma F_x = 0, \ \Sigma F_y = 0 \ \text{and} \ \Sigma M = 0$$

Can be written for any one portion of the truss and can be solved to determine the three unknowns.

Following points should be noted while using the method of sections:-

1. The section should be passed through the members and not through the joints.
2. A section should divide the truss into two clearly separate and connected portions.
3. A section should cut only three members since only three unknowns can be determined from the three equations of equilibrium. However in special cases more than three members may be cut by a section.
4. When using the moment equation, the moment can be taken about any convenient point which may or may not lie on the section under consideration

Q: A truss is loaded and supported as shown. Determine the axial forces in the members CE, CG and FG.



Charlie

Date

Page No.

Solution: Consider the equilibrium of the entire truss as a free-body.

Taking moment about A

$\Sigma M_A = 0$

$R_B(3) - 2000 \times AD - 1000 \times AC = 0$

$3R_B - 2000\, AF \cos 30^\circ - 1000\, AG \cos 30^\circ = 0$

$3R_B = 2000 \times \frac{\sqrt{3}}{2} + 1000 \times 2 \times \frac{\sqrt{3}}{2}$

$R_B = \frac{2000}{\sqrt{3}}\ N$

$\Sigma F_y = 0$

$R_B + y_A - 1000 \cos 30^\circ - 2000 \cos 30^\circ - 1000 \cos 30^\circ = 0$

$y_A = \frac{\sqrt{3}}{2}(4000) - \frac{2000}{\sqrt{3}}$

$y_A = \frac{4000}{\sqrt{3}}\ N$

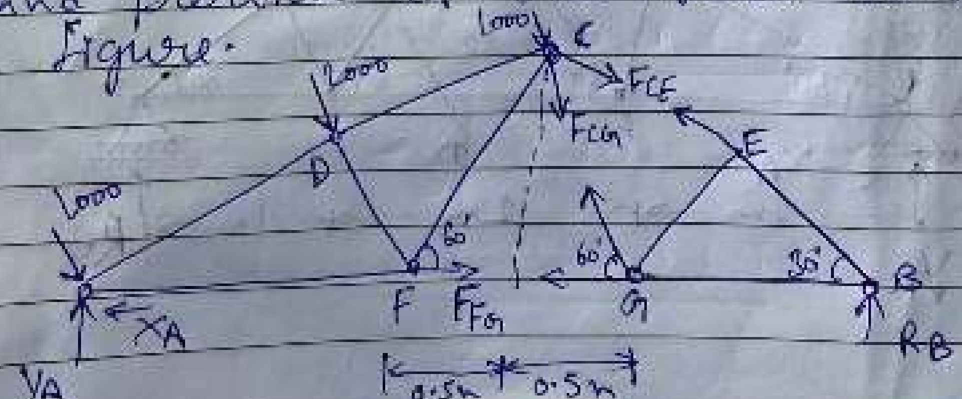
$\Sigma F_x = 0$

$-X_A + 1000 \sin 30^\circ + 2000 \sin 30^\circ + 1000 \sin 30^\circ = 0$

$X_A = 1000 \times 0.5 + 2000 \times 0.5 + 1000 \times 0.5$

$X_A = 2000\ N$

Pass a section 'mn' through the truss cutting the members CE, CG & FG. Consider the equilibrium of the right hand portion of the truss as show in figure.



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Date
Page No.

Taking moment about 'C'

$\Sigma M_C = 0$

$-F_{FG} \times 0.5 \tan 60° + R_B(1.5) = 0$

$\left\{ \frac{P}{B} = \tan 60°, \; P = B \times \tan 60°, \; P = 0.5 \times \sqrt{3} \right.$

$\underline{F_{FG} = 2000\,N\,(T)}$

Taking moment about 'G'

$\Sigma M_G = 0$

$F_{CE}(1 \times \sin 30°) + R_B(1) = 0$

$F_{CE} = -\frac{2000}{\sqrt{3}} \times \frac{1}{0.5} = -2309\,N$

Reverse the sign of the force F_{CE}

$\underline{F_{CE} = 2309\,N\,(C)}$

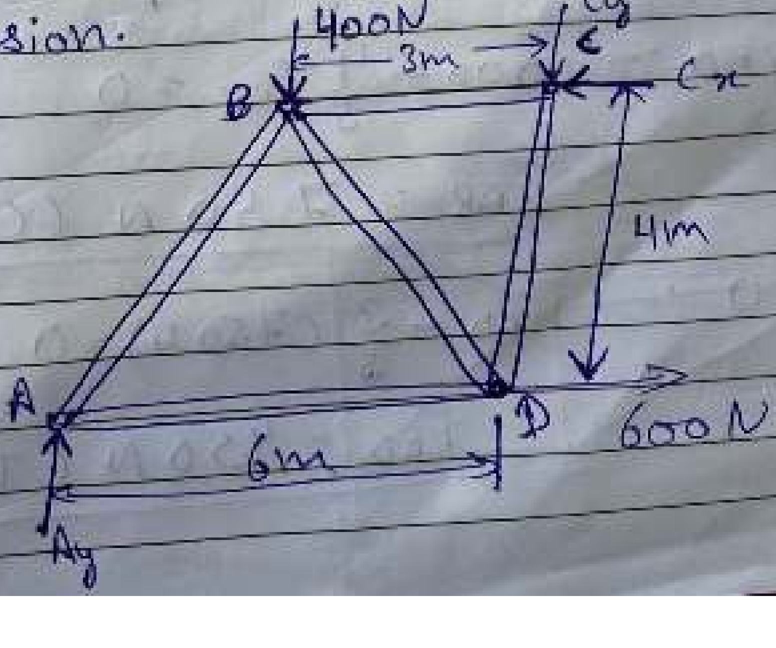
Taking moment about B,

$\Sigma M_B = 0$

$F_{CG}(1 \times \sin 60°) = 0$

$\underline{F_{CG} = 0}$

Q2 Determine the force in each member of the truss shown in figure. Indicate whether the members are in tension or compression.



Solution :-

Support Reactions :-

Applying Equation of equilibrium we have,

$\Sigma F_x = 0$:- $600 - C_x = 0$

$C_x = 600\,N$

$\Sigma F_y = 0$:- $600 - 400 - C_y = 0$

$C_y = 200\,N$

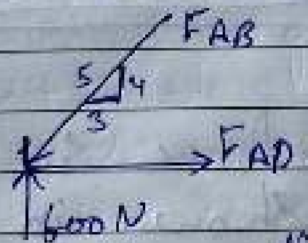
$\Sigma M_C = 0$:- $\cancel{600 - 400 - C_y = 0}$

$-A_y(6) + (3)400 + 600(4) = 0$

$A_y = 600\,N$

Joint A :-

F.B.D of Joint A



Applying Equation of Equilibrium

$\Sigma F_y = 0$:- $600 - \frac{4}{5} F_{AB} = 0$

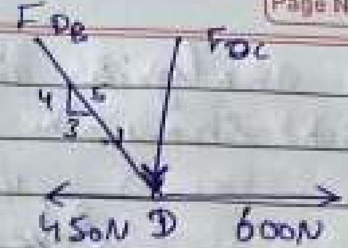
$\underline{F_{AB} = 750\,N\ (C)}$

$\Sigma F_x = 0$:- $F_{AD} - \frac{3}{5}(750\,N) = 0$

$\underline{F_{AD} = 450\,N\ (T)}$

Page No.

