ALTERNATING QUANTITIES

INTRODUCTION:

The alternating quantity is one whose value varies with time. This alternating quantity may be periodic and non-periodic. Periodic quantity is one whose value will be repeated for every specified interval. Generally to represent alternating voltage or current we prefer sinusoidal wave form, because below listed properties

1. Derivative of sine is an sine function only.
2. Integral of sine is an sine function only.
3. It is easy to generate sine function using generators.
4. Most of the 2nd order system response is always sinusoidal.

Alternating quantity:

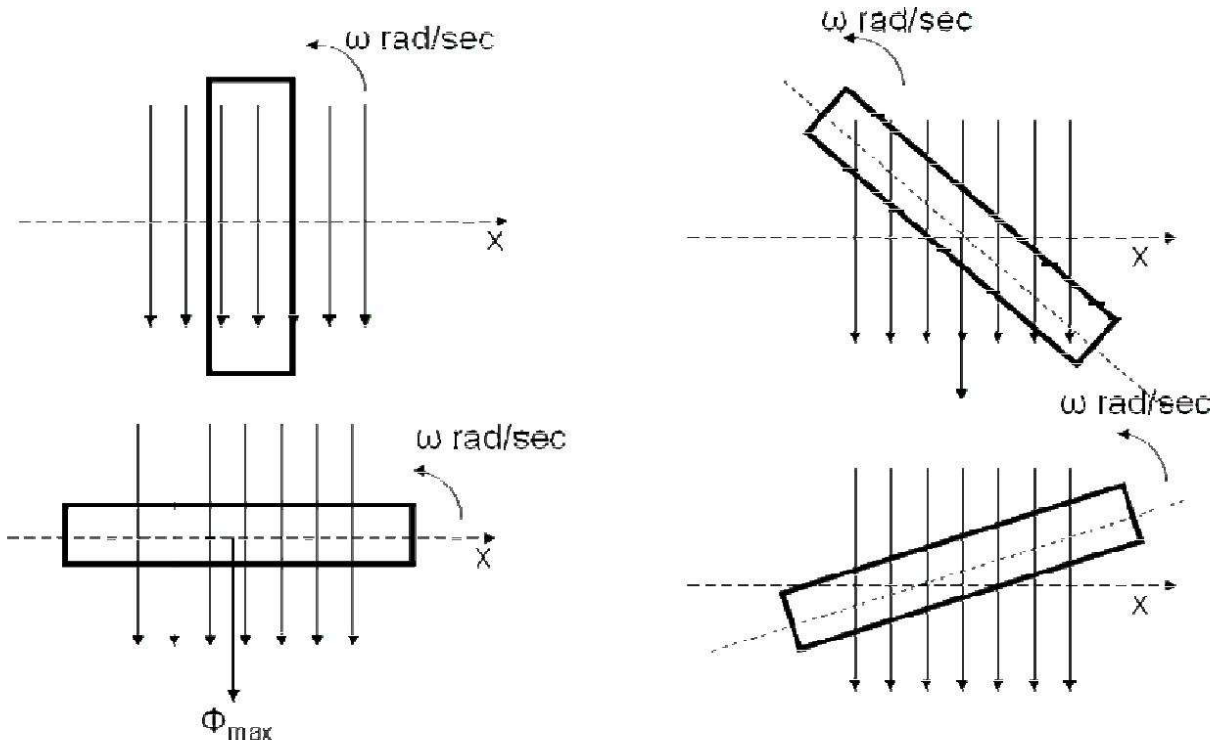
As said above an alternating voltage or current can be represented with sine wave. Sine wave can be defined with degree or radians as reference.

At, 0 degrees --- 0
90 degrees --- maximum
180 degrees --- 0
270 degrees --- maximum
360 degrees --- 0

i.e value of sine function varies with time, firstly increases from zero and reaches maximum and again falls to zero, there after tends to increase in opposite direction and reaches maximum value and falls to zero. This the variation of sine in 1st cycle is called as positive half cycle and other negative half cycle.(i.e during +ve half cycle direction is required one and during 2nd half cycle direction actual required direction.). Therefore one positive and negative cycle combinely forms one complete cycle

Generation of sinusoidal AC voltage

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.