AT Joint D :-



Applying Equation of Equilibrium

$\Sigma F_x = 0$

$$-450 + \frac{3}{5} F_{DB} + 600 = 0$$

$$F_{DB} = -250\,N$$

The Negative sign indicates that F_{DB} acts in the opposite sense to that shown in figure. Hence

$$\underline{F_{DB} = 250\,N\ (T)}$$

To Determine F_{DC}, we can either correct the sense of F_{DB} and then apply $\Sigma F_y = 0$ or apply this equation and retain the negative sign for F_{DB} i.e

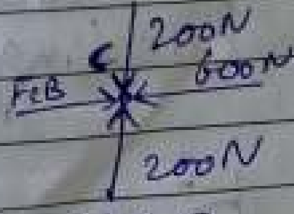
$\Sigma F_y = 0$

$$-F_{DC} - \frac{4}{5}(-250) = 0$$

$$\underline{F_{DC} = 200\,N\ (C)}$$

Joint C :-

Applying Equation of Equilibrium



F.B.D

$\Sigma F_x = 0$

$$F_{CB} - 600 = 0$$

$$F_{CB} = 600\,N\ (C)$$

$\Sigma F_y = 0$

$$200 - 200 = 0$$

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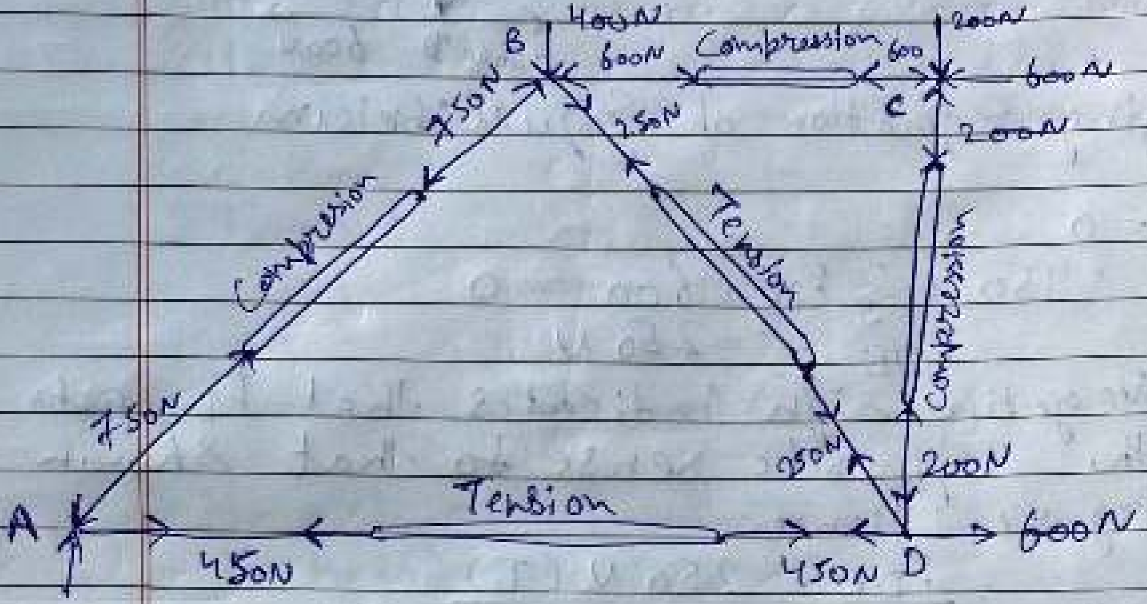
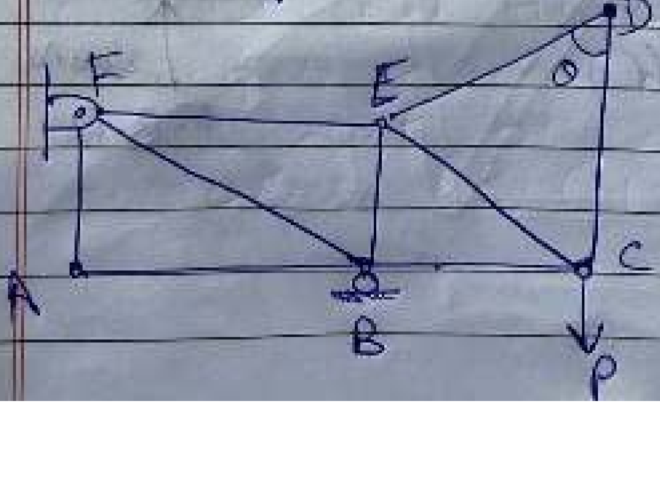
The analysis is summarized in figure which show the correct F.B.D

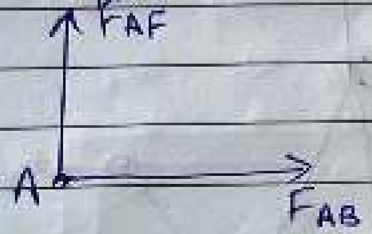


Zero force Members :-

Truss analysis using the method of Joints is greatly simplified if one is First able to determine those members which support no loading. These zero force members are used to increase the stability of the truss during construction and to provide support if the applied loading is changed.



Consider the truss shown in Figure. If the Free body diagram of the Joint A is drawn, It is seen that ~~two~~ members AB and AF are Zero Force members.



$\Sigma F_x = 0$:- $F_{AB} = 0$

$\Sigma F_y = 0$:- $F_{AF} = 0$

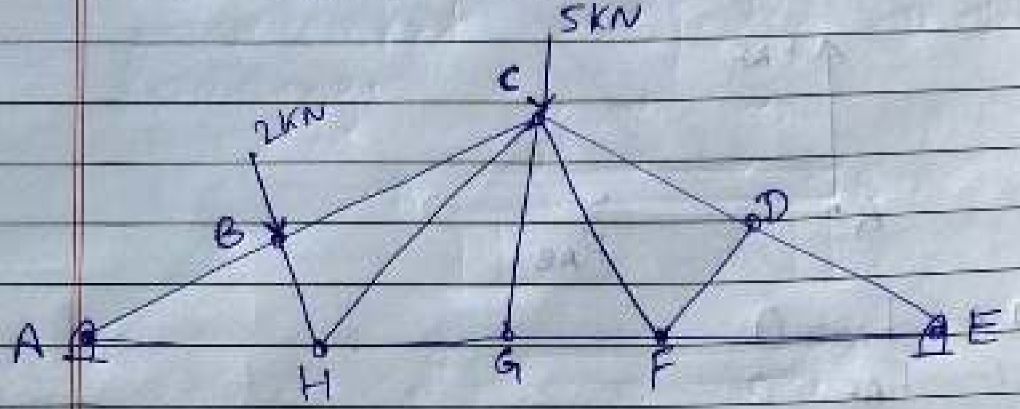
On the other hand, notice that we could not have come to this conclusion if we had consider the F.B.D of Joint F or B simply because there are five unknowns at each of these Joints.

General Rules :-

(i) If only two members form a truss Joint and no external load or support reaction is applied to the Joint, the members must be zero force members.

(ii) If three members form a truss Joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the Joint.

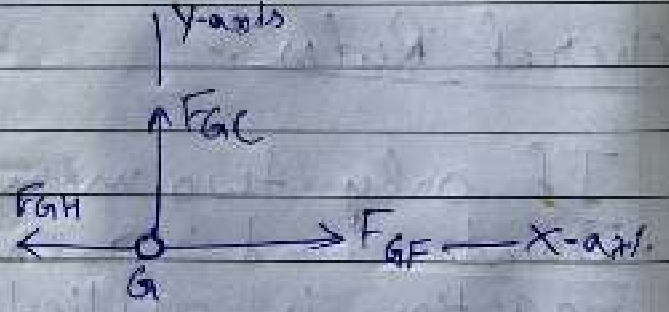
Q1:- Using the methods of Joints, determine all the Zero-force members of the roof truss shown in figure. Assume all Joints are pin connected.



Solution :-

Look for Joint geometries that have three members for which two are collinear. We have

Joint G :-

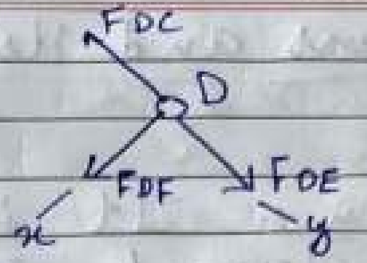


$\Sigma F_y = 0$

$F_{GC} = 0$

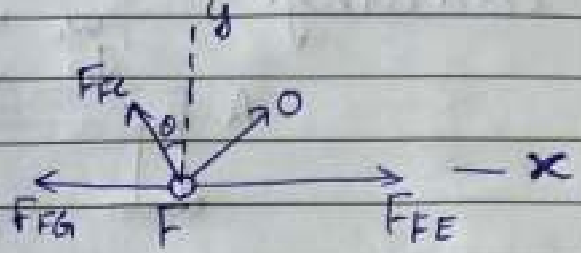
Realize that we could not conclude that GC is a zero force member by considering Joint C, where there are five unknowns. The fact that GC is a zero force member means that the 5 kN load at C must be supported by members CB, CH, CF and CD.

Joint D :-



$\Sigma F_x = 0$:- $F_{DF} = 0$

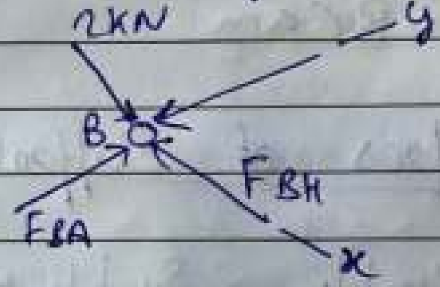
Joint F



$\Sigma F_y = 0$:- $F_{FC} \cos\theta = 0$ Since $\theta \neq 90°$

$F_{FC} = 0$

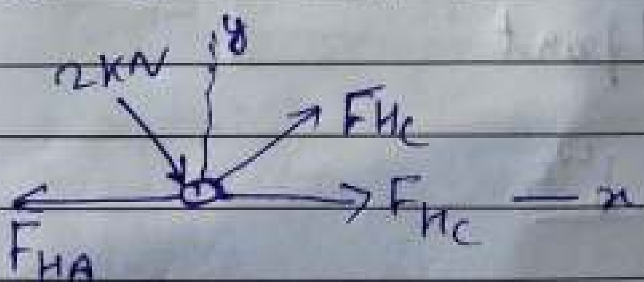
Not that If Joint B is analyzed.



$\Sigma F_x = 0$

$2KN - F_{BH} = 0$ i.e. $F_{BH} = 2KN$ (C)

Note that F_{HC} must satisfy $\Sigma F_y = 0$
From F.B.D of Joint H



By analysing equation of Equilibriums we find that $F_{HC} \neq 0$. Therefore HC is not a zero force member.

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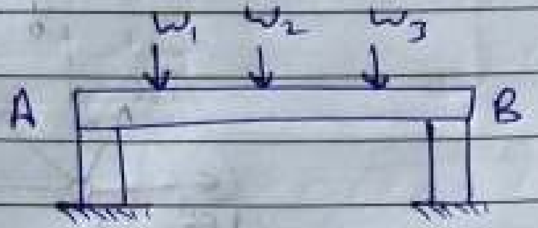
Beams and its types :-

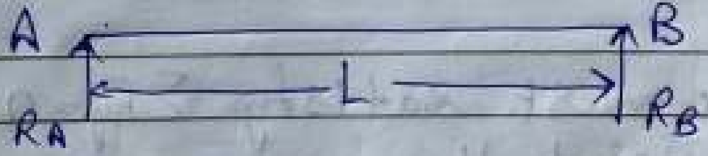
Beam :- It is a horizontal structural element subjected to transverse loading.



L = length of the Beam
It is also known as span of the Beam

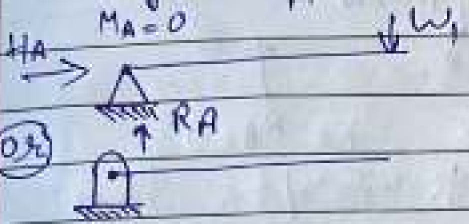
Types of Supports :-

(1) Simply Supported



(2) Roller Support

(3) Hinge Support



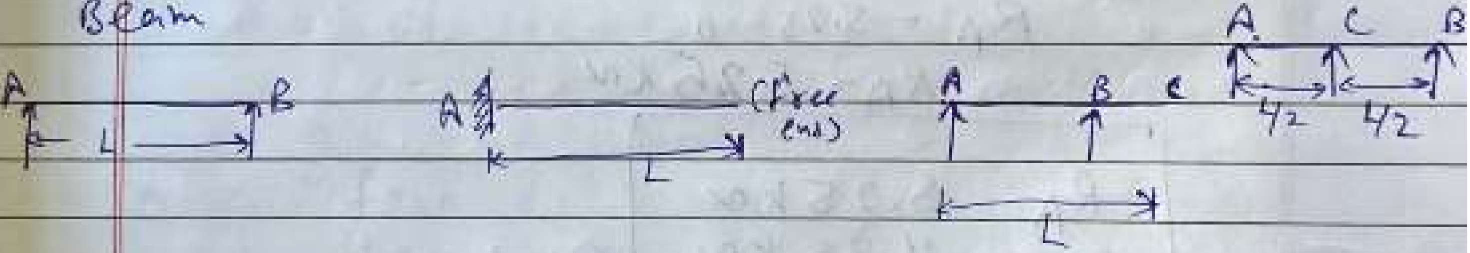
H_A = Horizondal Reaction
R_A = Vertical Reaction
M_A = Moment about A

(4) Fixed Support

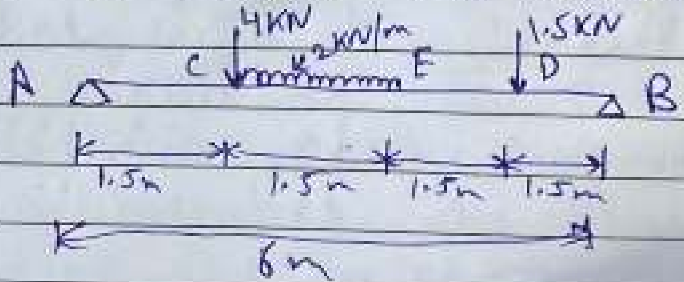
$H_A \neq 0$ $M_A \neq 0$ $R_A \neq 0$

Types of Beams :-

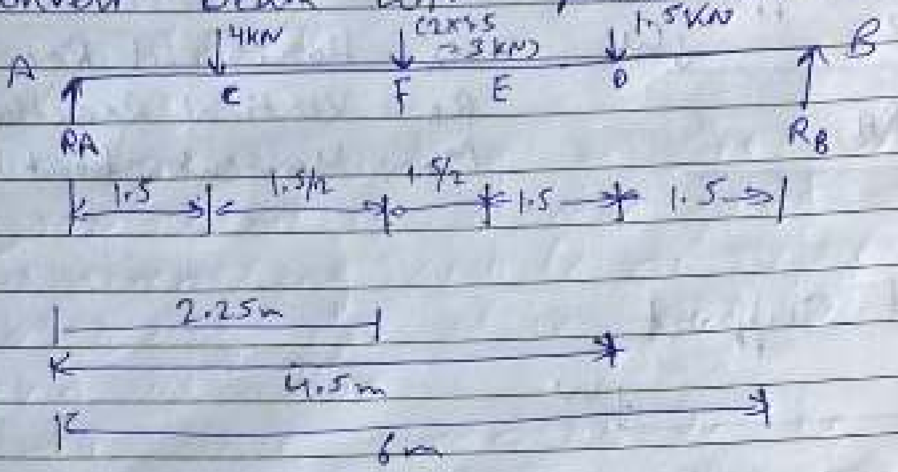
Beams
- Simply Supported Beam
- Cantilever Beam
- over hanging Beam
- Continuous Beam



Q1. Find the Support Reactions for the Simply supported beam under the action of loads as shown in figure.