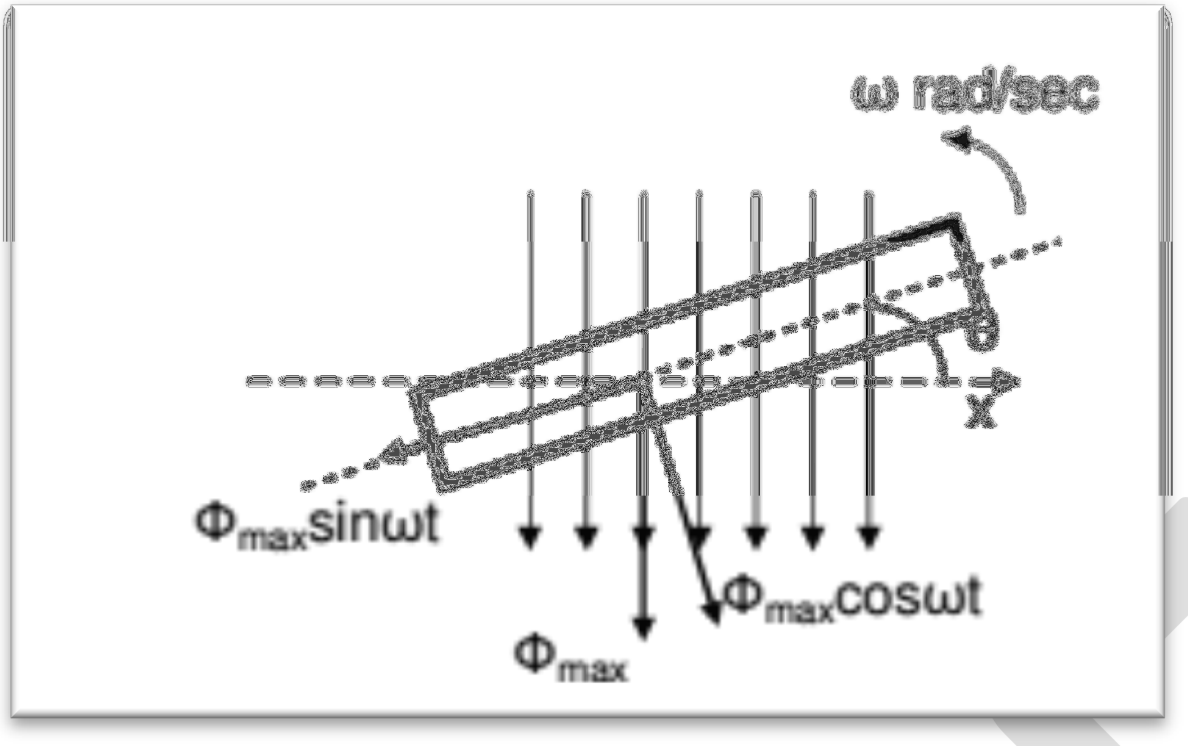


When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil according to Faradays Law. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations.

Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure.

This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil.
Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\varnothing = \Phi_{max}\cos\omega t.$$

$$e=-N d\varnothing/dt$$

$$e=-N d(\Phi_{max}\cos\omega t)/dt$$

$$e=N\,\Phi_{max}\,\omega\,\sin\omega t.$$

$$e=E_{max}\,\sin\omega t.$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current
in the circuit given by

$$i= i_m \sin\omega t.$$

where i= instantenous value
i_m = maximum value

Angular Frequency (ω)
Angular frequency is defined as the number of radians covered in one

second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

Peak to peak value: It is total value from positive peak to the negative peak.(2Vm)

Instantaneous value: It is the magnitude of wave form at any specified time. V(t)

Average value : It is ratio of area covered by wave form to its length.(Vd)

$Vd = (1/T) \int V(t)\, dwt.$

$Vd = (1 / 2\Pi) \int Vm \sin wt.dwt$

$= - Vm / 2\Pi. \cos wt$---with limits of 2Π and 0

= 0. (i.e. average value of sine wave over a full cycle is zero)

Hence it is defined for half cycle.