Solution: Convert beam with point load



Taking moments @ A

$\Sigma M_A = 0$

$4\times1.5 + 3\times2.25 + 1.5\times4.5 - R_B\times6 = 0$

$\therefore R_B = \frac{19.5}{6} = 3.25\ KN$

$\Sigma V = 0$

$R_A + 3.25 - 4 - 3 - 1.5 = 0$

$R_A - 5.25 = 0$

$R_A = 5.25\ KN$

$$\boxed{\begin{aligned} R_A &= 5.25\ KN \\ R_B &= 3.25\ KN \end{aligned}}$$ Ans

Date
Page No.

Frames :-

Frames are structures consisting of several bars or members pinned together and in which one or more than one of its members are subjected to more than two forces. They are designed to support loads and are usually stationary structures.

In case of plane frames forces can be applied anywhere on the members whereas in the case of plane trusses forces can be applied only at the Joints. Therefore, the members or the bars of the frames are subjected to bending as well as simple tension or compression.

Method of Analysis :-

The general problem in the analysis of a frame is to find the magnitudes and directions of the forces transmitted to another through the connecting pins, under a given loading. For this we have to consider the conditions of equilibrium of the entire frame together with the conditions of equilibrium of its members. To consider the equilibrium of the member we have to dismember the frame. while representing the forces acting on the members.

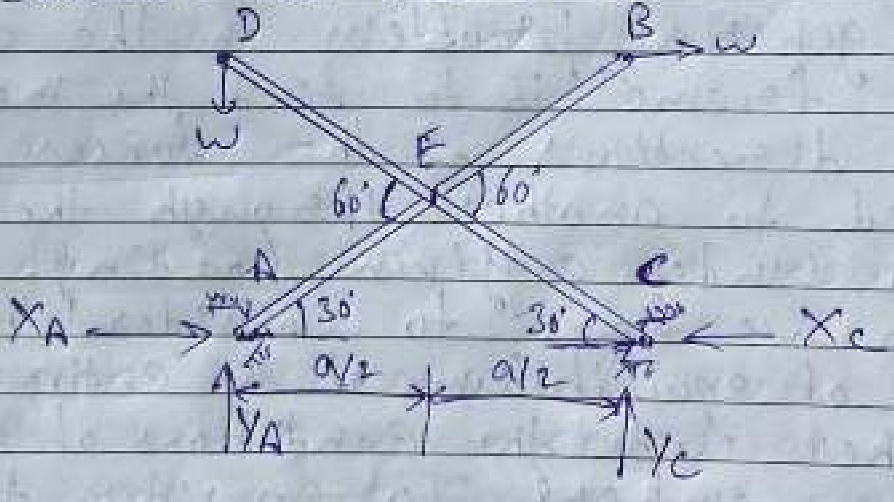
Charlie
Date
Page No.

The following rules should be observed:-

(i) The forces acting on the two members at a common joint after dismembering should be represented observing the principal of action and reaction

(ii) Two force members in the frame should be identified. The forces acting at each end of such a member should have the same magnitude, same line of action and opposite sense.

Q1- An X-frame is loaded and supported as shown in figure. Find the Horizontal and vertical components of the reaction at 'A' and 'C'.



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Date
Page No.

Solution:- Consider the Equilibrium of the entire Frame as a Free body.

$\Sigma F_x = 0$:- $X_A + W - X_C = 0$

$X_C - X_A = W$ ⟵ (i)

$\Sigma F_y = 0$:- $Y_A + Y_C - W = 0$ ⟵ (ii)

Taking moments about A

$\Sigma M_A = 0$:- $Y_C(a) - W(a \tan 30^\circ) = 0$

$Y_C = W \tan 30^\circ$

$Y_C = 0.577 W$

from (ii)

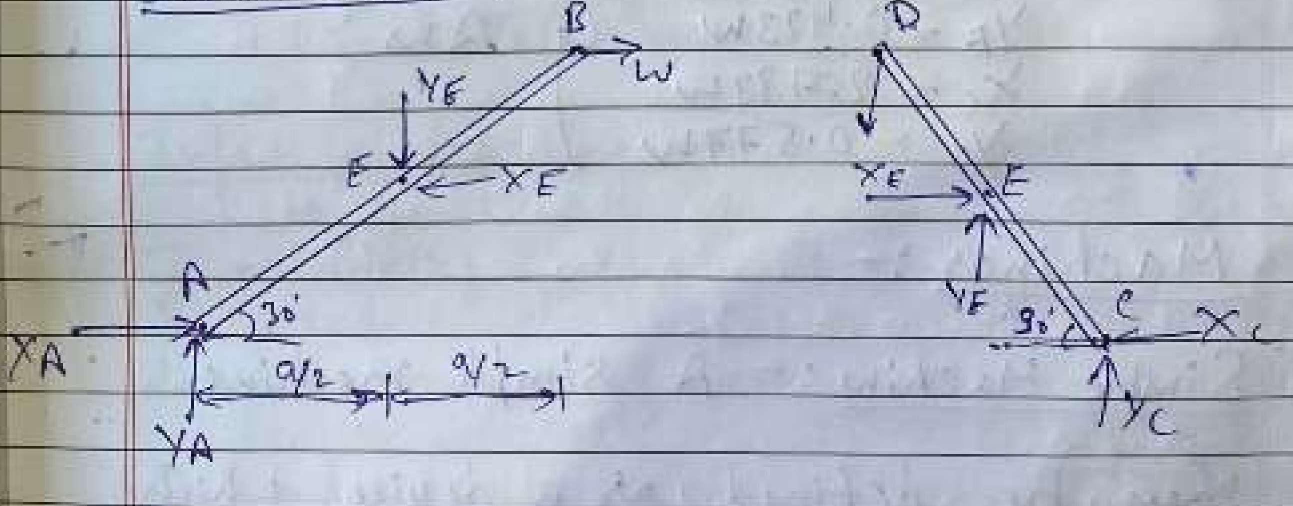
$Y_A + W + Y_C = 0$

$Y_A = W - Y_C$

$Y_A = W - 0.577W$

$Y_A = 0.423 W$

Dismember :-



Member AB

$\Sigma F_x = 0$ $X_A + W - X_E = 0$

$\Sigma F_y = 0$ $Y_A - Y_E = 0$

Taking moment about A,

$\Sigma M_A = 0$

$$-Y_E\left(\frac{a}{2}\right) - W(a\tan 30^\circ) + X_E\left(\frac{a}{2}\tan 30^\circ\right) = 0$$

$$X_E = \frac{2W \times 0.577 + 0.423W}{0.577}$$

$$X_E = 2.733W$$

from (iii)

$$X_A = -W + \cancel{X_E}$$

$$X_A = -W + 2.733W$$

$$X_A = 1.733W$$

from (i)

$$X_C = X_A + W$$

$$X_C = 1.733W + W$$

$$X_C = 2.733W$$

$$\left.\begin{aligned} X_A &= 1.733W \\ Y_A &= 0.423W \\ X_C &= 2.733W \\ Y_C &= 0.577W \end{aligned}\right\} \text{Ans}$$

<u>Machines</u> :-

<u>Simple Machine</u> :- A simple machine may be defined as a device, which enables us to do some useful work at some point or to over come some resistance, when an effort or Force is applied to it, at some other convenient point.

Date
Page No.

Compound Machine:-

A compound machine may be defined as a device consisting of a number of simple machines which enables us to do some useful work at a faster speed.

Lifting Machine :-

It is a device which enables us to lift a heavy load by applying a comparatively smaller effort.

Mechanical Advantage :-

It is the ratio of weight lifted to the effort applied

$$M.A = \frac{W}{P}$$

Input of Machine :- The input of a machine is the work done on the Machine. In a lifting Machine, it is measured by the product of effort and the distance through which it has moved.

Output of a Machine :-

The output of a machine is the actual work done by the machine.

Date
Page No.

Efficiency of Machine :-

It is the ratio of output to the input of a machine

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100$$

Velocity Ratio :-

It is the ratio of distance moved by the effort (y) to the distance moved by the load (x).

$$V.R = \frac{y}{x}$$

Q:- In a certain weight lifting machine a weight of 1 KN is lifted by an effort of 25N. While the weight moves up by 100mm, the point of application of effort moves by 8 m. Find Mechanical advantage, Velocity Ratio and efficiency of the machine.

Soln:- Given :-

$W = 1\,KN = 1000\,N$ Effort $(P) = 25N$

$x = 100\,mm = 0.1\,m$ $y = 8\,m$

Mechanical advantage

$$M.A = \frac{W}{P} = \frac{1000}{25} = \underline{40}$$

Velocity ratio of the Machine

$$V.R = \frac{y}{x} = \frac{8}{0.1} = 80$$

Efficiency of Machine

$$\eta = \frac{M.A}{V.R} = \frac{40}{80} = 0.5 = 50\%$$

Reversibility of a Machine :-

Sometimes a machine is also capable of doing some work in the reversed direction, after the effort is removed. Such a machine is called a reversed machine, and its action is known as reversibility of the Machine.

The condition for a Machine to be reversible is that its efficiency should be more than 50%.

Q1. A certain weight lifting machine of V.R 30 can be lift a load of 1500 N with the help of 125 N effort. Determine if the machine is reversible.

Sol :- Mechanical Advantage

$$M.A = \frac{W}{P} = \frac{1500}{125} = 12$$

Efficiency

$$\eta = \frac{M.A}{V.R} = \frac{12}{30} = 0.4 = 40\%$$

Since efficiency of Machine is less than 50%. Therefore the machine is not reversible.

Unit-2 Part-II

Centroid and Moment of Inertia

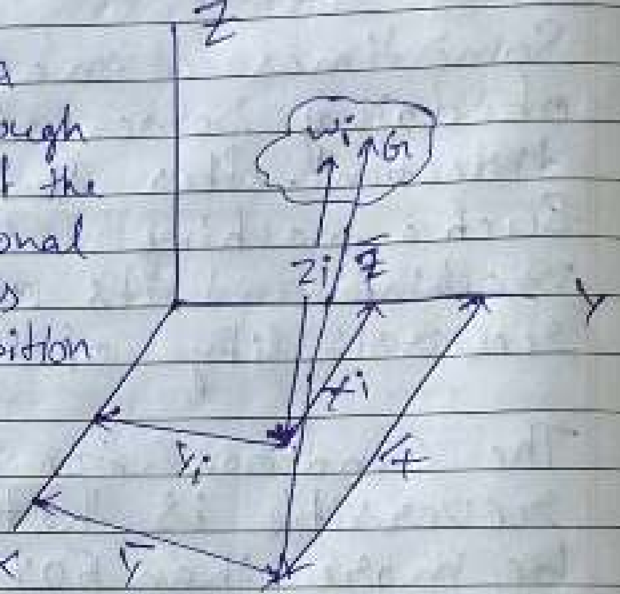
Introduction :-

Centroid, centre of gravity and moment of inertia are the important properties of a section which are required in the analysis of several Engineering problem.

Centre of Gravity :-

Centre of gravity of a body is the point through which the resultant of the distributed gravitational parallel forces passes Irrespective to the position of the body.

Centre of gravity is the point where whole weight of the body is assumed to be concentrated.



Let W_i be the weight of an element of the given body of weight W. Let the co-ordinates of this element be (x_i, y_i, z_i) and that of centre of gravity G be $(\bar{x}, \bar{y}, \bar{z})$

Now, $W = W_1 + W_2 + \text{----} + W_3$ — ①

$$\boxed{W = \sum W_i}$$

We know that the resultant of all elemental weight passes through G, so we have

$$W\bar{x} = W_1x_1 + W_2x_2 + \cdots W_nx_n \quad -(2)$$

$$W\bar{x} = \Sigma W_ix_i$$

$$\bar{x} = \frac{\Sigma W_ix_i}{W} = \frac{\Sigma W_ix_i}{\Sigma W_i}$$

$$\bar{Y} = \frac{\Sigma W_iy_i}{\Sigma W_i}$$

$$\bar{Z} = \frac{\Sigma W_iz_i}{\Sigma W_i}$$

The expression for co-ordinate of C.G can also be written as

$$\bar{x} = \frac{\int x\,dw}{\int dw} = \frac{\int x\,dw}{W}$$

$$\bar{y} = \frac{\int y\,dw}{W}$$

$$\bar{Z} = \frac{\int z\,dw}{W}$$

If ρ is the density or mass per unit volume

$$dm = \rho\,dv$$

$$\bar{x} = \frac{\int x\rho\,dv}{\int \rho\,dv} = \frac{\int x\,dv}{V}$$

$$\bar{y} = \frac{\int y\,dv}{V}, \quad \bar{Z} = \frac{\int z\,dv}{V}$$

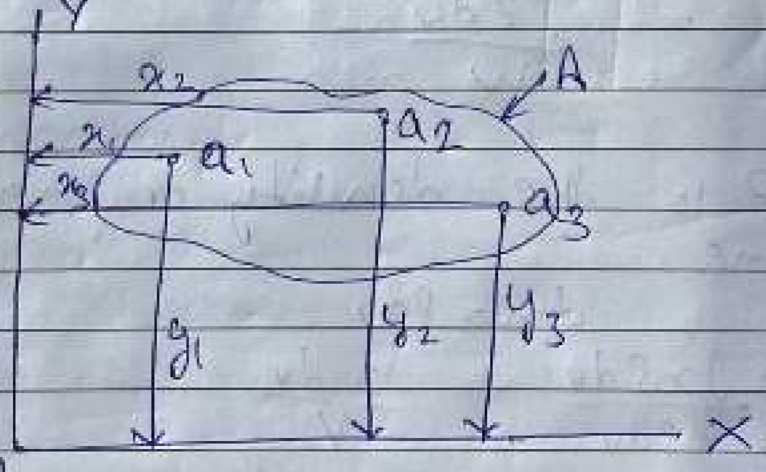
Date
Page No.

Centroid :-

Centroid of an area is a point in a plane such that the moment of area about any axis through the point is Zero or entire area assumed tobe concentrated at point known as centroid.

This is the where the whole area of the body is assumed to be concentrated.

Thus, when the thickness of the body is not consider, then the centre of gravity and centroid of body is same and passes through the same point. The method of finding centre of gravity and centroid is also same



$A = a_1 + a_2 + a_3 + \text{-----}$

Moment of the area A about OY axis

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{A}$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$\boxed{\bar{x} = \frac{\Sigma ax}{\Sigma a}}$$

Moment of area A about Ox-axis

$$A\bar{y} = a_1y_1 + a_2y_2 + a_3y_3$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$\boxed{\bar{y} = \frac{\Sigma ay}{\Sigma a}}$$

Difference b/w centre of gravity and Centroid:-

(i) Centre of gravity applies the body with mass and weight while centroid applies to plane areas.

(ii) Centre of gravity of a body is a point through which the resultant gravitational forces passes for any orientation

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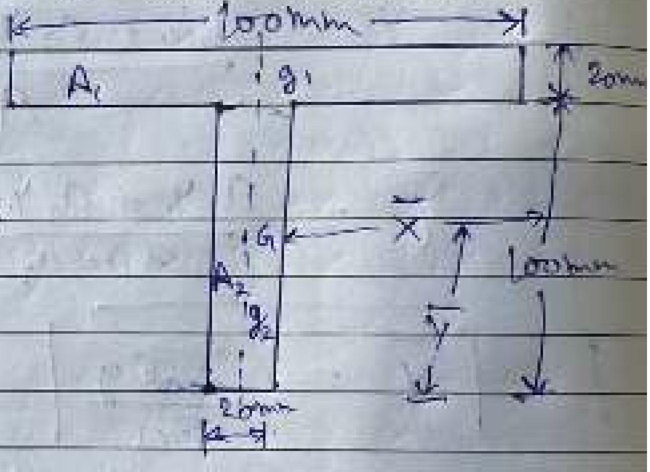
of the body whereas Centroid is a point in a plane area such that the moment of area about any axis through that point is zero.