$Vd = (1 / \Pi) \int Vm \sin wt.dwt$

$= - Vm / \Pi . \cos wt$ with limits of Π and 0

$= 2Vm / \Pi$

RMS value:

It is the root mean square value of the function, which given as

$Vrms = (1/T) \int V(t)]2\, dwt.$

$= (1/2\Pi) \int Vm2[(1- \cos 2wt)/2]dwt.$

$= (1/2\Pi) .Vm2[(wt- \sin 2wt / 2wt)/2]$

= Vm / = effective value.

Peak factor:

It is the ratio of peak value to the rms value.

$Pp = Vp / Vrms = \sqrt{2}$

Form factor:

It is the ratio of average value to the rms value.

$Fp = Vd / Vrms = 2\sqrt{2} / \Pi = 1.11$

Phasor Representation:

An alternating quantity can be represented using

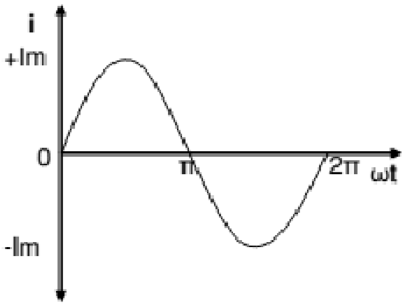
i) Waveform

ii) Equations

iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a Phasor. A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

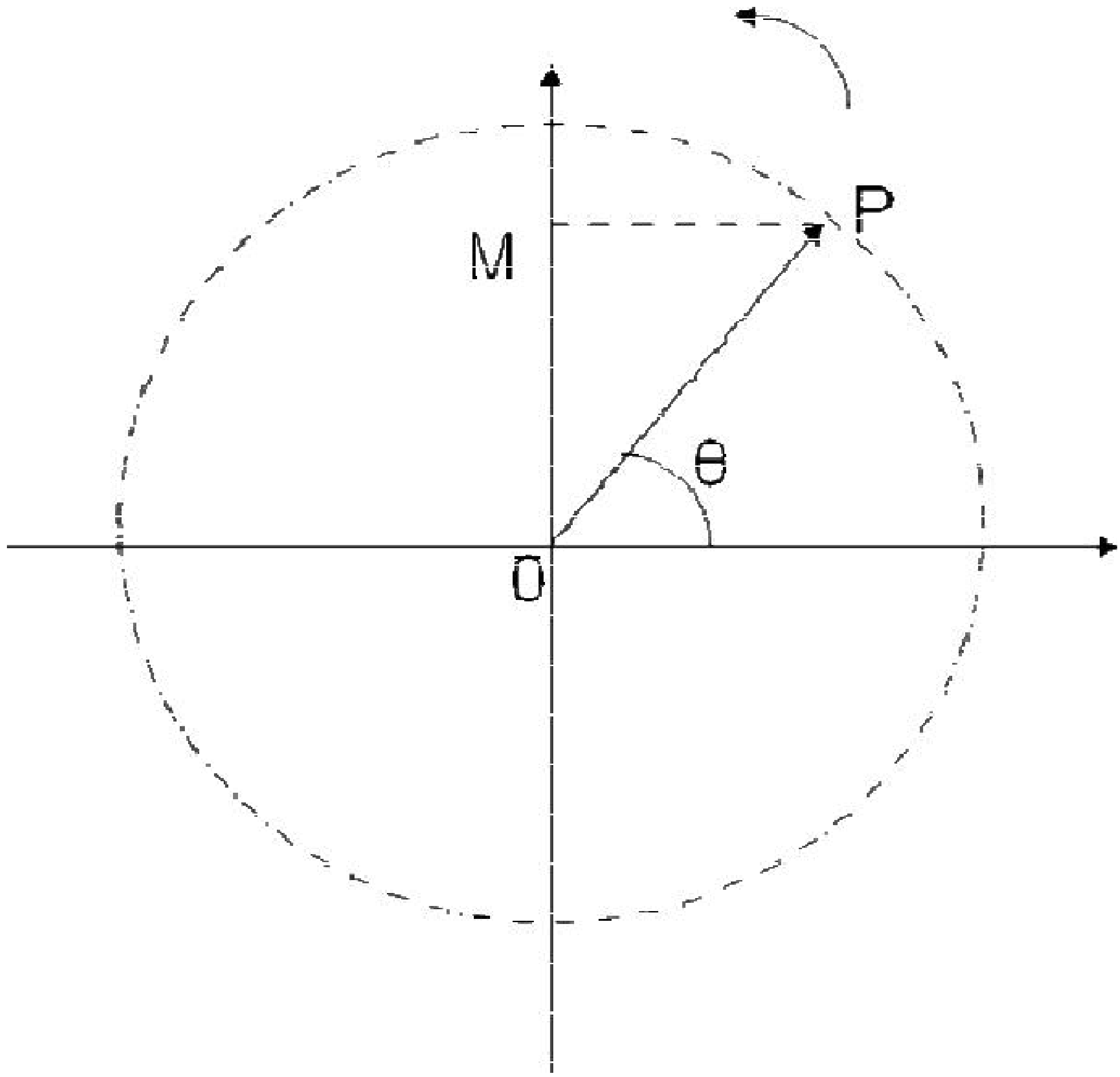
The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.