Note:- Centroid of an area lies on the axis of symmetry, if it exists. If there is one axis of symmetry Centroid lies on it. If there are two axis of symmetry, then the centroid lies on the intersection of the axis of symmetry

Q: Locate the centroid of the T-section.

Solution:



Area of rectangle

$A_1 = 100 \times 20 = 2000 mm^2$

$x_1 = \frac{b}{2} = \frac{100}{2} = 50mm$

$y_1 = 100 + \frac{20}{2} = 110mm$

Area of rectangle

$A_2 = 100 \times 20 = 2000mm$

$x_2 = \frac{b}{2} = 50mm$

$y_2 = \frac{h}{2} = \frac{100}{2} = 50mm$

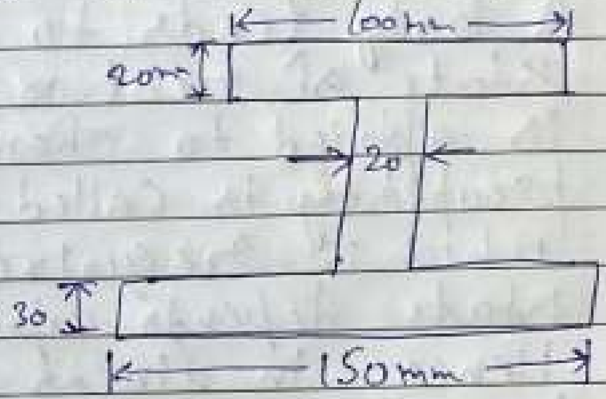
$$\bar{x} = \frac{A_1 X_1 + A_2 X_2}{A_1 + A_2}$$

$$\bar{x} = \frac{2000 \times 50 + 2000 \times 50}{2000 + 2000} = \underline{50\,mm} \text{ Ans}$$

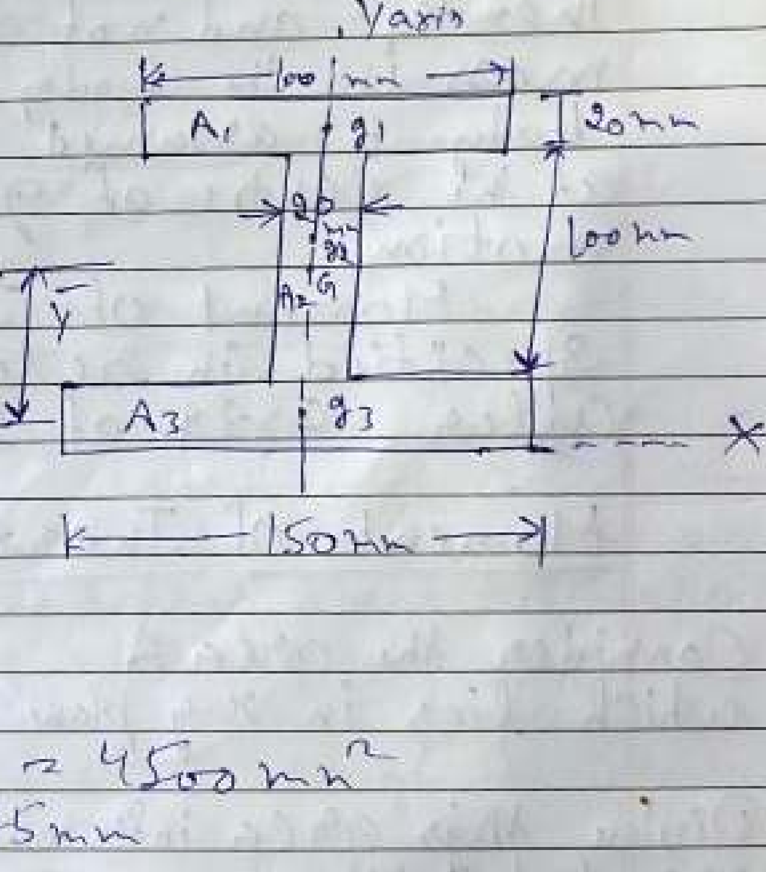
$$\bar{y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}$$

$$= \frac{2000 \times 110 + 2000 \times 50}{2000 + 2000} = \underline{80\,mm} \text{ Ans}$$

Q:- Locate the centroid of I-section.



Soln:- Due to symmetry C.G lies in y axis i.e $\bar{x} = 0$



$A_1 = 100 \times 20 = 2000\,mm^2$

$Y_1 = 30 + 100 + \frac{20}{2}$

$\boxed{Y_1 = 140\,mm}$

$A_2 = 100 \times 20 = 2000\,mm^2$

$Y_2 = 30 + \frac{100}{2}$

$\boxed{y_2 = 80\,mm}$

$A_3 = 150 \times 30 = 4500\,mm^2$

$y_3 = \frac{150}{2} = 75\,mm$

$$\bar{Y} = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3}{A_1 + A_2 + A_3}$$

$$\bar{Y} = \frac{2000 \times 140 + 2000 \times 80 + 4500 \times 75}{2000 + 2000 + 4500}$$

$$\boxed{\bar{Y} = 59.71 \, mm}$$

Moment of Inertia :-

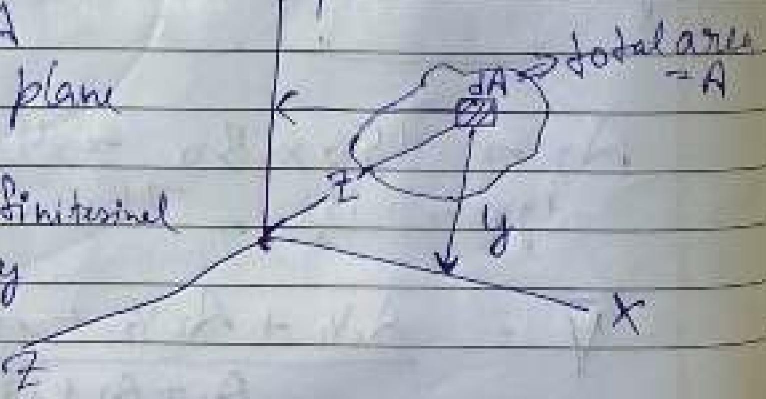
A body tends to remain in its own state of rest or motion unless a force is applied to change the state. This property is called inertia. The force which is required to change the state depends upon the mass of body. Hence in linear motion mass governs inertia and not the distribution of mass in the body. The mass of the body is assumed to be concentrated at the centre of gravity in linear motion.

Moment of Inertia is always specified in relocation to a particular axis of rotation.