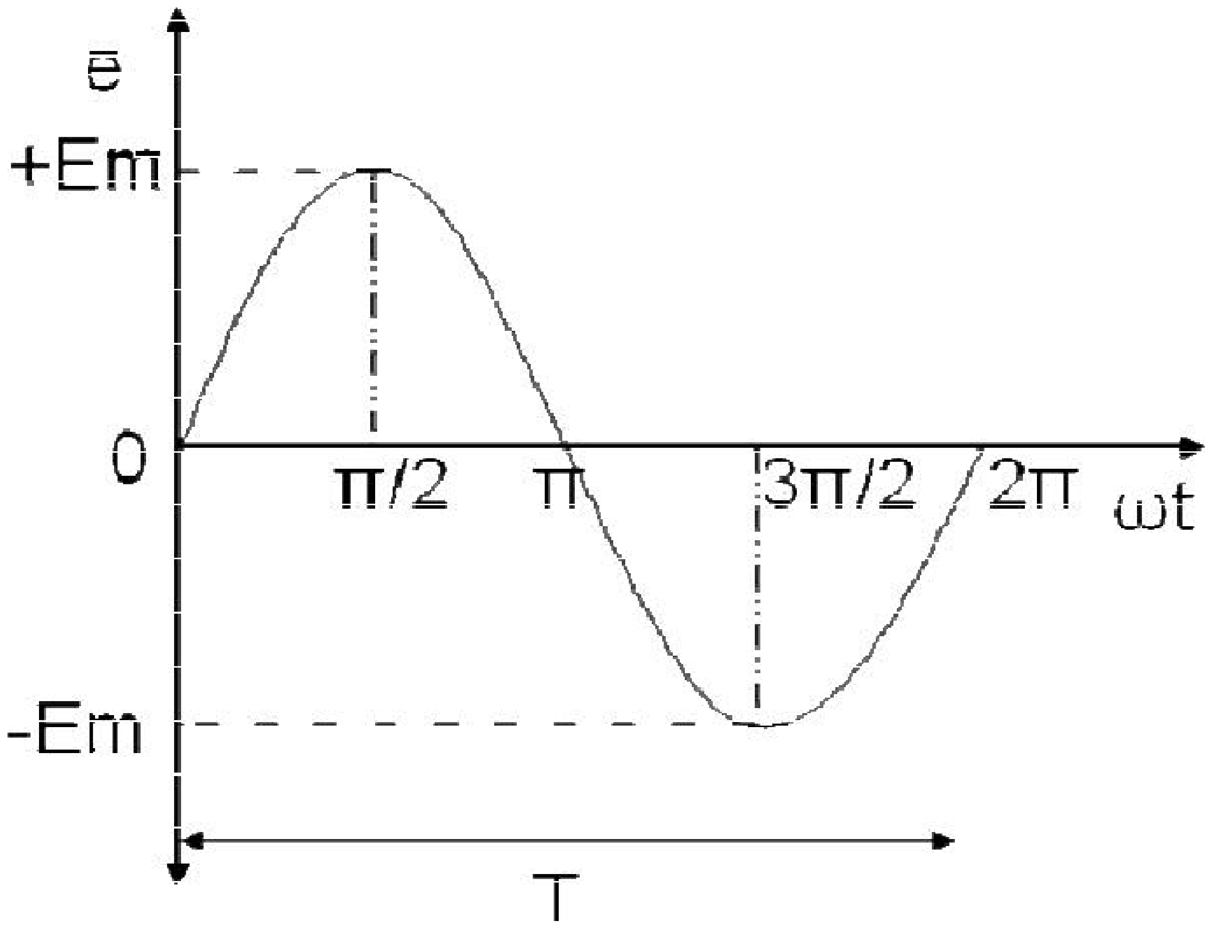
$$i = I_m \sin \omega t$$

In phasor form the above wave is written as ¯ = ∠0

Draw a line OP of length equal to Im. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = Imsinωt. Hence the line OP is the phasor representation of the sinusoidal current.

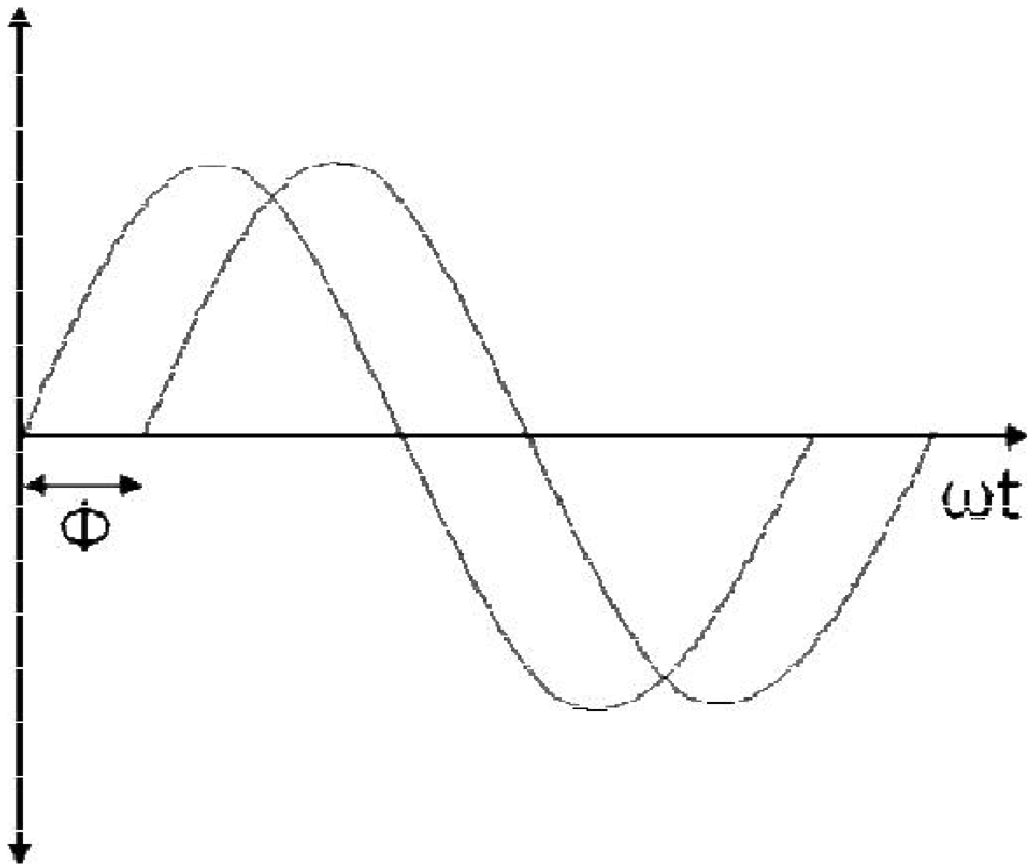


Phase



Phase is defined as the fractional part of time period or cycle through which the
quantity has advanced from the selected zero position of reference
Phase of +Em is π/2rad or T/4 sec
Phase of -Em is π/2rad or 3T/4 sec