Moment of Inertia of an area :-

Consider the area A which lies in x-y plane

Divide this area infinitesimal area. Let dA be any element of area



Situated at a distance (x, y) from the respective axes
The MOI of the differential area dA about x-axis is

$$d\bar{I}_{xx} = dA y^2$$

for entire area A, the MOI about x-axis

$$\boxed{\bar{I}_{xx} = \int_A y^2 \cdot dA}$$

Similarly, MOI about Y-axis

$$dI_{yy} = dA x^2$$

for entire area,

$$\boxed{\bar{I}_{yy} = \int_A x^2 \cdot dA}$$

Theorems of moment of Inertia :-

Theorem of perpendicular Axis :-

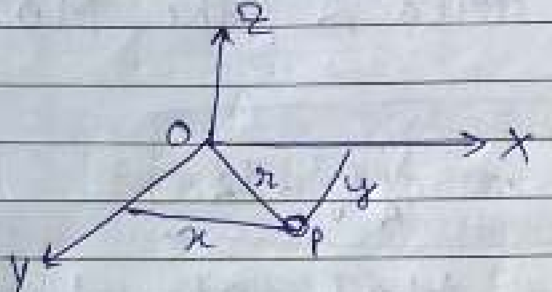
It states, If I_{xx} and I_{yy} be the moments of Inertia of a plane section about two perpendicular axis meeting at O, the moment of Inertia I_{zz} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by

$$I_{zz} = I_{xx} + I_{yy}$$

Date
Page No.

Proof :-

Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in figure.



Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that $OP = r$

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

We know that the moment of Inertia of the lamina P about X-X axis.

$$I_{xx} = da \cdot y^2 \qquad [\because I = \text{area} \times (\text{Distance})^2]$$

Similarly

$$I_{yy} = da \cdot x^2$$

$$I_{zz} = da \cdot r^2 = da\,(x^2 + y^2)$$

$$= da \cdot x^2 + da\, y^2 = I_{yy} + I_{xx}$$

Theorem of parallel axis :-

It states, If the moment of Inertia of a plane area about an axis through its Centre of gravity is denoted by I_G then moment of Inertia of the area about any other axis AB, parallel to the first and at a distance h from the centre of gravity is given by

$$I_{AB} = I_G + ah^2$$

where I_{AB} = Moment of Inertia of the area about an axis AB

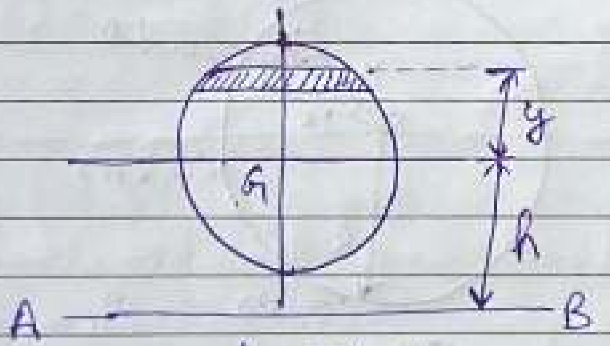
I_G = M.O.I of the area about its C.G

a = Area of the section

h = Distance between Centre of gravity of the section and axis AB.

Proof:-

Consider a strip of a circle, whose moment of Inertia is required to be found out about a line AB as shown in figure



Let δa = area of strip

y = Distance of the strip from the Centre of gravity of the section

h = Distance between centre of gravity of the section and the axis AB.

We know that moment of Inertia of the whole section about an axis passing through the centre of gravity of the section

$= \delta a \cdot y^2$

moment of Inertia of the whole section about an axis passing through its centre of gravity

$I_G = \Sigma \delta a \cdot y^2$

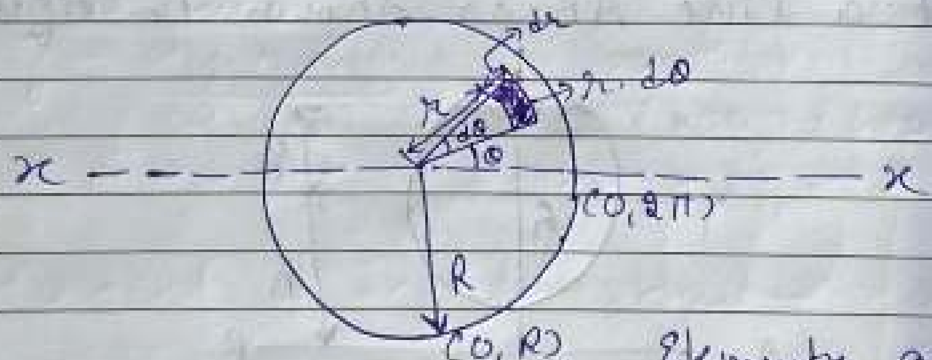
$\therefore$ M.O.I of the section about the axis AB,

$I_{AB} = \Sigma \delta a (h+y)^2 = \Sigma \delta a (h^2 + y^2 + 2hy)$

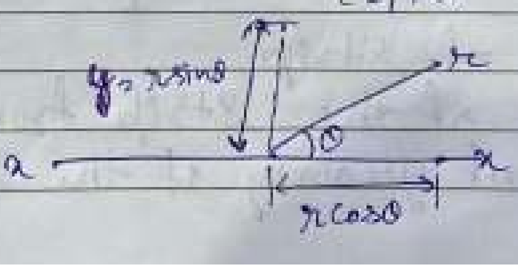
$= (\Sigma h^2 \cdot \delta a) + (\Sigma y^2 \cdot \delta a) + (\Sigma 2hy \cdot \delta a)$

$= ah^2 + I_G + 0$

<u>Mass moment area of Circular plate</u> :-



Elementry area $= r \cdot d\theta \cdot dr$ —①



Charlie
Date
Page No.

Moment of Inertia for Elementry area :-

$$I_{xx} = Y^2 \cdot dA$$
$$= (r\sin\theta)^2 \cdot r \cdot d\theta \cdot dr$$
$$= r^2 \sin^2\theta \cdot r \cdot d\theta \cdot dr$$
$$= r^3 \sin^2\theta \cdot d\theta \cdot dr$$

for whole Body :-

$$I_{xx} = \int_0^R \int_0^{2\pi} r^3 \sin^2\theta \cdot d\theta \cdot dr$$
$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1-\cos 2\theta}{2} \right) \cdot d\theta \cdot dr$$
$$= \int_0^R \frac{r^3}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} \cdot dr$$
$$= \int_0^R \frac{r^3}{2} [2\pi] \cdot dr$$
$$= \frac{\pi [r^4]_0^R}{4}$$
$$I_{xx} = \frac{\pi \cdot R^4}{4} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4$$
$$\boxed{I_{xx} = \frac{\pi d^4}{64}}$$

Similarly $\boxed{I_{yy} = \frac{\pi d^4}{64}}$

Q: Find the moment of Inertia of a T-section with flange as 150mm x 50mm and Web as 150mm x 50mm about X-X and Y-Y axes through the Centre of gravity of the section.

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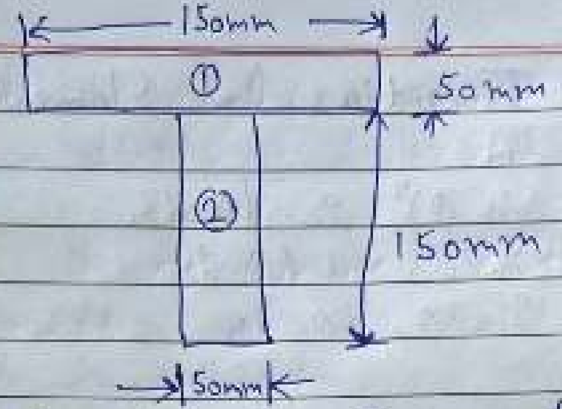


Sol: As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis.

Split up the whole section into two rectangles as shown in figure.

(i) Rectangle (1)

$$a_1 = 150 \times 50 = 7500 mm^2$$
$$y_1 = 150 + \frac{50}{2} = 175 mm$$

(ii) Rectangle (2)

$$a_2 = 150 \times 50 = 7500 mm^2$$
$$y_2 = \frac{150}{2} = 75 mm$$

We know that Distance between centre of gravity of the section and bottom of the web

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500}$$
$$\bar{y} = 125 mm$$

Moment of Inertia about X-X axis

M.I of Rectangle (1) about an axis through its centre of gravity and parallel to X-X axis

Date
Page No.