Phase Difference

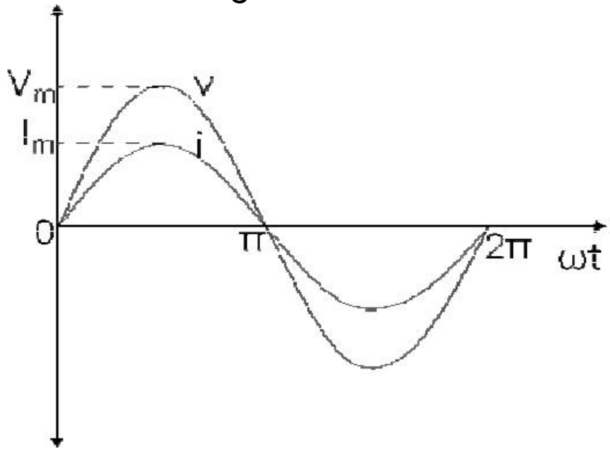


When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero.

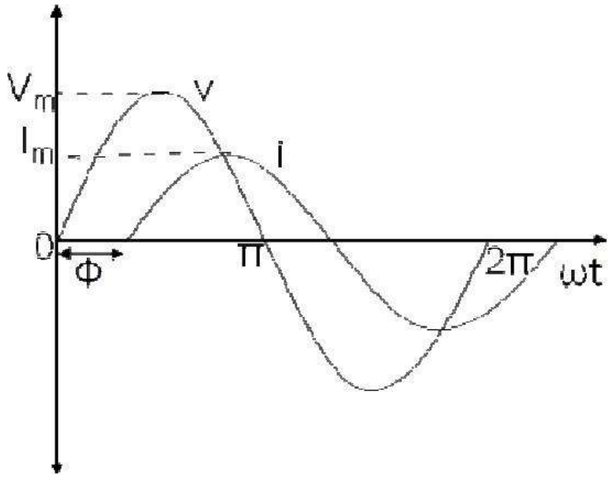
That is the zero points of both the waveforms are same. The waveform, phasor and equation
representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

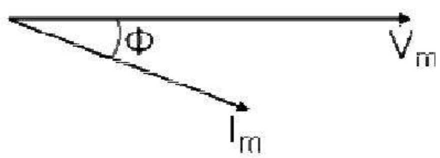


$v = v_m \sin\omega t$
$i = i_m \sin\omega t.$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.

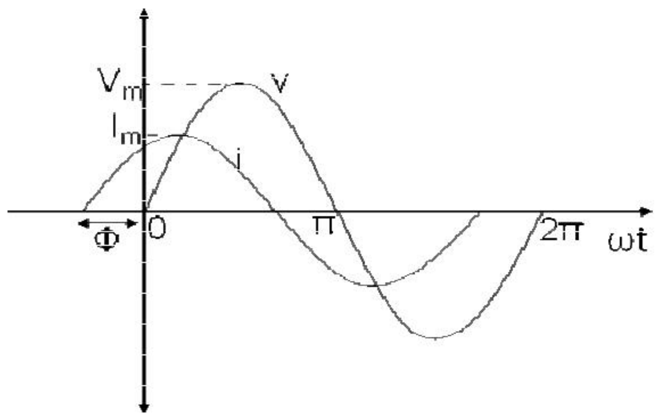


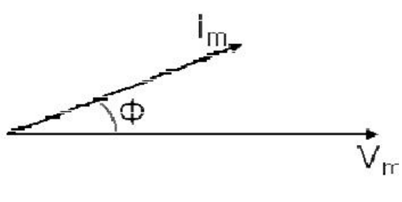


$v = v_m \sin\omega t$
$i = i_m \sin(\omega t - \varnothing).$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.





$v = v_m \sin\omega t$
$i = i_m \sin(\omega t + \varnothing).$

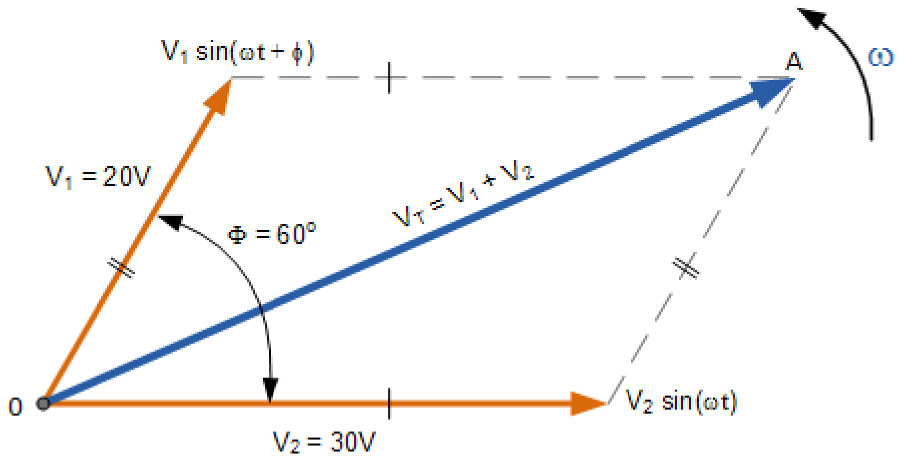
Phasor Addition

Sometimes it is necessary when studying sinusoids to add together two alternating waveforms, for example in an AC series circuit, that are not in-phase with each other. If they are in-phase that is, there is no phase shift then they can be added together in the same way as DC values to find the algebraic sum of the two vectors. For example, if two voltages of say 50 volts and 25 volts respectively are together "in-phase", they will add or sum together to form one voltage of 75 volts (50 + 25).

If however, they are not in-phase that is, they do not have identical directions or starting point then the phase angle between them needs to be taken into account so they are added together using phasor diagrams to determine their **Resultant Phasor** or **Vector Sum** by using the *parallelogram law*.

Consider two AC voltages, V_1 having a peak voltage of 20 volts, and V_2 having a peak voltage of 30 volts where V_1 leads V_2 by 60^o. The total voltage, V_T of the two voltages can be found by firstly drawing a phasor diagram representing the two vectors and then constructing a parallelogram in which two of the sides are the voltages, V_1 and V_2 as shown below.

Phasor Addition of two Phasors



Their phasor sum $V_1 + V_2$ can be easily found by measuring the length of the diagonal line, known as the "resultant r-vector", from the zero point to the intersection of the construction lines 0-A.

Mathematically we can add the two voltages together by firstly finding their "vertical" and "horizontal" directions, and from this we can then calculate both the "vertical" and "horizontal" components for the resultant "r vector", V_T. This analytical method which uses the cosine and sine rule to find this resultant value is commonly called the Rectangular Form.

In the rectangular form, the phasor is divided up into a real part, x and an imaginary part, y forming the generalised expression $Z = x \pm jy$. (we will discuss this in more detail in the next tutorial). This then gives us a mathematical expression that represents both the magnitude and the phase of the sinusoidal voltage as:

Definition of a Complex Sinusoid

$$v = V_m \cos(\phi) + jV_m(\sin\phi)$$

So the addition of two vectors, A and B using the previous generalised expression is as follows:

$$A = x + jy \qquad B = w + jz$$

$$A + B = (x + w) + j(y + z)$$

Phasor Addition using Rectangular Form

Voltage, V_2 of 30 volts points in the reference direction along the horizontal zero axis, then it has a horizontal component but no vertical component as follows.

- • Horizontal Component = 30 cos 0^o = 30 volts
- • Vertical Component = 30 sin 0^o = 0 volts
- This then gives us the rectangular expression for voltage V_2 of: 30 + j0

Voltage, V_1 of 20 volts leads voltage, V_2 by 60^o, then it has both horizontal and vertical components as follows.

- • Horizontal Component = 20 cos 60^o = 20 x 0.5 = 10 volts
- • Vertical Component = 20 sin 60^o = 20 x 0.866 = 17.32 volts
- This then gives us the rectangular expression for voltage V_1 of: 10 + j17.32

The resultant voltage, V_T is found by adding together the horizontal and vertical components as follows.