$I_{G1} = \frac{150(50)^3}{12} = 1.5625 \times 10^6 mm^4$

Distance between centre of gravity of rectangle (1) and X-X axis

$h_1 = 175-125 = 50mm$

$\therefore$ Moment of Inertia of rectangle (1) about X-X axis

$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2]$

$= 20.3125 \times 10^6 mm^4$

Similarly

M.O.I of Rectangle (2) about an axis through its centre of gravity and parallel to X-X axis

$I_{G2} = \frac{50(150)^3}{12} = 14.0625 \times 10^6 mm^4$

Distance between C.G of rectangle (2) and X-X axis

$h_2 = 125-75 = 50mm$

$\therefore$ Moment of Inertia of rectangle (2) about X-X axis

$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2]$

$= 32.8125 \times 10^6 mm^4$

Now M.O.I of the whole section about X-X axis

$I_{xx} = (20.3125 \times 10^6) + (32.8125 \times 10^6)$

$I_{xx} = \underline{53.125 \times 10^6 mm^4}$

Moment of Inertia about Y-Y axis :-

M.I of rectangle (1) about Y-Y axis

$= \frac{50(150)^3}{12} = 14.0625 \times 10^6 mm^4$

Date
Page No.

M.O.I of rectangle (2) and Y-Y axis

$$= \frac{150(50)^3}{12} = 1.5625 \times 10^6\ mm^4$$

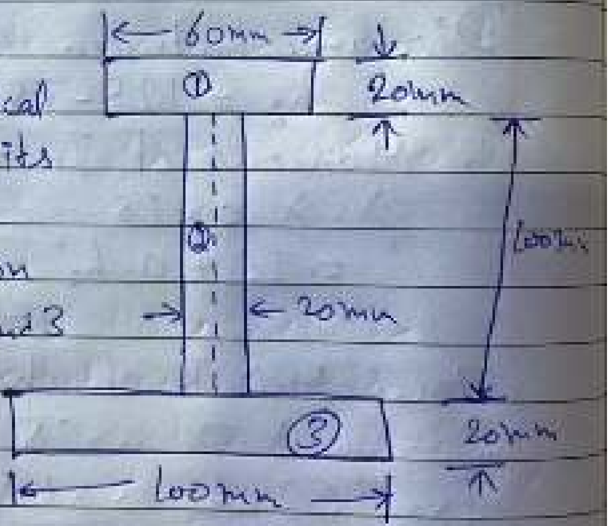
Now, M.O.I of the whole section about Y-Y axis

$$I_{yy} = (14.0625 \times 10^6) + (1.5625 \times 10^6)$$

$$\underline{I_{yy} = 15.625 \times 10^6\ mm^4}$$

Q:1 An I-section is made up of three rectangles as shown in figure. Find the M.O.I of the section about the horizontal axis passing through the centre of gravity of the section.

Soln



As the section is symmetrical about Y-Y axis, therefore its C.G will lie on this axis. Split up the whole section into three rectangles 1, 2 and 3 as shown in figure.

(i) Rectangle 1

$$a_1 = 60 \times 20 = 1200\ mm^2$$

$$y_1 = 20 + 100 + \frac{20}{2} = 130\ mm$$

(ii) Rectangle 2

$$a_2 = 100 \times 20 = 2000\ mm^2$$

$$y_2 = 20 + \frac{100}{2} = 70\ mm$$

Charlie
Date
Page No.

(iii) Rectangle 3

$a_3 = 100 \times 20 = 2000\ mm^2$

$y_3 = \frac{20}{2} = 10\ mm$

We know that the distance between C.G of the section and bottom face

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000}$$

$$= 60.8\ mm$$

M.O.I of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis

$$I_{G_1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3\ mm^4$$

Distance between C.G of rectangle (1) & X-X axis

$h_1 = 130 - 60.8 = 69.2\ mm$

$\therefore$ Moment of Inertia of rectangle (1) about X-X axis.

$= I_{G_1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2]$

$= 5786 \times 10^3\ mm^4$

Similarly

M.O.I of rectangle (2) about an axis through its C.G and parallel to X-X axis

$$I_{G_2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3\ mm^4$$

Distance between C.G of rectangle (2) and X-X axis

$h_2 = 70 - 60.8 = 9.2\ mm$

M.O.I of rectangle (2) about X-X axis

$= I_{G_2} + a_2 h_2^2 = (1666.7\times10^3) + [2000\times(9.2)^2]$

$= 1836\times10^3\ mm^4$

M.O.I of rectangle (3) about an axis through its C.G and parallel to X-X axis

$I_{G_3} = \frac{100\times(20)^3}{12} = 66.7\times10^3\ mm^4$

Distance between C.G of rectangle (3) & X-X axis

$h_3 = 60.8 - 10 = 50.8\ mm$

Moment of Inertia of Rectangle (3) about X-X axis

$= I_{G_3} + a_3 h_3^2 = (66.7\times10^3) + [2000\times(50.8)^2]$

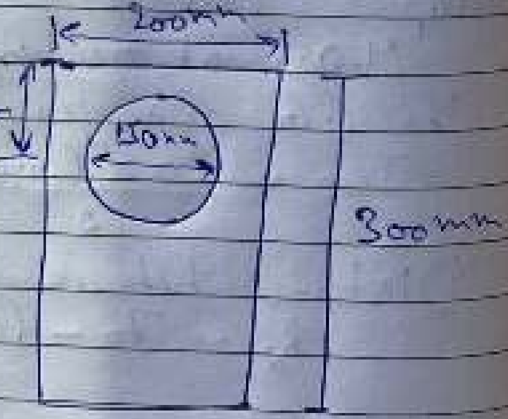
$= 5228\times10^3\ mm^4$

M.O.I of the whole section about X-X axis

$I_{xx} = (5786\times10^3) + (1836\times10^3) + (5228\times10^3)$

$= 12850\times10^3\ mm^4$

Q:- Find the M.O.I of a hollow section shown in figure about an axis passing through its centre of gravity or parallel X-X axis.



Sol: As the section is symmetrical about Y-Y axis therefore C.G of the section will lie on this axis. Let $\bar{y}$ be the distance b/w C.G of section from the bottom face.

Charlie
Date
Page No.

(I) Rectangle

$$a_1 = 300 \times 200 = 60\,000\ mm^2$$
$$y_1 = \frac{300}{2} = 150mm$$

(II) Circular hole

$$a_2 = \frac{\pi}{4} \times (150)^2 = 17670\ mm^2$$

$$y_2 = 300 - 100 = 200\ mm$$

We Know that distance between C.G of section and its bottom face,

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(60000 \times 150) - (17670 \times 200)}{60000 - 17670}$$
$$\bar{y} = 129.1mm$$

M.O.I of rectangular section about an axis through its C.G and parallel to X-X axis

$$I_{G_1} = \frac{200 \times (300)^3}{12} = 450 \times 10^6\ mm^4$$

Distance of C.G of rectangular section and X-X axis

$$h_1 = 150 - 129.1 = 20.9mm$$

M.O.I of rectangle about X-X axis

$$= I_{G_1} + ah^2 = (450 \times 10^6) + [(300 \times 200) \times (20.9)^2]$$
$$= 476.21 \times 10^6\ mm^4$$

M.O.I of Circular section about an axis through its C.G and parallel to X-X axis

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$I_{G_2} = \frac{\pi}{64} \times (150)^4 = 24.85 \times 10^6 \, mm^4$

Distance b/w C.G of section and X-X axis

$h_2 = 200 - 129.1 = 70.9 mm$

M.O.I of the circular section about X-X axis

$= I_{G_2} + ah^2 = (24.85 \times 10^6) + [(17670) \times (70.9)^2]$

$= 113.67 \times 10^6 \, mm^4$

M.O.I of the whole section about X-X axis

$= (476.21 \times 10^6) - (113.67 \times 10^6)$

$= \underline{362.54 \times 10^6 \, mm^4}$

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