- $V_{Horizontal}$ = sum of real parts of V_1 and V_2 = 30 + 10 = 40 volts

- $V_{Vertical}$ = sum of imaginary parts of V_1 and V_2 = 0 + 17.32 = 17.32 volts

Now that both the real and imaginary values have been found the magnitude of voltage, V_T is determined by simply using **Pythagoras's Theorem** for a 90° triangle as follows.

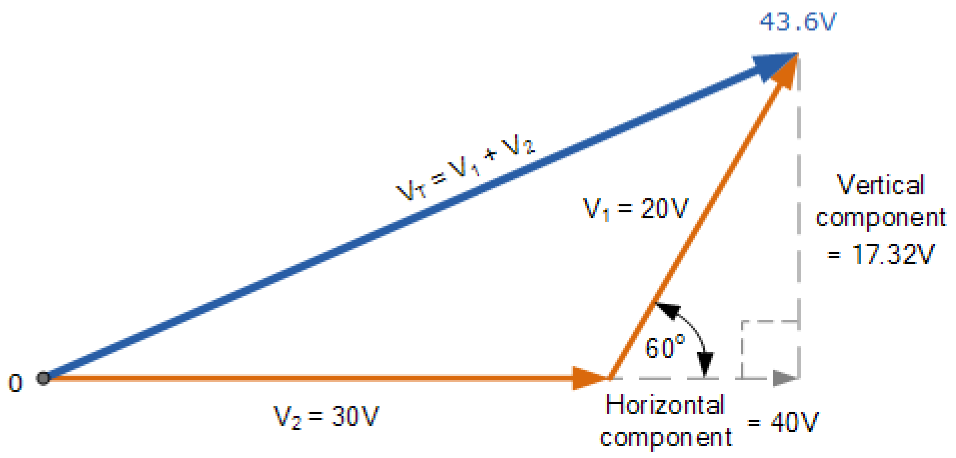
$$V_T = \sqrt{\begin{pmatrix}\text{Real or Horizontal} \\ \text{Component}\end{pmatrix}^2 + \begin{pmatrix}\text{Imaginary or Vertical} \\ \text{Component}\end{pmatrix}^2}$$

$$V_T = \sqrt{40^2 + 17.32^2}$$

$$\therefore V_T = 43.6\,\text{volts}$$

Then the resulting phasor diagram will be:

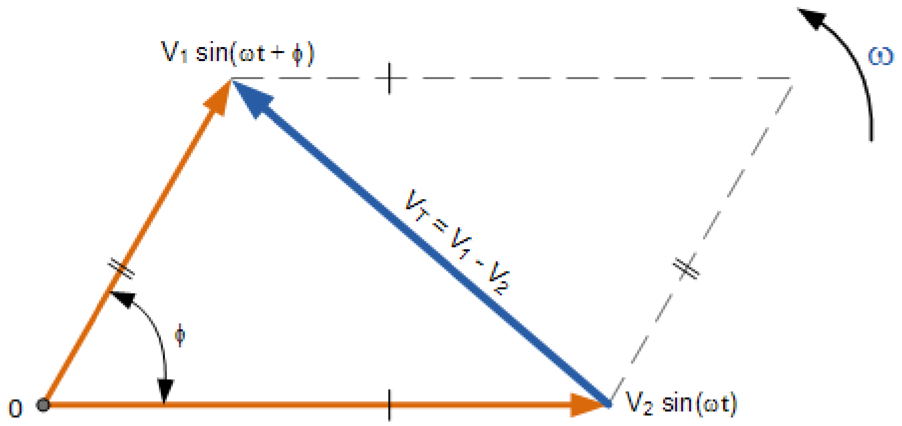
Resultant Value of V_T



Phasor Subtraction

Phasor subtraction is very similar to the above rectangular method of addition, except this time the vector difference is the other diagonal of the parallelogram between the two voltages of V_1 and V_2 as shown.

Vector Subtraction of two Phasors



This time instead of “adding” together both the horizontal and vertical components we take them away, subtraction.

$$A = x + jy \qquad B = w + jz$$

$$A - B = (x - w) + j(y - z)$$